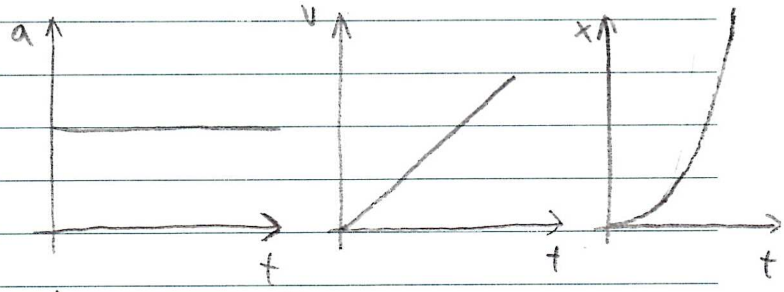
Premo gibanje: $a = A + Bt$



$a = \frac{dv}{dt}$

$dv = a\,dt$

$\int_{v_0}^{v} dv = \int_0^t a\,dt$

$v - v_0 = \int_0^t (A + Bt)\,dt = At + \frac{Bt^2}{2}$

$v = At + \frac{Bt^2}{2} - v_0$

$v = \frac{dx}{dt} \Rightarrow dx = v\,dt$

$x - x_0 = \int_{x_0}^{x} dx = \int_0^t \left(\left(At + \frac{Bt^2}{2}\right) - v_0\right) dt = \frac{At^2}{2} + \frac{Bt^3}{6} - v_0 t$

$x = \frac{At^2}{2} + \frac{Bt^3}{6} - v_0 t + x_0$

Gibanje sistema točkastih teles, gibalna količina.

$\vec{F} = m\vec{v} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{G}}{dt} \Rightarrow \vec{G} = \int \vec{F}\,dt$

Če ni vpliva sil iz okolice, potem sistem ohranja gibalno količino.

$\vec{G_i} = m_i \vec{v_i} \Rightarrow \frac{d\vec{G_i}}{dt} = \vec{F_i}$

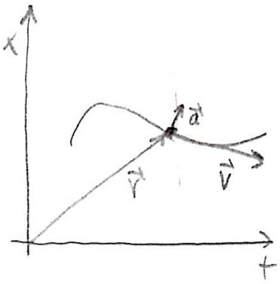
$\vec{G} = \vec{v_i}\, m_i = m_i \cdot \frac{d\vec{r_i}}{dt} = m \cdot \vec{v_c}$, kot da bi bila ~~vsa~~ masa zbrana v masnem središču

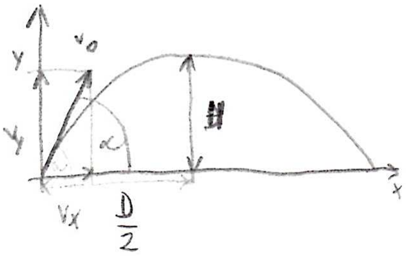
Sila curka: $G = m \cdot v$; $\phi_m = \frac{dm}{dt} \Rightarrow G = \phi_m t \cdot v$

$F = m \cdot a = \phi_m t \cdot \frac{u}{t} = \phi_m \cdot u$ $\qquad F = \frac{dG}{dt} = \frac{\phi_m t \cdot v}{dt} = \phi_m \cdot v$

Klancek.si

Ploskovno gibanje / Poševni met:





$$\cos\alpha = \frac{v_x}{v_0} \Rightarrow v_x = \cos\alpha \; v_0$$

$$v_y = \sin\alpha \; v_0$$

Eksplicitna oblika:

$$x = v_x \cdot t = v_0 \cdot t \cdot \cos\alpha$$

$$y = v_0 \sin\alpha \cdot t - \frac{1}{2} g t^2$$

Parametrična oblika:

$$t = \frac{x}{v_0 \cos\alpha}$$

$$y = v_0 \sin\alpha \cdot \frac{x}{v_0 \cos\alpha} - \frac{1}{2} g \cdot \left(\frac{x}{v_0 \cos\alpha}\right)^2$$

$$\boxed{y = x \cdot \tan\alpha - \frac{1}{2} g \frac{x^2}{v_0^2 \cos^2\alpha}}$$

Domet, Dvig:

$$\frac{dy}{dx} = \tan\alpha - \frac{1}{2} g \frac{x}{v_0^2 \cos^2\alpha} \cdot 2 = \tan\alpha - \frac{g \cdot x}{v_0^2 \cos^2\alpha}$$

$$x = \frac{\tan\alpha \, v_0^2 \cos^2\alpha}{g} = \frac{\frac{\sin\alpha}{\cos\alpha} v_0^2 \cos^2\alpha}{g} =$$

$$= \frac{v_0^2 \sin\alpha \cos\alpha}{g} = \frac{D}{2}$$

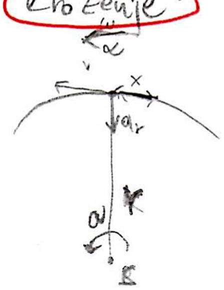
$$\boxed{D = 2 \frac{v_0^2 \sin\alpha \cos\alpha}{g}}$$

$$H\left(x = \frac{D}{2}\right) = \frac{D}{2} \cdot \tan\alpha - \frac{1}{2} g \frac{\left(\frac{D}{2}\right)^2}{v_0^2 \cos^2\alpha} =$$

$$= \frac{v_0 \sin\alpha \cancel{\cos\alpha}}{g} \cdot \frac{\sin\alpha}{\cancel{\cos\alpha}} - \frac{1}{2} g \frac{v_0^{\cancel{4}\,2} \sin^2\alpha \cancel{\cos^2\alpha}}{g^2 \cancel{v_0^2} \cancel{\cos^2\alpha}} =$$

$$= \frac{v_0 \sin^2\alpha}{g} - \frac{1}{2} \frac{v_0^2 \sin^2\alpha}{g} = \boxed{\frac{v_0^2 \sin^2\alpha}{2g}}$$

Kroženje:



$$v = \frac{dx}{dt} \Rightarrow dx = r \cdot d\varphi$$

$$\boxed{v = r \cdot \frac{d\varphi}{dt} = r \cdot \omega}$$

$$\omega = \frac{d\varphi}{dt}$$

$$\boxed{\omega = \frac{v}{r}}$$

$$\boxed{\alpha = \frac{d\omega}{dt}}$$

$$a_t = \frac{dv}{dt} = \frac{d(r\omega)}{dt} = \frac{d\omega}{dt} r$$

$$\boxed{a_t = r\alpha}$$

$$a_r = \frac{dv}{dt} = \cancel{2} v \cdot \frac{d\varphi}{\cancel{2} dt} = v \frac{d\varphi}{dt} = v\omega$$

$$= r \cdot \omega \cdot \omega \Rightarrow \boxed{a_r = \omega^2 r}$$

$$\boxed{a = \sqrt{a_t^2 + a_r^2} = \sqrt{r^2 \alpha^2 + \omega^4 r^2} =}$$

Newtnovi zakoni:

1.) Vsako telo vztraja v enakomernem premem gibanju ali pa miruje če nanj ne vpliva nobena sila.

2.) Vzrok za pospešeno gibanje je sila: $\vec{F} = m \cdot \vec{a}$. Z njo definirana tudi teža: $\vec{F_g} = m\vec{g}$,

3.) Zakon o vzajemnem učinku. Akcija je enaka reakciji. $m_1 \; \vec{F_1} \rightarrow \quad \leftarrow \vec{F_2} \; m_2$

$$\vec{F_1} = -\vec{F_2}$$

1. kozmična hitrost

$v = \omega r \Rightarrow \omega = \frac{v}{r}$

$a_r = \omega^2 r = \left(\frac{v}{r}\right)^2 r = \frac{v^2}{r}$

$$mg = F = m \cdot a_r = m\frac{v^2}{r}$$

$$g = \frac{v^2}{r} \Rightarrow v = \sqrt{gr}, \quad r = 6400\,km = R$$

$$v_1 = \sqrt{gR} \approx 8\,^{km}/_s$$

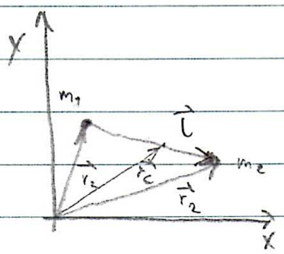
2. kozmična hitrost:

$$F = G\frac{Mm}{r^2} = \frac{1}{2}m v_2^2$$

$$v_2 = \sqrt{\frac{2GM}{R^2}}, \quad r = 6400\,km = R \quad \approx 11{,}7\,^{km}/_s$$

Masno središče:

$$(m_1 + m_2)\cdot\vec{r_c} = m_1\vec{r_1} + m_2\vec{r_2} \quad \text{– enakost navorov}$$



$$\vec{r_c} = \frac{m_1\vec{r_1} + m_2\vec{r_2}}{m_1 + m_2}, \quad \vec{r_2} = \vec{r_1} + \vec{l} \Rightarrow \vec{r_c} = \frac{m_1\vec{r_1} + m_2(\vec{r_1} + \vec{l})}{m_1 + m_2}$$

$$\vec{r_c} = \vec{r_1} + \left(\frac{m_2}{m_1 + m_2}\right)\cdot\vec{l}$$

$$\vec{r_c} = \frac{1}{m}\sum_{i=1}^{N} m_i\vec{r_i}$$

Zvezna porazdelitev mase:

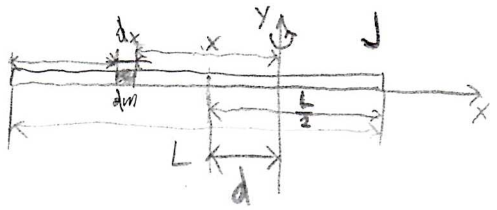
$$\frac{dm}{dV} = \rho, \quad \vec{r_c} = \frac{1}{m}\int \vec{r}\,dm \Rightarrow dm = \rho\,dV \Rightarrow \vec{r_c} = \frac{1}{m}\int \vec{r}\rho\,dV$$

Vrtenje togega telesa:

$$\vec{M} = J\vec{\alpha} \Rightarrow J = \frac{\vec{M}}{\vec{\alpha}} \quad , \quad \vec{\Gamma} = J\vec{\omega} \Rightarrow J = \frac{\vec{\Gamma}}{\vec{\omega}}$$

Steinerjev izrek: $J = J_c + md^2$

Palica:



$$J = \int x^2 \, dm$$

$$dm = \frac{m}{N} = \frac{m}{\frac{L}{dx}} = \frac{m}{L} dx$$

$$J = \int x^2 \frac{m}{L} dx = \frac{m}{L} \int_{-\frac{L}{2}+d}^{\frac{L}{2}+d} x^2 \, dx = \frac{m}{L} \frac{x^3}{3} \Bigg|_{-(\frac{L}{2}-d)}^{\frac{L}{2}+d}$$

$$J = \frac{m}{L}\left(\frac{\frac{L^2}{4} + Ld + d^2}{3} + \frac{d^2 + Ld + \frac{L^2}{4}}{3} \right) =$$

$$= \frac{m}{L} \frac{\frac{L^2}{2} + 2Ld + 2d^2}{3} = \frac{1}{12} mL^2 + md^2$$

Delo, moč:

Kinetična energija: $W_k = \frac{1}{2} m v^2$

Potencialna energija: $W_p = mgh$

Kotaljenje: $W_k = \frac{1}{2} m v_c^2 + \frac{1}{2} J_c \omega^2$

Izrek o kinetični energiji: $\Delta W_p + \Delta W_k = A$

Konservativne sile: $A = \int_{\vec{r}_1}^{\vec{r}_2} F \, d\vec{r} = W_p(\vec{r}_1) - W_p(\vec{r}_2)$

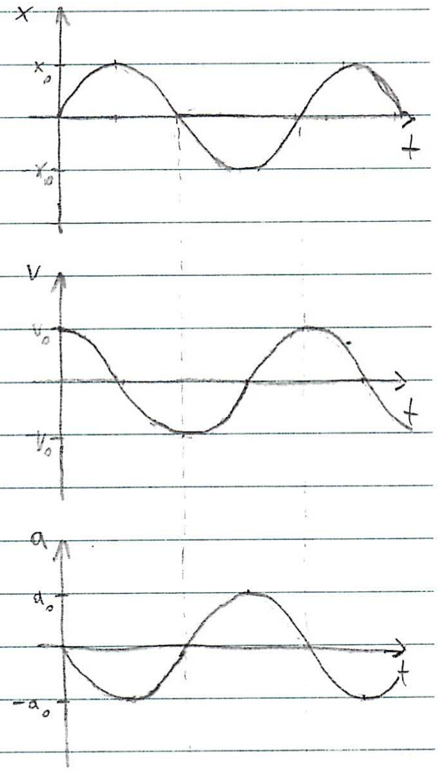
Moč: $P = \frac{dA}{dt} = \frac{\vec{F} \cdot d\vec{R}_c + \vec{M} \cdot d\vec{\varphi}}{dt} = \vec{F} \cdot \vec{v}_c + \vec{M} \cdot \vec{\omega}$

samo translatorno: $P = \vec{F} \cdot \vec{v}_c$

samo vrtenje: $P = \vec{M} \cdot \vec{\omega}$

Delo konservativne sile je na koncu enako 0 in je neodvisna od oblike poti

Harmonično nihanje:



Enačba: $$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$x(t) = x_0 \sin(\omega_0 t + \delta)$$

Ohranitev energije:

$$W_{nih} = \frac{1}{2}mv^2 + W_p(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

ker ni pretvorb v notranjo energijo, se prožnostna spremija v celoti v kinetično in obratno.

Prožno nihalo: $\omega^2 = \frac{k}{m} \Rightarrow t_0 = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

Polžasto nihalo: $M = -D\varphi = J\alpha$

$$-D\varphi = J\frac{d^2\varphi}{dt^2}, \quad \omega_0 = \sqrt{\frac{D}{J}} \Rightarrow t_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{J}{D}}$$

Vsiljeno nihanje, resonanca:

$F_{(t)} = F_0 \sin(\Omega t)$, Ω - vsiljena frekvenca

$X_{(vsilj)} = A \sin(\Omega t - \phi)$, A - amplituda vsiljenega nihanja
ϕ - fazni zamik nihala glede na zunanjo silo

$\omega_0 = \Omega \Rightarrow$ nihalo nihamo z lastno frekvenco, to je smo v resonanci z nihalom

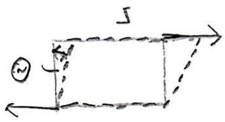
Sestavljena nihala: če je nihalo sestavljeno iz N nihal, potem ima N lastnih frekvenc in N lastnih nihanj.

Deformacije teles:

Elastična - po prenehanju obremenitve se telo vrne v prvotno stanje.

Plastična - po prenehanju obremenitve se telo ne povrne v prvotno stanje.

Strižna napetost: $\vec{\tau} = \frac{d\vec{F}_\parallel}{dS}$



torzija - zasuk

$\tau = G\Theta$ - Hooke-ov zakon za strig, G-strižni modul snovi

Tlačna napetost: $p = \frac{d\vec{F}_\perp}{dS}$ $\qquad \frac{dV}{V} = -\chi p$ - Hooke-ov zakon za tlak, χ-stisljivost snovi

Natezna napetost: $\sigma = \frac{d\vec{F}_\perp}{dS}$ $\qquad dl = \left(\frac{1}{E}\right)\cdot\frac{F}{S}\cdot l$ - Hooke-ov zakon za nateg

$E = \frac{\sigma}{\varepsilon}$ - modul elastičnosti, do meje elastičnosti še elastične spremembe

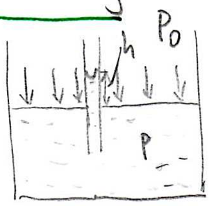
$\frac{da}{a} = \frac{db}{b} = -\frac{dl}{l}\cdot\mu$ - Poissonovo število, absolutna vrednost razmerja med skrčkom v prečni smeri in raztezkom v vzdolžni smeri.

Površinska napetost: (γ)

Gladina kapljevine se obnaša kot elastična membrana.

$F = \gamma l$, $\qquad A = F\cdot\Delta x = \gamma\cdot l\cdot\Delta x = \gamma\cdot\Delta S$

Kapilarni dvig:

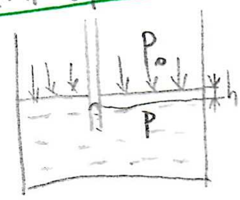


$P > P_0$

$h = \frac{2\gamma}{\rho g r}$

Če so sile med steno in molekulami večje kot sile med molekulami.

Kapilarni spust:



$P_0 > P$

$h = \frac{2\gamma}{\rho g r}$

Če so sile med molekulami večje kot sile med molekulami in steno.

Plinska enačba: $pV = NkT$, $N = \frac{m}{M} N_A$ $\Rightarrow$ $pV = \frac{m}{M} RT$

$T = const \Rightarrow pV = const.$

$p = const. \Rightarrow \frac{V}{T} = const.$

Prvi zakon termodinamike:

To je zakon o ohranitvi energije: $Q + A = \Delta W_p + \Delta W_k + \Delta W_n$ ...

oz.

$$W_n = A + Q$$

Notranja energija: $W_n = W_k$ (termično gibanje) + W_p (sile med molekulami)

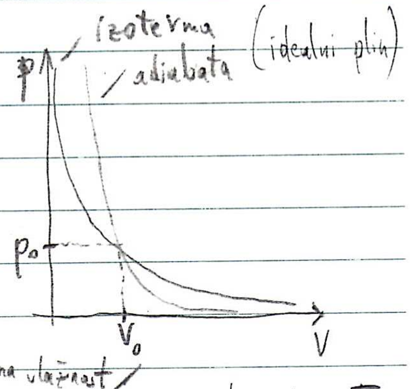
hitrost delcev "temperatura" — vrste in moč sil med molekulami

idealni plin: $W_n = W_k$ (termično gibanje) – razdalje med molekulami so tako velike, da W_p zanemarimo

$$W_n = N\cdot\frac{1}{2} m_0 \overline{v^2} = N\frac{3}{2}kT = \frac{N}{N_A}\cdot\frac{3}{2}RT\frac{m_0}{m_0} = m\frac{3}{2}\left(\frac{R}{M}\right)T$$

Specifična toplota:
c_p – stalen tlak $= \frac{5}{2}\frac{R}{M}$
c_v – stalna temperatura $= \frac{3}{2}\frac{R}{M}$
$\Rightarrow c_p - c_v = \frac{R}{M}$

Enačba diabate: $p\cdot V^{\gamma} = const.$, $\gamma = \frac{c_p}{c_v}$

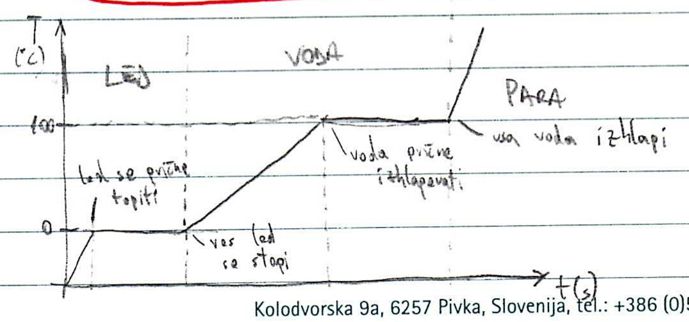


Spremembe agregatnega stanja:

Relativna vlažnost = $\frac{\text{absolutna vlažnost}}{\text{nasičena vlažnost pri } T}$

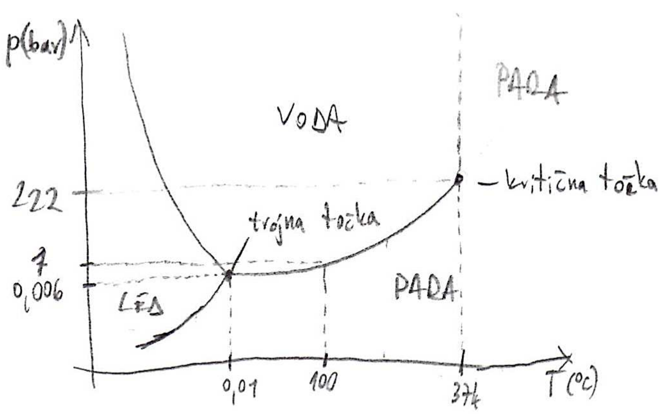
Absolutna vlažnost = $\frac{\text{masa vode}}{m^3 \text{ zraka}}$

Rosišče – temperatura, pri kateri se začne iz vlažnega nasičenega zraka izločati voda v obliki kapljic.



Klancek.si

Fazni diagram vode:

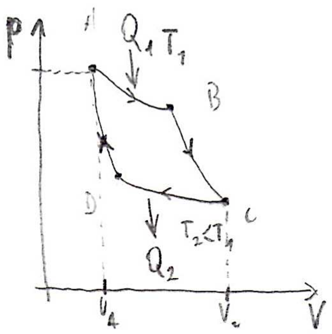


Drugi zakon termodinamike

Toplota teče od mesta z višjo temperaturo k mestu z nižjo temperaturo.

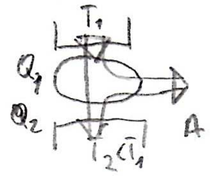
Carnotov toplotni stroj: idealen plin, ki je podvržen krožni spremembi iz dveh adiabat in dveh izoterm.

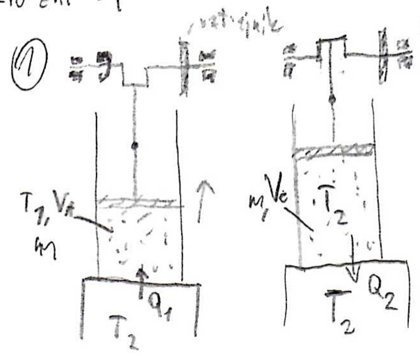
Deloma snov je izoterm.



B → C in D → A - adiabate

A - B in C → D - izoterme

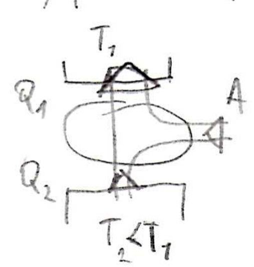




1. Malo toplejšo snov, se plin raztegne in pri tem opravi mehansko delo,
2. nato pristavimo malo hladnejše telo, plin krči, spet mehansko delo.

$$\eta = 1 - \frac{T_2}{T_1}$$

Hladilni stroji delujejo na obratnem postopku.



$$\gamma_{H} = \frac{T_2}{T_1 - T_2}$$, nam pove koliko J dela moramo vložiti, da snovi odvzamemo 1 J toplote.

$$\gamma_{TČ} = \frac{T_1}{T_1 - T_2}$$, nam pove koliko J dela pridobimo iz enega J toplote.

Klancek.si

Prevajanje toplote:

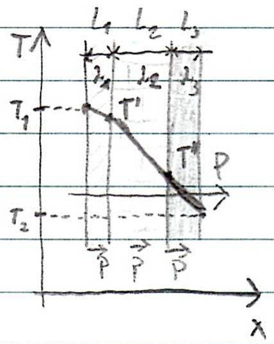
Zakon za prevajanje toplote: $j(x) = -\lambda \frac{dT}{dx}$, λ - koeficient toplotne prevodnosti

Stacionarno stanje: $P(x) = -\lambda S \frac{dT}{dx}$ $\qquad j(x) = \frac{P(x)}{S}$

$$T(x) = T_1 - \frac{T_1 - T_2}{L} x$$

$$P = \lambda S \frac{\Delta T}{L} \Rightarrow P = \frac{\Delta T}{R} = k S \Delta T, \quad k = \frac{\lambda}{L}$$

Toplotni upor večplastne stene:



$$P = \lambda_1 S \frac{T_1 - T'}{L_1} = \frac{T_1 - T'}{R_1} \Rightarrow R_1 = \frac{L_1}{\lambda_1 S}$$

$$P = \lambda_2 S \frac{T' - T''}{L_2} = \frac{T' - T''}{R_2} \Rightarrow R_2 = \frac{L_2}{\lambda_2 S}$$

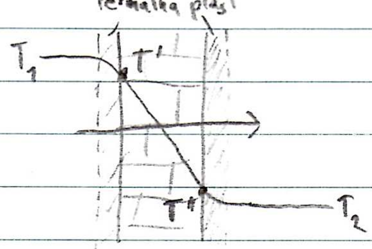
$$P = \lambda_3 S \frac{T'' - T_2}{L_3} = \frac{T'' - T_2}{R_3} \Rightarrow R_3 = \frac{L_3}{\lambda_3 S}$$

$$PR_1 + PR_2 + PR_3 = T_1 - T_2 \Rightarrow P = \frac{T_1 - T_2}{R_1 + R_2 + R_3}$$

$$R = R_1 + R_2 + R_3 \Rightarrow \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

Stena, zrak:

termalna plast



$$\left.\begin{aligned} P &= S\alpha_L (T_1 - T') \\ P &= S\alpha_D (T'' - T_2) \end{aligned}\right\} \text{prestopna koeficienta}, \alpha_L, \alpha_D$$

$$R_{cel} = \frac{1}{S\alpha_L} + R + \frac{1}{S\alpha_D}$$

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