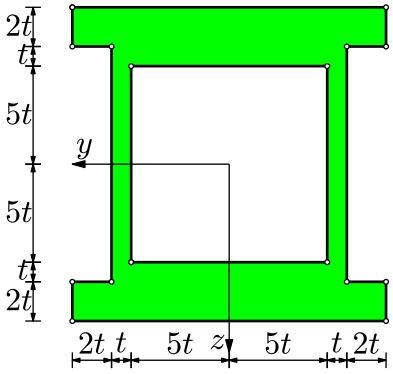
Drugi kolokvij iz TRDNOSTI (UNI), 26. januar 2015

1. Prečni prerez na sliki je obtežen z osno silo N_x, prečno silo N_z in upogibnim momentom M_y. Določi:

   - največjo normalno napetost v prerezu.
   - največjo strižno napetost v prerezu in potek strižnih napetosti po prerezu.
   - velikost dodatne osne sile ΔN_x, s katero moramo dodatno obtežiti prečni prerez, da bodo normalne napetosti po celotnem prerezu tlačne.
   - velikost dodatne osne sile ΔN_x, s katero moramo dodatno obtežiti prečni prerez, da bodo normalne napetosti po celotnem prerezu natezne.

   Podatki: $N_x = 100$ kN, $N_z = 100$ kN, $M_y = 1$ MN m, $t = 2$ cm.



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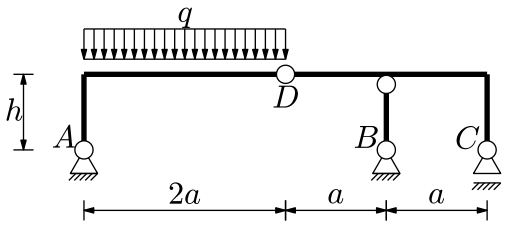
2. Ravninski okvir na sliki je obtežen z enakomerno linijsko zvezno obtežbo q. Določi:

   - reakcije, notranje sile in diagrame notranjih sil,
   - vodoravni in navpični pomik vozlišča D.

   Podatki: $a = 4$ m, $h = 3$ m, $A_x = 500$ cm^2, $I_y = 20\,000$ cm^4, $E = 200\,000$ MPa.



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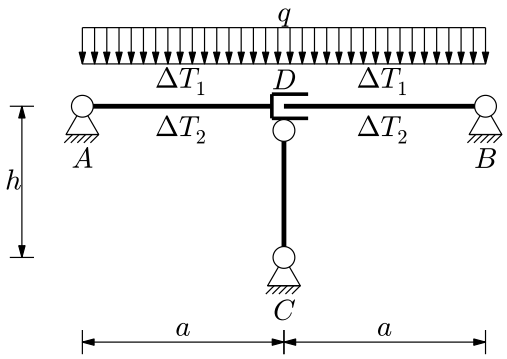
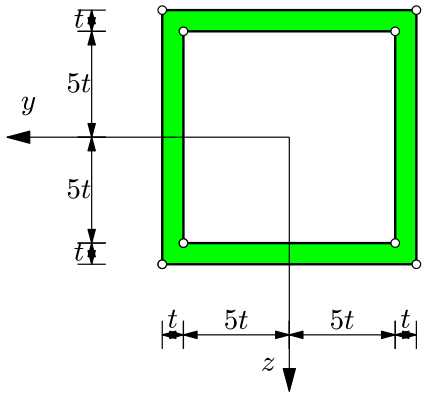
3. Ravninski okvir iz linearno elastičnega homogenega izotropnega materiala je obtežen z enakomerno linijsko zvezno obtežbo q in s spremembo temperature na nosilcih AD and DB kot prikazuje slika. Vez D dopušča medsebojni vzdolžni pomik nosilcev AD in DB in medsebojni vzdolžni zasuk nosilcev AD in CD. Vsi nosilci imajo enak prečni prerez prikazan na sliki spodaj.

   - Določi reakcije, notranje sile in skiciraj diagrame notranjih sil.

   Podatki: $a = 4$ m, $h = 3$ m, $t = 3$ cm, $E = 200\,000$ MPa, $q = 10\,\frac{\text{kN}}{\text{m}}$, $\Delta T_1 = 15$ K, $\Delta T_2 = 45$ K, $\alpha_T = 10^{-5}\,\frac{1}{\text{K}}$.

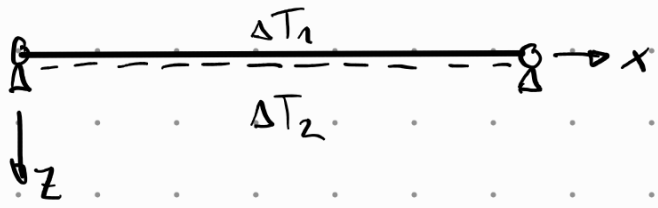


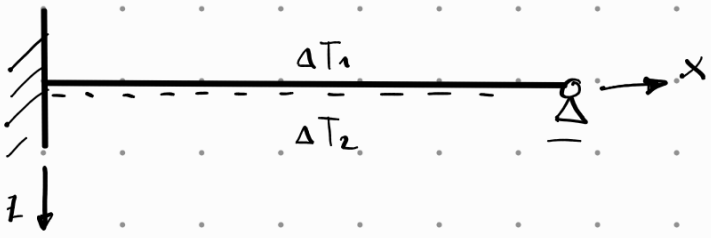


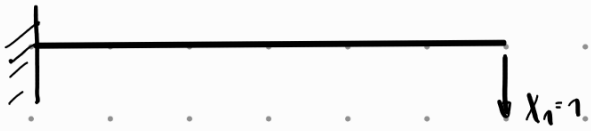
Točkovanje: 35 % + 30 % + 35 % = 100%.

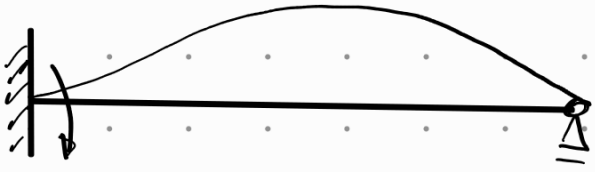
$E, A_x, I_y, \Delta T_1 = 30$

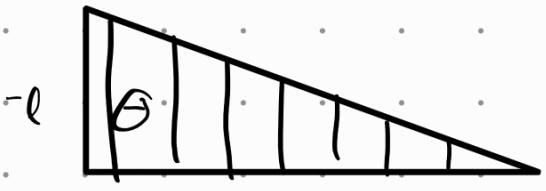
$L, \alpha_T \qquad \Delta T_2 = 10$

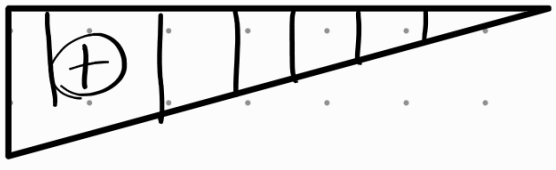












$a_{11} = \frac{1}{}$

1. Prečni prerez na sliki je obtežen z osno silo N_x, prečno silo N_z in upogibnim momentom M_y. Določi:

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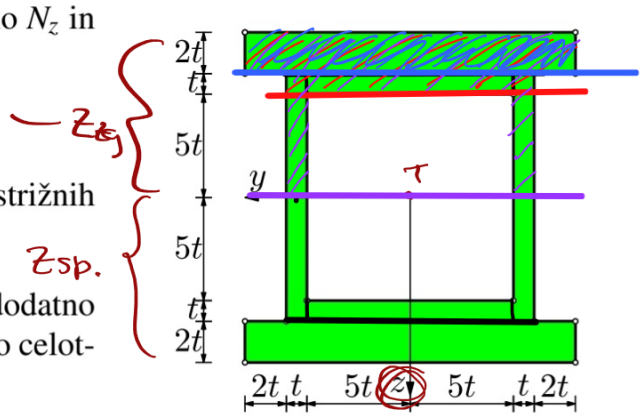


Podatki: $N_x = 100$ kN, $N_z = 100$ kN, $M_y = 1$ MN m, $t = 2$ cm.

$$I_y^T = \frac{(16t)^3 \cdot 16t}{12} - \frac{(10t)^3 \cdot 10t}{12} - 2\left(\frac{(12t)^3 \cdot 2t}{12}\right) = 64832 \text{ cm}^3$$

$N_x + N$

$$\sigma_{xx} = \frac{\Delta N_x}{A_x} + \frac{M_y}{I_y} z + \frac{M_z}{I_z} y$$

$$A_x = (2t \cdot 16t)2 + 2(10t^2) + 2(12t^2)$$

$$A_x = 432$$

$$\sigma_{xx} = \frac{100\,\text{kN}}{432\,\text{cm}^2} + \frac{100000\,\text{kNcm}}{64832\,\text{cm}^3} \cdot 8t$$

Preskok $b^* = 2t$ ali $12t$

$$S_y^* = \left|(2t \cdot 16t \cdot (-7t) + (t \cdot 12t \cdot 11)\right|$$

$$= -2320$$

$\sigma_{xx,sp} = 24{,}912 \ \frac{\text{kN}}{\text{cm}^2}$

$\sigma_{xx,zg} = -24{,}448 \ \frac{\text{kN}}{\text{cm}^2}$

$b^* = 2t$ ali $12t$

Največja

$$S_y^* = 2\left(5t \cdot t \cdot \left(-\tfrac{5}{2}t\right)\right) + \left(12t^2 \cdot \left(\tfrac{11}{2}t\right)\right) + \left(32t^2 \cdot (-7t)\right) = -2520$$

$$\tau_{xz} = \frac{-N_z \cdot S_y^*}{I_y \, b^*} = \underline{0{,}8946} \quad b^* = 2t$$

$$= 0{,}1491 \quad b^* = 12t$$

Največja

$\tau_{xz} = 0{,}9717$

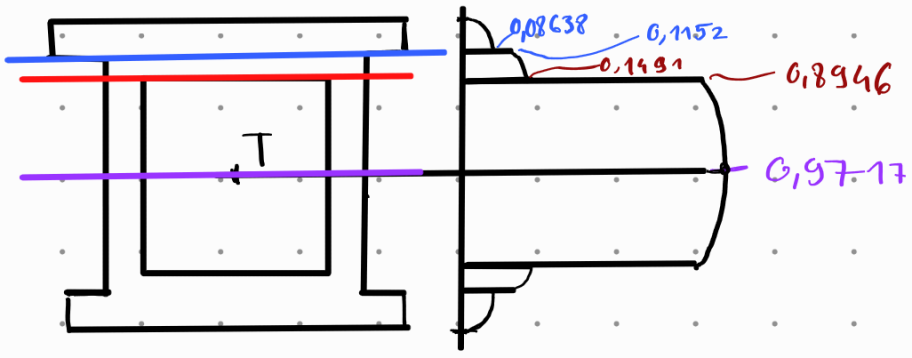
Odsek

$$S_y^* = (32t^2 \cdot (-7t)) = -1792$$

Odsek

$\tau_{xz} = 0{,}08638 \quad b^* = 16t$

$0{,}1152 \quad b^* = 12t$



$$\sigma_{xx,sp} = 0 : \frac{N_x + \Delta N_x}{A_x} + \frac{M_y}{I_y} \cdot 16 \quad (\text{Tlak}) \qquad \frac{100 + \Delta N_x}{A_x} + \frac{M_y \cdot 16}{I_y} = 0$$

$24,679$

$$\sigma_{xx,zy=0} = \frac{N_x + \Delta N_x}{A_x} - \frac{M_y}{I} 16 \quad (\text{Napetost}) \qquad \frac{100}{A_x} + \frac{\Delta N_x}{A_x} = -\frac{M_y \cdot 16}{I_y}$$

$$\frac{\Delta N_x}{A_x} = -\frac{M_y \cdot 16}{I_y} - \frac{100}{A_x}$$

$$\Delta N_x = -\frac{M_y \cdot 16 \cdot A_1}{I_y} - 100$$

$$\Delta N_x = 10761{,}40$$

$$\Delta N_x = 10561{,}40$$

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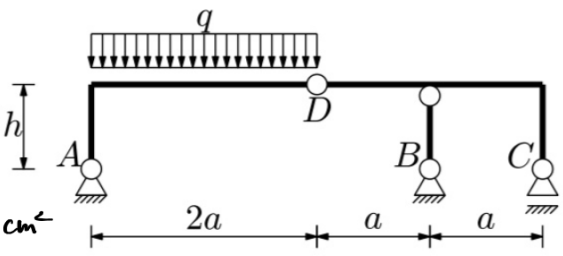
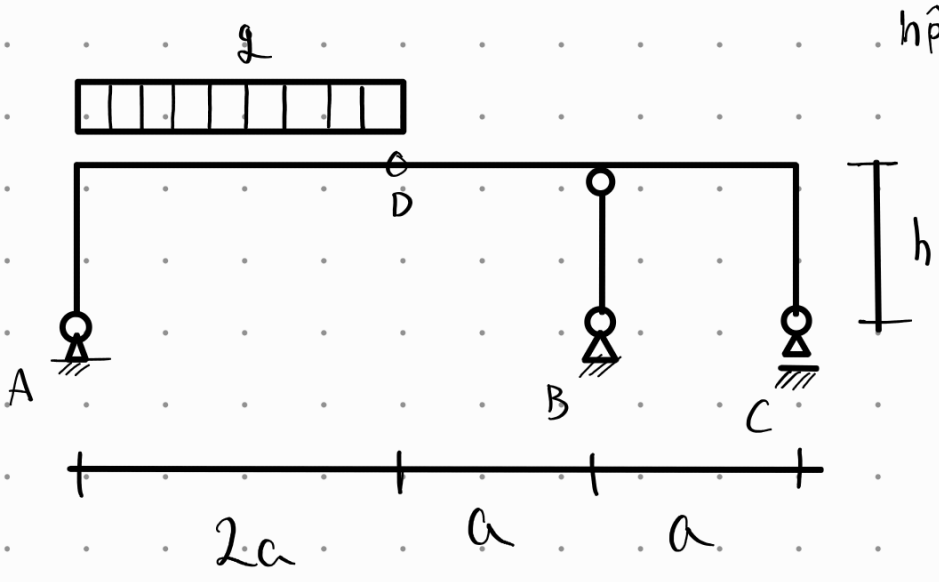


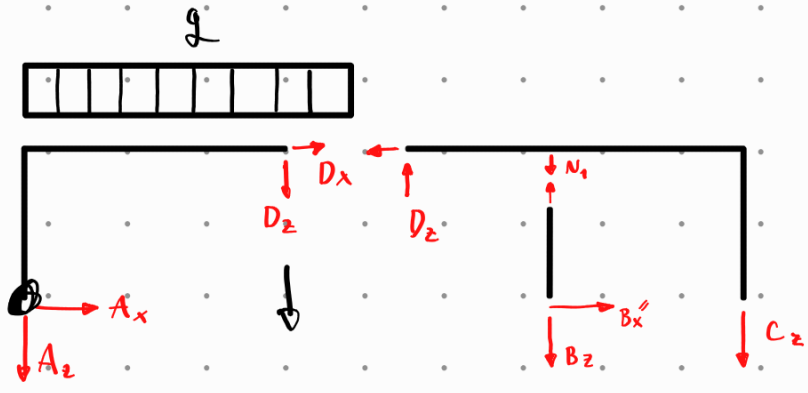
$1m = 100cm$

$1m^2 = 10000cm^2$

Podatki: $a = 4$ m, $h = 3$ m, $A_x = 500$ cm^2, $I_y = 20000$ cm^4, $E = 20000$ MPa. M $\frac{N}{m^2}$ → 10000 cm^2



hps = g - 5 - 2 - 2 = 0



$B_x = 0$

$A_x = 0$

$N_1 = B_z$

$-D_z \cdot 2a - q\,2a^2 = 0$

$D_z = -2qa$

$N_1 \cdot a - D_z \cdot 2a = 0$

$N_1 = D_z \cdot 2$

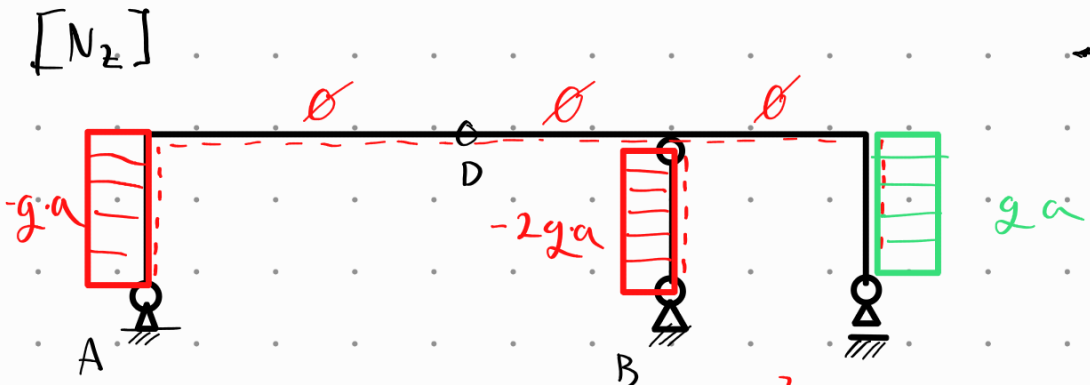
$N_1 = -2qa$

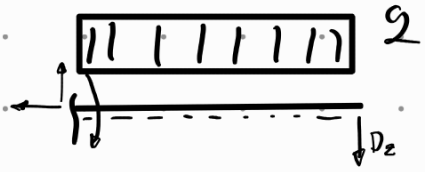
$A_z = -q \cdot 2a + qa$

$A_z = -qa$

$C_z = -A_z - N_1 = q \cdot a + 2qa = 3qa$

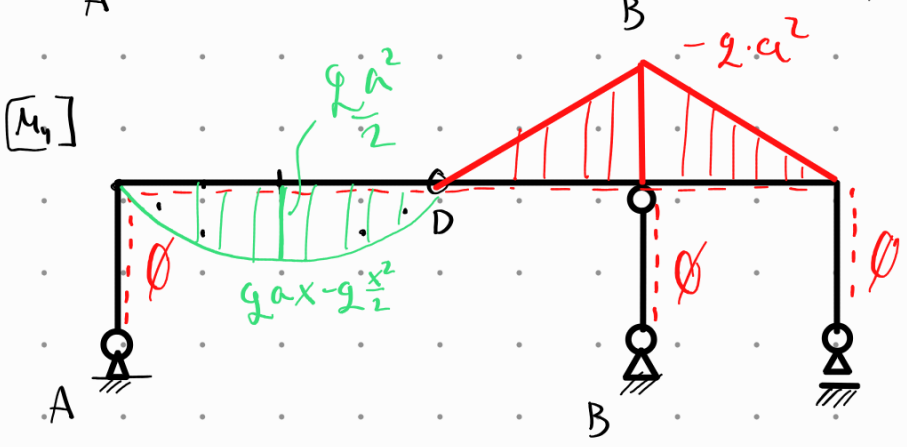
$q \cdot 2a + A_z + C_z + B_z =$ $\quad C_z = -q2a + qa + 2qa = qa$





$M_y = qax - q\frac{x^2}{2}$

$20x - \frac{5}{2}x^2$



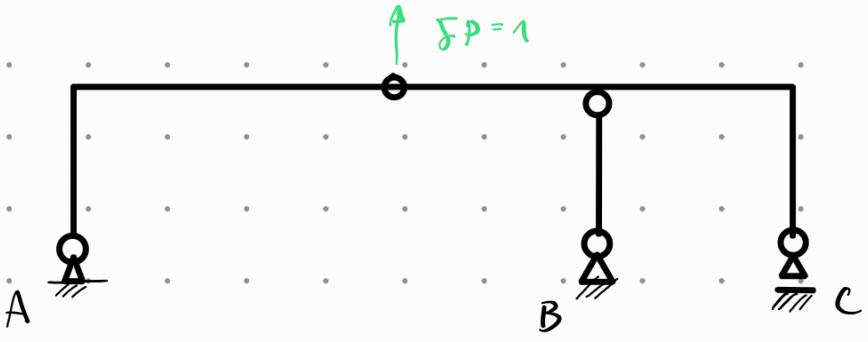
$M_y = D_z \cdot x$

$M_y = -qa \cdot x$

$M_y = -D_z \cdot (a+x) + (-2qa) \cdot x$

$M_y = q \cdot a^2 - qax$

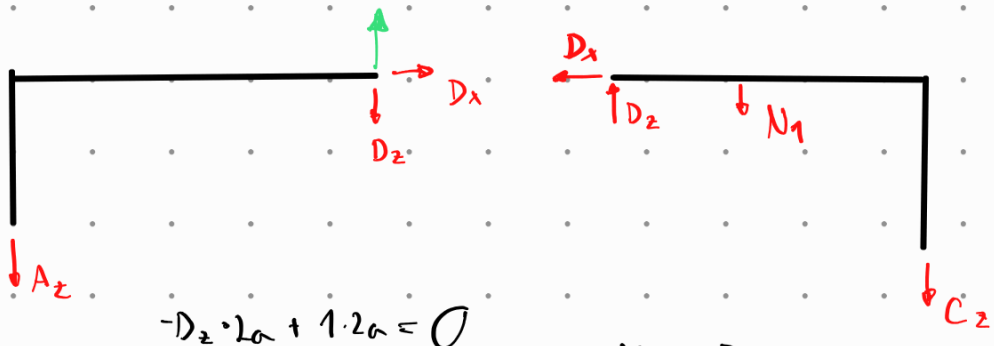
$80 - 20x$



$B_x = 0$

$A_x = 0$

$D_x = 0$



$-D_z \cdot 2a + 1 \cdot 2a = 0$

$D_z = 1$

$A_z = 0$

$N_1 \cdot a - D_z \cdot 2a = 0$

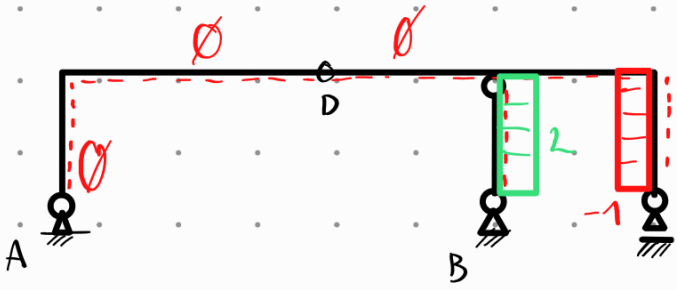
$N_1 = D_z \cdot 2$

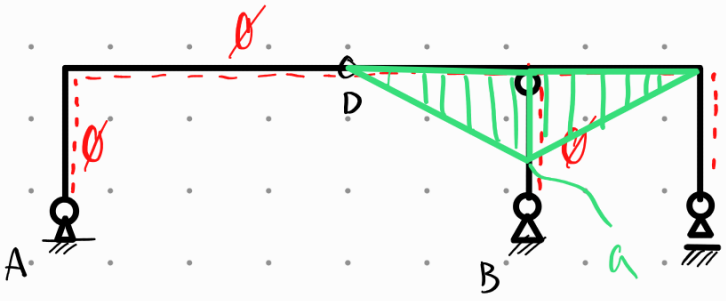
$N_1 = 2$

$C_z + B_z - 1 = 0$

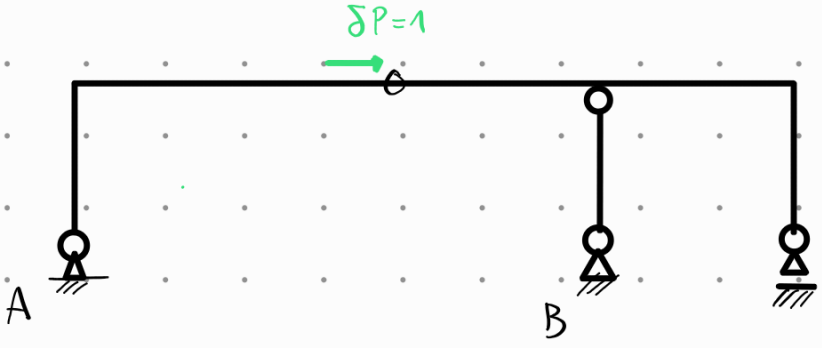
$C_z = 2 - 1 - 0$

$C_z = -1$



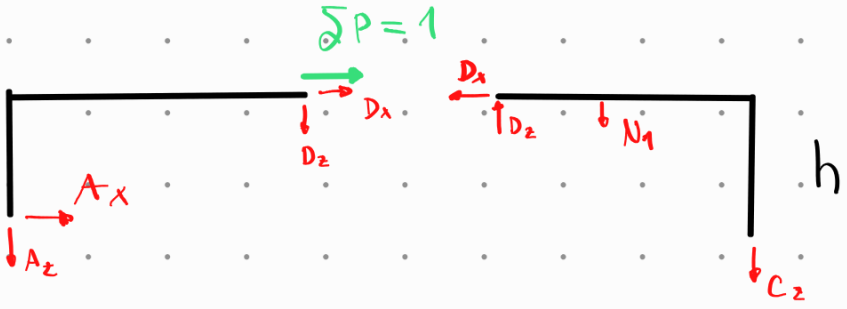


$-M_y - C_z \cdot x$



$B_x = 0$

$A_x = -1$



$A_x + D_x + 1 = 0$

$D_x = 0$

$-\cancel{D_x h} - D_z \cdot 2a - 1 \cdot h = 0$

$D_z = -\frac{h}{2a} = -\frac{3}{8}$

$A_z = \frac{3}{8}$

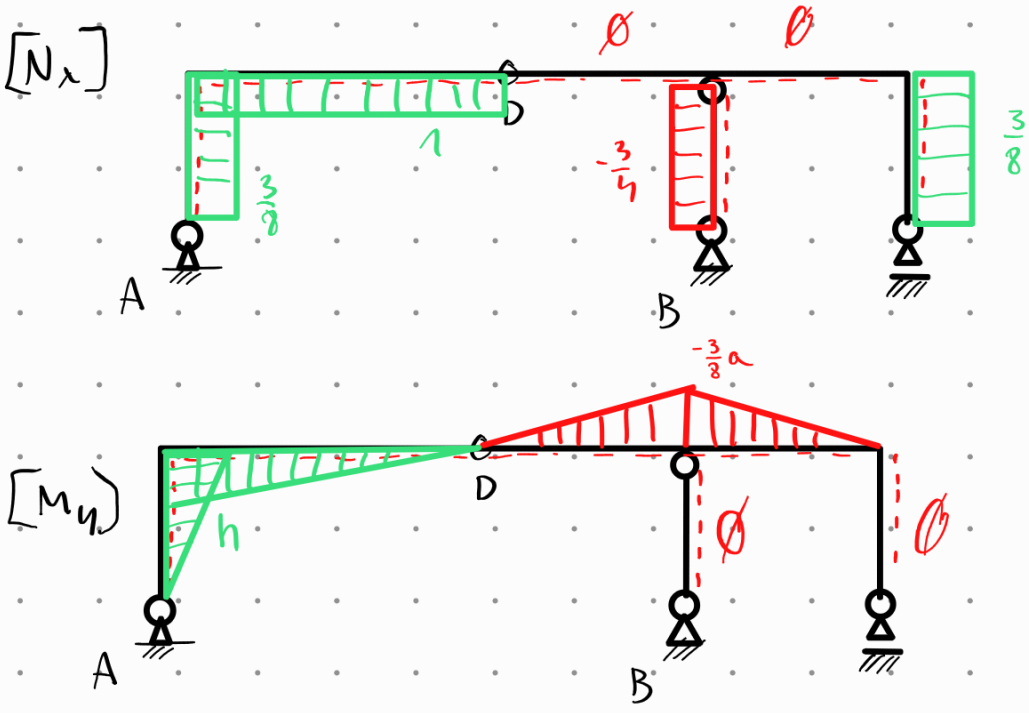
$-D_z \cdot 2a + N_1 \cdot a = 0$

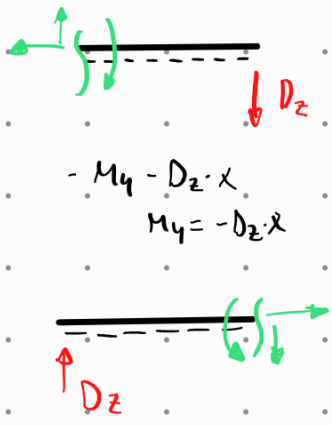
$N_1 = D_z \cdot a$

$N_1 = -\frac{3}{4}$

$C_z + A_z + B_z = 0$

$C_z = -A_z - B_z = +\frac{3}{8}$





$$u_D^H = \sum \int_0^\ell \frac{\overline{N_x} \cdot N_x}{E A_x} + \frac{\overline{M_y} M_y}{I_y E} \, dx =$$

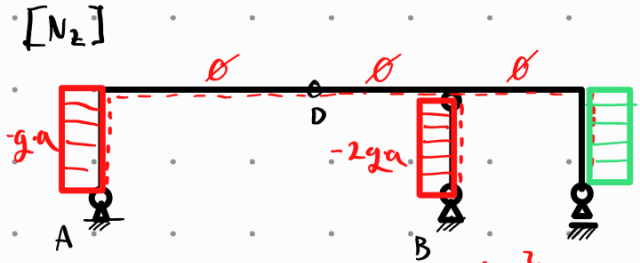
$$u_D^V = \sum \int_0^\ell \frac{\overline{N_x} \cdot N_x}{E A_x} + \frac{\overline{M_y} M_y}{I_y E} \, dx$$

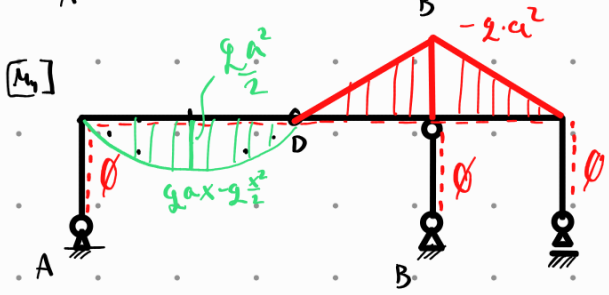
$5 \frac{kN}{m} = 0{,}05$

$$u_D^V = \frac{\overbrace{\left(-2qa\right)\left(\frac{h}{a}\right) \cdot h}^{-18 \cdot 10^4 \cdot q}}{E A_x} + \frac{\overbrace{\left(\frac{1}{3} h \cdot q\frac{a^2}{2}\right) + 2\left(\frac{1}{3} \cdot \frac{h}{2} \cdot qa^2\right)}^{8 \cdot 10^6 \cdot q \; + \; 16 \cdot 10^6 q}}{I_y E} = 42q = 2{,}1$$

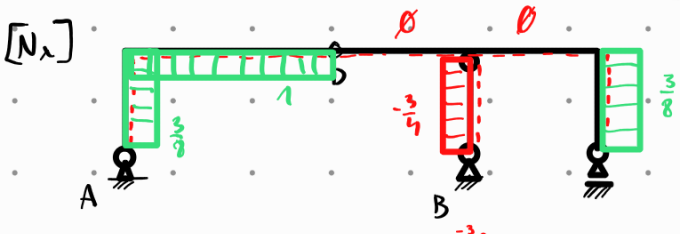
$$u_D^h = \frac{\overbrace{\left(-2qa \cdot 2h\right)\left(-1 \cdot qa \cdot h\right)}^{-60\,q}}{E A_x} + \frac{\overbrace{2 \cdot \left(\frac{1}{3}\left(-qa^2\right) \cdot a \cdot a\right)}^{-42666{,}66\,q}}{I_y E} =$$

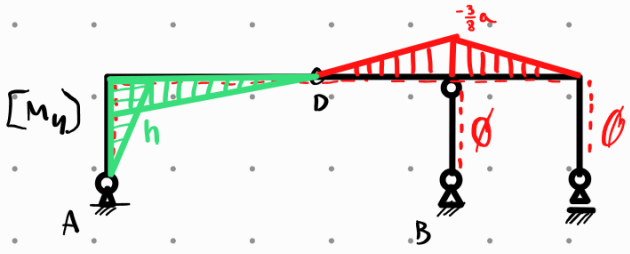
Síle, zatěžbe, ...



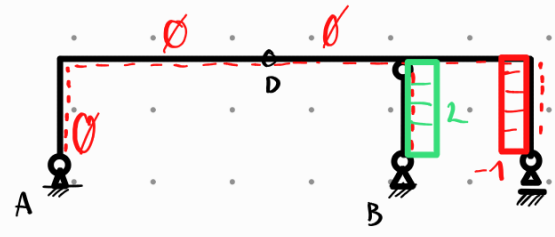


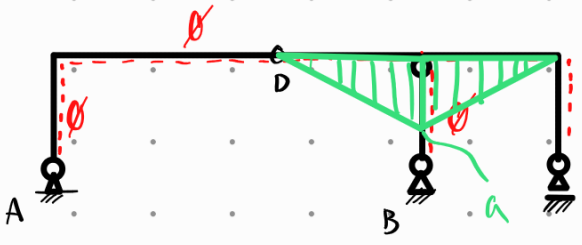
Horizontální





Vertikální





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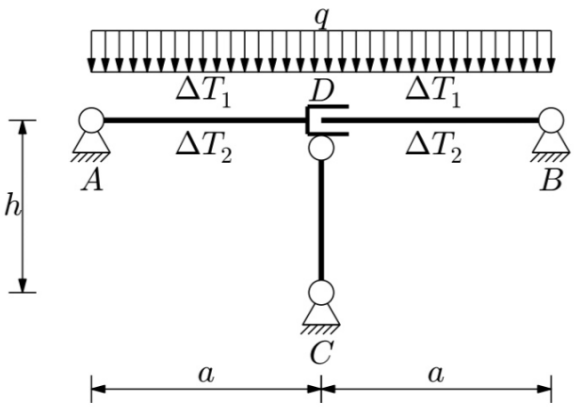

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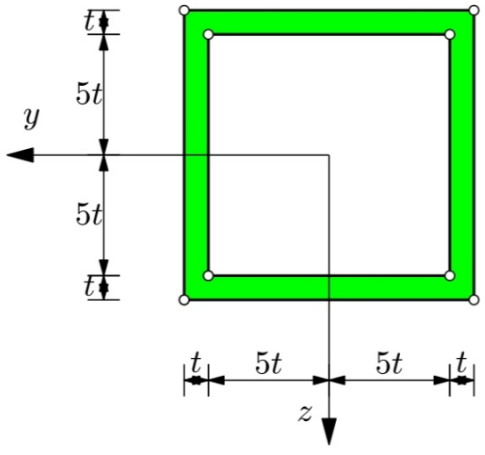
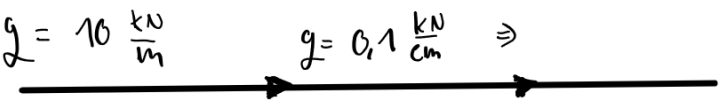
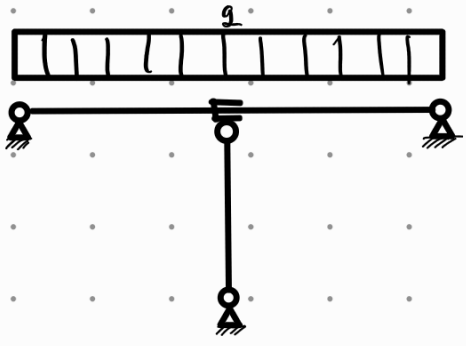


$q = 10\ \frac{kN}{m}$ $\quad q = 0{,}1\ \frac{kN}{cm} \Rightarrow$

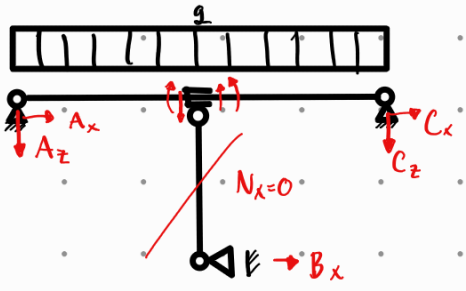


$A_x = (12t)^2 - (10t)^2 = 396\ cm^2$

$I_y = \frac{(12t)^4}{12} - \frac{(10t)^4}{12} = 72468\ cm^4$



$\tilde{n}_{ps} = 9 - 6 - 2 - 2 = -1$



$A_x = 0$
$C_x = 0$
$B_x = 0$

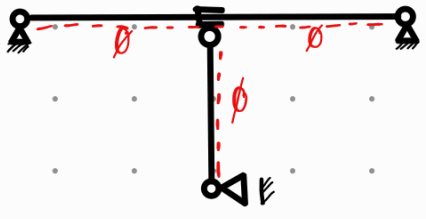
$-C_z \cdot 2a - q \cdot 2a^2$

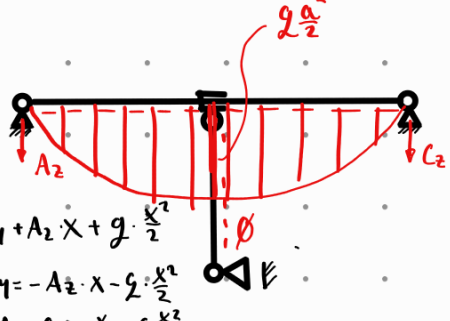
$C_z = -2qa$

$A_z + C_z + q \cdot 2a = 0$

$A_z = -q \cdot 2a + 2qa$

$A_z = -qa$

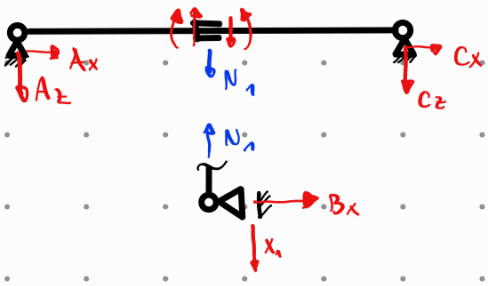




$M_y + A_z \cdot x + q \cdot \frac{x^2}{2}$

$M_y = -A_z \cdot x - q \frac{x^2}{2}$

$M_y = qa \cdot x - q \frac{x^2}{2}$

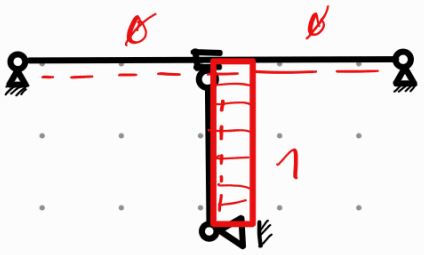


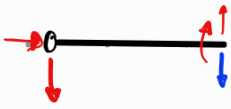
$N_1 = 1$

$B_x = 0$

$A_x = 0$

$C_x = 0$

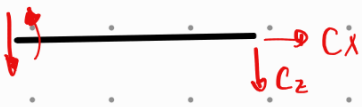




$$-N_1 \cdot a + D_z \cdot a + D_z \cdot a$$

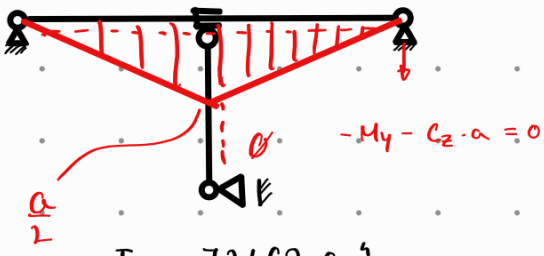
$$D_z = \frac{1}{2}$$

$$0 = A_z + N_1 - D_z \Rightarrow A_z = -\frac{1}{2}$$



$$C_z + D_z = 0$$

$$C_z = -\frac{1}{2}$$



$$I_y = 72468\ cm^4$$

$$A_x = 396\ cm^2$$

$$E = 20000 \qquad 20\,000$$

$$g = 0{,}1\ \frac{kN}{cm}$$

$$D_z \cdot a + M_D = 0$$

$$M_D = -D_z \cdot a = -\frac{a}{2}$$

$$a_{11} = \int_0^l \frac{\overline{N_{x1}}\,\overline{N_{x1}}}{EA_x}dx + \int_0^l \frac{\overline{M_{y1}}\,\overline{M_y}}{EI_y}dx = \underset{3{,}78 \cdot 10^{-5}}{\frac{(1)^2 h}{EA_x}} + \underset{7{,}359\ldots \cdot 10^{-3}}{\frac{\left(\frac{1}{3}\cdot\left(\frac{a}{2}\right)^2 \cdot a\right)\cdot 2}{EI_y}}$$

$$b_1 = \int_0^l \frac{N_x \overline{N_{x1}}}{EA_x}dx + \int_0^l \frac{M_y \overline{M_{y1}}}{EI_y} + \int_0^l \alpha_T \Delta T_x \overline{N_{x1}}\,dx + \int_0^l \alpha_T \Delta T_z \overline{M_{y1}}\,dx$$

$$b_1 = \underset{=0{,}36}{\frac{2\left(\frac{5}{12}\cdot a \cdot q\frac{a^2}{2}\cdot\frac{a}{2}\right)}{EI_y}} + \alpha_T \Delta T_x \cdot 1 + \left(\alpha_T \Delta T_z \cdot \frac{a}{2}\right)\cdot 2$$

$$\Delta T_x = \frac{15+45}{2} = 30$$

$$\Delta T_z = \frac{45-15}{h} = 0{,}8\overline{33}$$

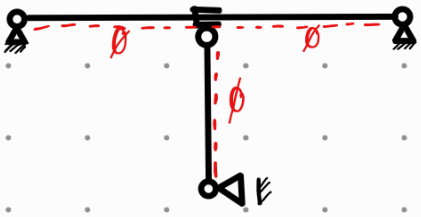
glede na vlakna (iz + vzameo katera ΔT je)

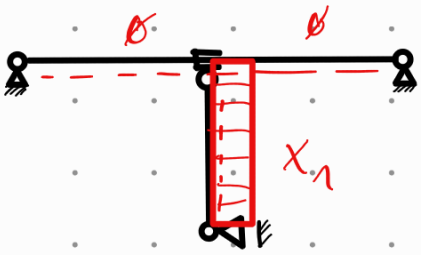
$$b_1 = 0{,}364945$$

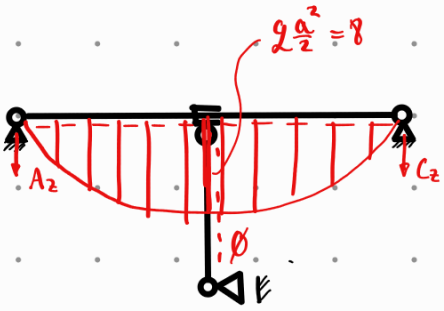
$$a_{11} = 7{,}3974 \cdot 10^{-3}$$

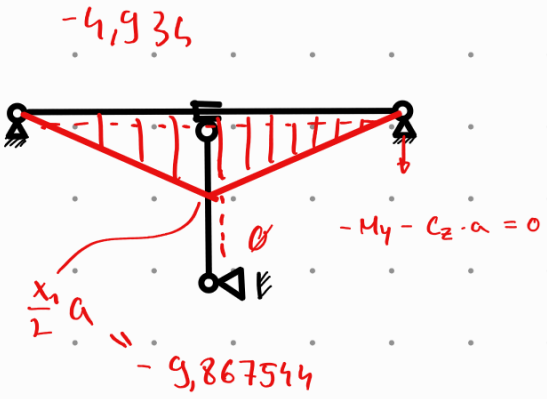
$$a_{11} x_1 + b_1 = 0$$

$$x_1 = \frac{-b_1}{a_{11}} = 4{,}93342 \cdot 10^{-5}$$









$$\sum \int_0^{\ell} \alpha_T \overset{\Delta T}{\overline{\Delta T_x}} \cdot \overline{N_x} \, d$$