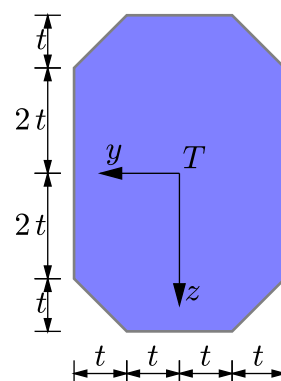


Drugi kolokvij iz TRDNOSTI (UNI-B), 9. januar 2017

1. Prečni prerez na sliki je obtežen z osno silo $N_x = 10 \text{ kN}$, s prečno silo $N_z = 10 \text{ kN}$ in z upogibnim momentom $M_y = 10 \text{ kNm}$.

- Določi vztrajnostna momenta I_y^T in I_z^T prereza.
- Določi po absolutni vrednosti največjo normalno napetost σ_{xx} v prerezu.
- Določi po absolutni vrednosti največjo strižno napetost σ_{xz} v prerezu.
- Ali se prijemališče $(0, t)$, nahaja v jedru prereza? Odgovor utemelji!

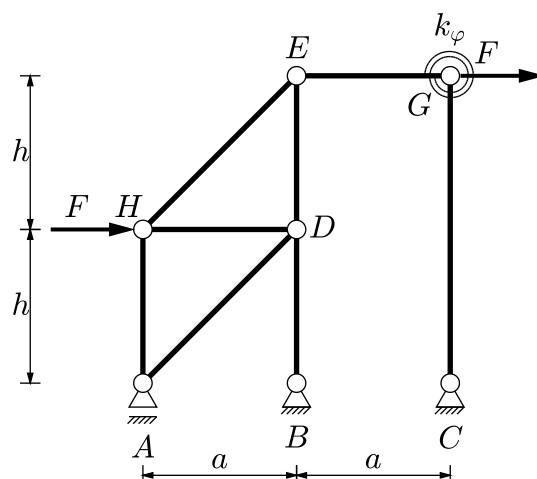


Podatki: $t = 3 \text{ cm}$.

2. Ravninski okvir na sliki je obtežen z vodoravnima silama F . Vsi nosilci so iz enakega linearno elastičnega materiala.

- Določi notranje sile in skiciraj diagrame notranjih sil.
- Določi vodoravni pomik u_G točke G .

Podatki: $a = 4 \text{ m}$, $h = 4 \text{ m}$, $F = 10 \text{ kN}$, $I_y = 20000 \text{ cm}^4$, $A_x = 200 \text{ cm}^2$, $E = 20000 \frac{\text{kN}}{\text{cm}^2}$, $k_\varphi = \frac{10000 \text{ kNm}}{\text{rad}}$.

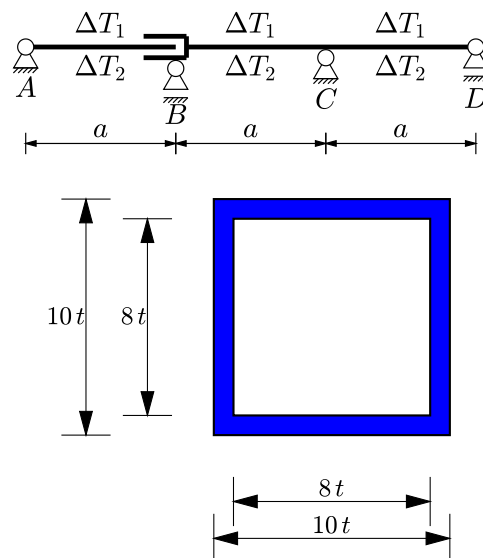


3. Gredo škatlastega prereza iz linearno elastičnega materiala na gornji strani segrejemo za ΔT_1 , na spodnji pa za ΔT_2 . Prečni prerez grede je prikazan na sliki spodaj.

- Določi notranje sile in skiciraj diagrame notranjih sil.

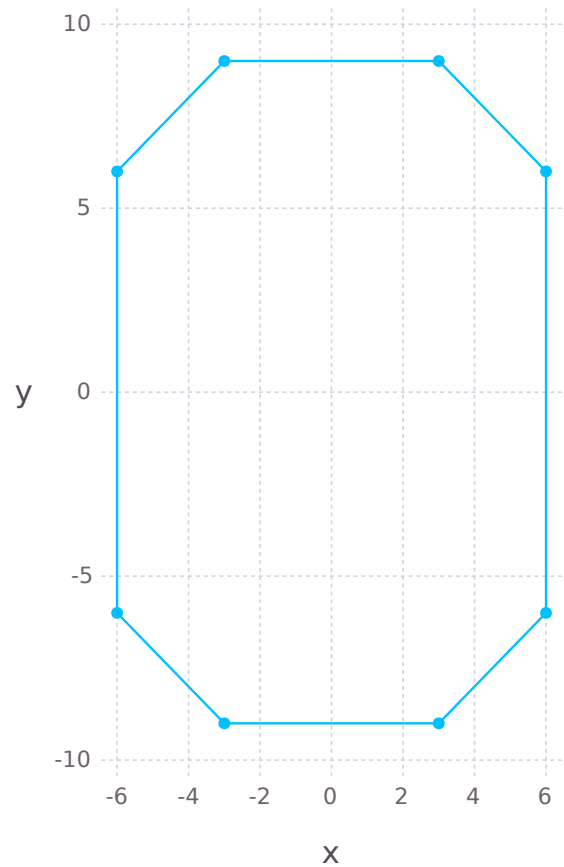
Pri reševanju privzemi linearen potek spremembe temperature po višini prečnega prereza.

Podatki: $a = 4 \text{ m}$, $t = 3 \text{ cm}$, $\Delta T_1 = 45 \text{ K}$, $\Delta T_2 = 15 \text{ K}$, $\alpha_T = \frac{10^{-5}}{\text{K}}$, $E = 20000 \frac{\text{kN}}{\text{cm}^2}$.



1. Dict{Any,Any} with 23 entries:

```
"IyT"    => 4671.0
"I2"     => 2133.0
"Ieta"   => 4671.0
"Iz"     => 2133.0
"I1"     => 4671.0
"yT"     => 0.0
"Iy"     => 4671.0
"IzT"    => 2133.0
"alphadeg" => -0.0
"Iyz"    => -0.0
"Ax"     => 198.0
"Sy"     => -0.0
"IyzT"   => -0.0
"Izeta"  => 2133.0
"eeta"   => [1.0,-0.0]
"Ietazeta" => 0
"Sz"     => 0.0
"alphag" => -0.0
"zT"     => -0.0
"e2"     => [0.0,1.0]
"J"      => 2x2 Array{Float64,2}:
"ezeta"  => [0.0,1.0]
"e1"     => [1.0,-0.0]
```

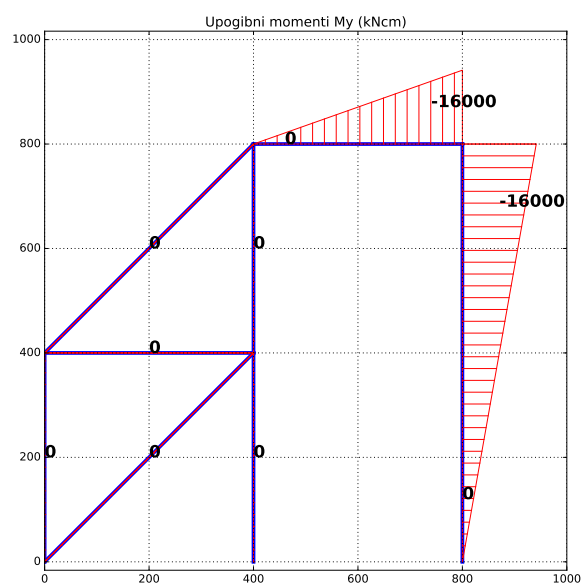
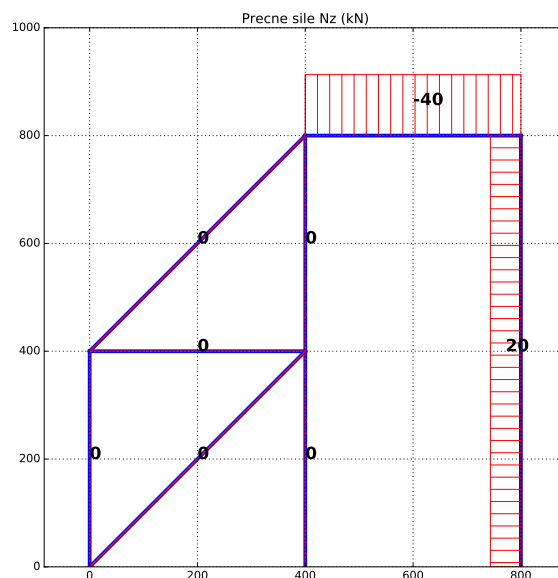
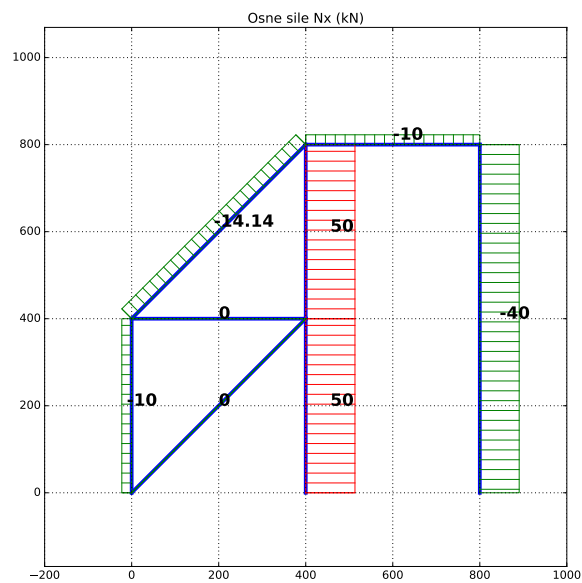


Dict{ASCIIString,Float64} with 6 entries:

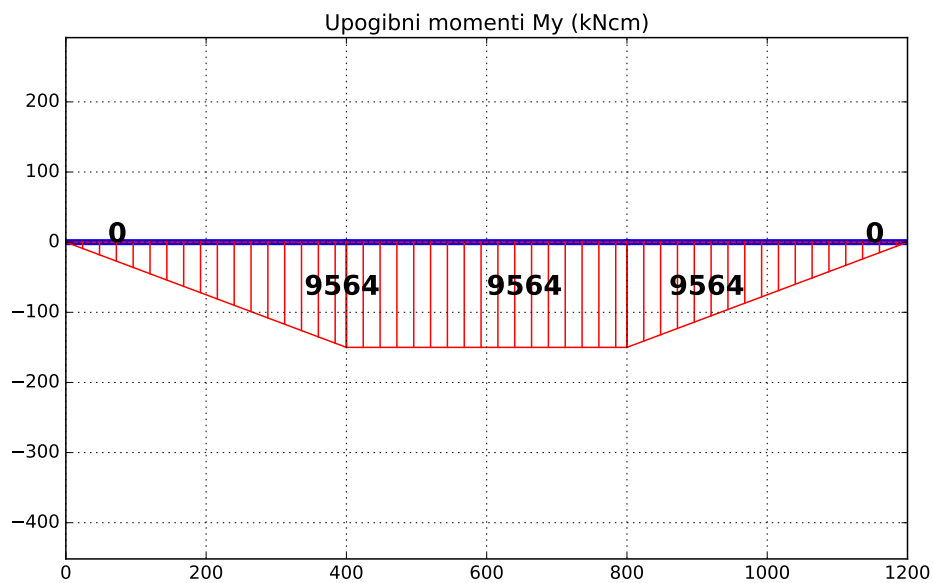
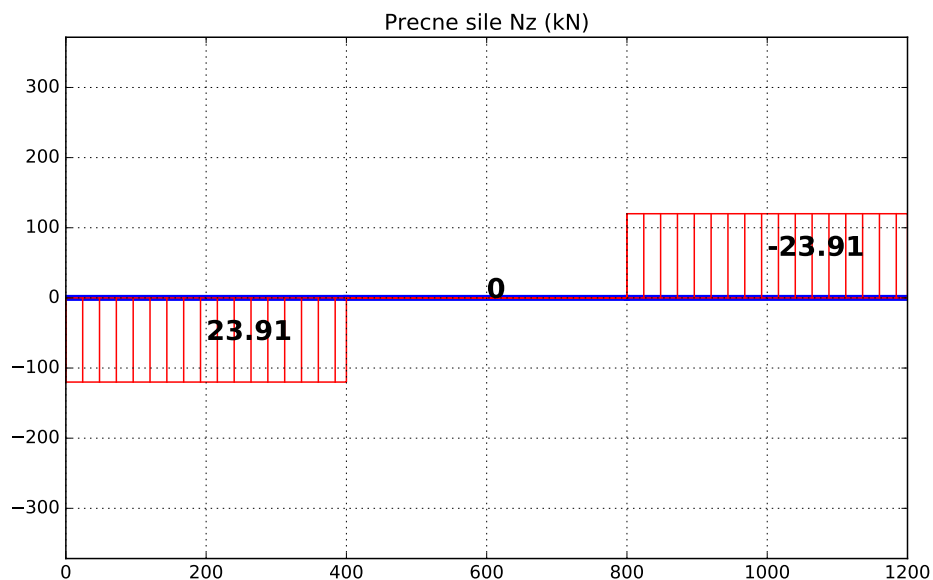
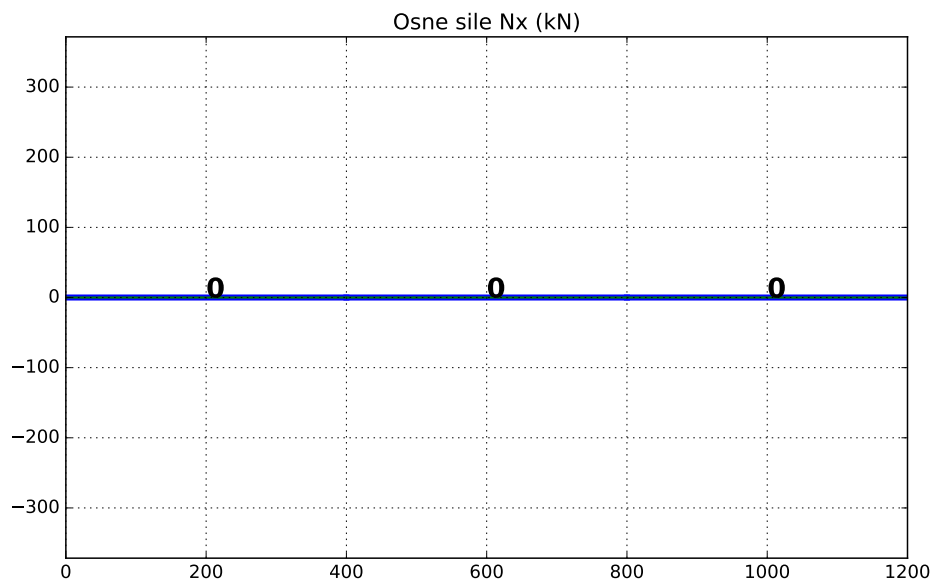
```
"sxz"    => 0.07385998715478484 kN/cm^2
"Ax"     => 198.0 cm^2
"Iy"     => 4671.0 cm^4
"sxx2"   => -1.8762772230980322
"sxx1"   => 1.9772873241081335 kN/cm^2
"Iz"     => 2133.0 cm^4
```

Prijemališče sile ni v jedru, ker nevtralna os seka prerez.

2. Diagrami notranjih sil N_x (kN), N_z (kN), M_y (kNcm), $u_{k\phi=\infty} = 12.836$ cm, $u_{k\phi} =$ cm.

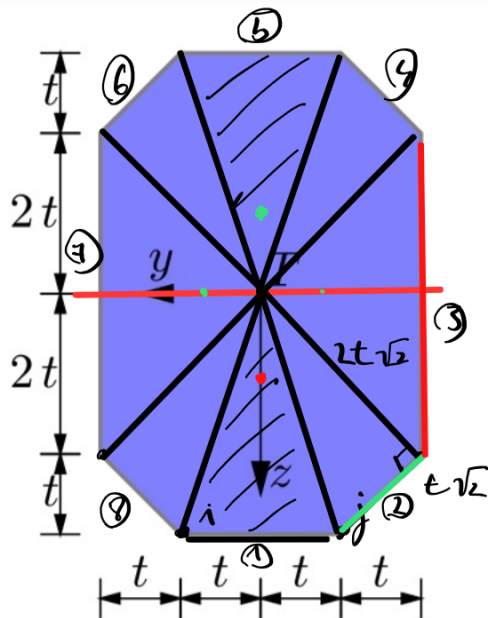
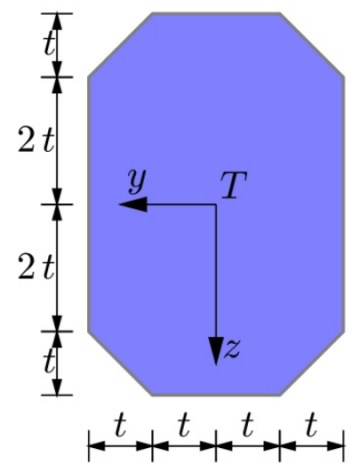


3. Diagrami notranjih sil N_x (kN), N_z (kN), M_y (kNcm):



1. Prečni prerez na sliki je obtežen z osno silo $N_x = 10 \text{ kN}$, s prečno silo $N_z = 10 \text{ kN}$ in z upogibnim momentom $M_y = 10 \text{ kNm}$.

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- Ali se prijemališče $(0, t)$, nahaja v jedru prereza? Odgovor utemelji!



$$I_y^T = \frac{(6t)^3 \cdot 4t}{12} + 4 \left(\frac{t^3 \cdot t}{36} - t^2 \frac{1}{2} \cdot \left(\frac{2}{3}t + 2t \right)^2 \right)$$

$$I_y^T = 5832 + 4 \left(-2,25 - 4,5 \cdot (64) \right) = 4671 \text{ cm}^4$$

$$I_z^T = \frac{(4t)^3 \cdot 6t}{12} - 4 \left(\frac{t^4}{36} + \frac{t^2}{2} \cdot \left(\frac{2}{3}t \right)^2 \right) = 2133 \text{ cm}^4$$

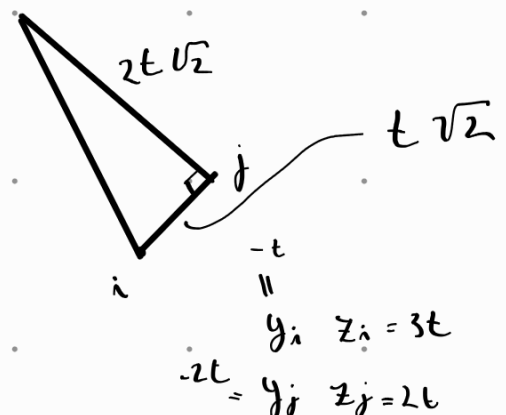
$$A_x = 6t \cdot 4t - 4 \frac{t^2}{2}$$

$$\sigma_{xx_{sp}} = \frac{N_x}{A_x} + \frac{M_y}{I_y} \cdot z = \frac{10}{198} + \frac{1000}{4671} \cdot 3 = 1,977$$

$$\sigma_{xx_{zg}} = \frac{10}{198} - \frac{1000}{4671} \cdot 3 = -1,8762$$

$$\sigma_{xz} = \frac{-N_z \cdot S_y^*}{I_y \cdot b^x}$$

$$S_y^* = \underbrace{(3t \cdot 4t \cdot (-1,5t))}_{-486} - \underbrace{t^2 \cdot ((2t + 2))}_9 = -495$$



$$I_y = \frac{bh^3}{12} \quad I_z = \frac{b^3h}{12}$$

$$i_y^2 = \frac{I_y}{A_x} = \frac{h^2}{12}, \quad i_z^2 = \frac{I_z}{A_x} = \frac{b^2}{12}$$

$$y_j = -t \quad z_j = 3t \\ y_i = t \quad z_i = 3t$$

rob	$y_j - y_i$	$z_j - z_i$	$2pl \Delta$	y_N	z_N
①	$-2t$	$\emptyset$	$6t^2$	$\emptyset$	$-2,621$
⑤	$2t$	$\emptyset$	$6t^2$	$\emptyset$	$2,621$
②	t	$-t$	36	$-0,358$	$-1,97$

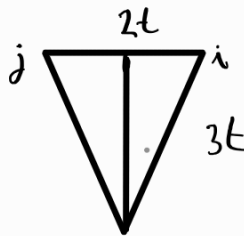
$$y_N = \frac{-i_z^2(z_j - z_i)}{2pl \Delta}, \quad z_N = \frac{i_y^2(y_j - y_i)}{2pl \Delta}$$

$$T(y_N, z_N)$$

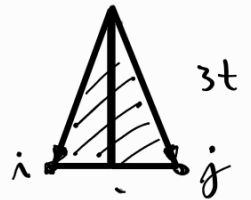
$$z_N = \frac{23,591 \cdot (-2t)}{3t^2} = -2,621$$

$$i_y^2 = \frac{I_y}{A_x} = 23,591$$

$$i_z^2 = \frac{I_z}{A_x} = 10,773$$



$$y_j = t \\ y_i = -t$$

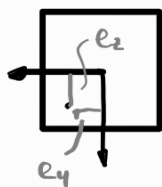
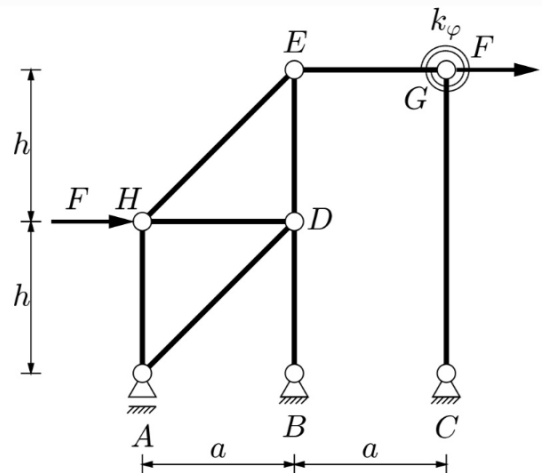


2. Ravninski okvir na sliki je obtežen z vodoravnima silama F . Vsi nosilci so iz enakega linearno elastičnega materiala.

- Določi notranje sile in skiciraj diagrame notranjih sil.
- Določi vodoravni pomik u_G točke G .

Podatki: $a = 4 \text{ m}$, $h = 4 \text{ m}$, $F = 10 \text{ kN}$, $I_y = 20000 \text{ cm}^4$,
 $A_x = 200 \text{ cm}^2$, $E = 20000 \frac{\text{kN}}{\text{cm}^2}$, $k_\varphi = \frac{10000 \text{ kNm}}{\text{rad}}$.

$$h_{ps} = 8 \cdot 3 \cdot 4 \cdot 2 \cdot 5$$



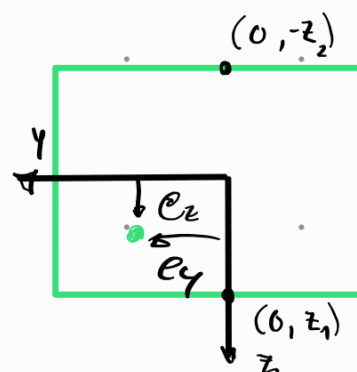
$$N_x = N$$

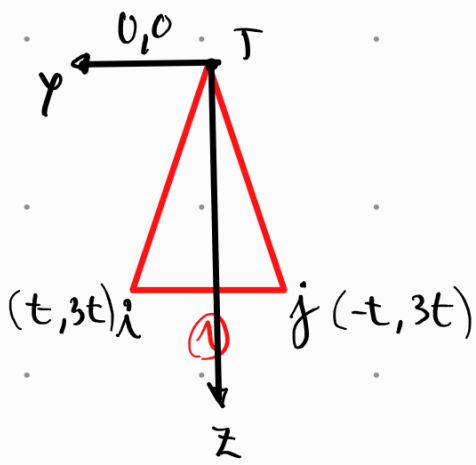
$$N \cdot e_2$$

$$N \cdot e_1$$

$$b_{xx} = \frac{N_x}{A_x} + \frac{M_y}{I_y} \cdot z - \frac{M_z}{I_z} \cdot y = 0$$

$M_y = +$ $M_z = +$	$M_y = -$ $M_z = +$
$M_y = +$ $M_z = -$	$M_y = -$ $M_z = -$





$$T_j(-t, 3t)$$

$$T_i(t, 3t)$$

$$\underline{z_j - z_i} = 0$$

$$\underline{y_j - y_i} = -2t$$

$$\underline{iy^2 = \frac{I_y}{A_x} = 23,591}$$

$$\underline{iz^2 = \frac{I_z}{A_x} = 10,773}$$

$$\underbrace{y_N = \frac{-iz^2(z_j - z_i)}{2pI_A}}_{}, \quad \underbrace{z_N = \frac{iy^2(y_j - y_i)}{2pI_A}}_{}$$

$$b_{xx, sp} = +$$

$$b_{xx, zg} = +$$

$$b_{xx} = \frac{N_x}{A_x} + \frac{M_y}{I_y} \overset{N \cdot e_z^3}{\textcircled{z}} = -$$

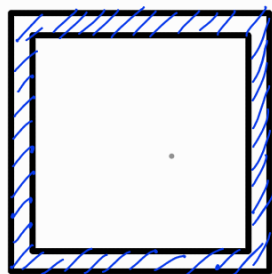
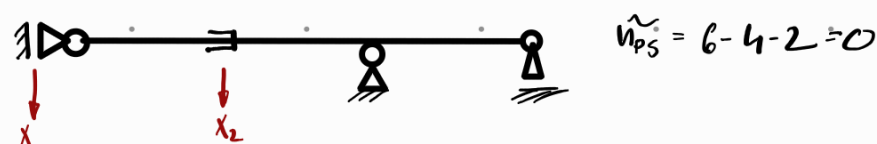
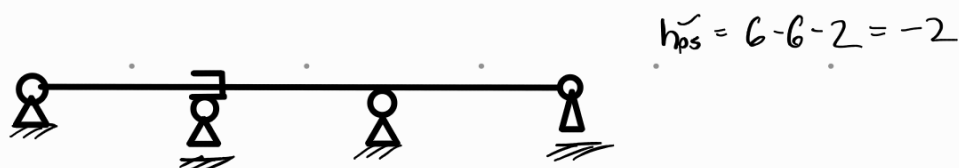
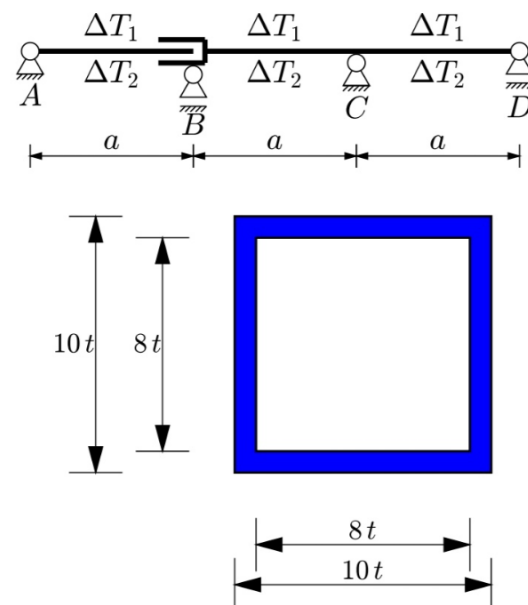
$$b_{xx} = \frac{N_x}{A_x} - \frac{M_y}{I_y} = +$$

3. Gredo škatlastega prereza iz linearno elastičnega materiala na gornji strani segrejemo za ΔT_1 , na spodnji pa za ΔT_2 . Prečni prerez grede je prikazan na sliki spodaj.

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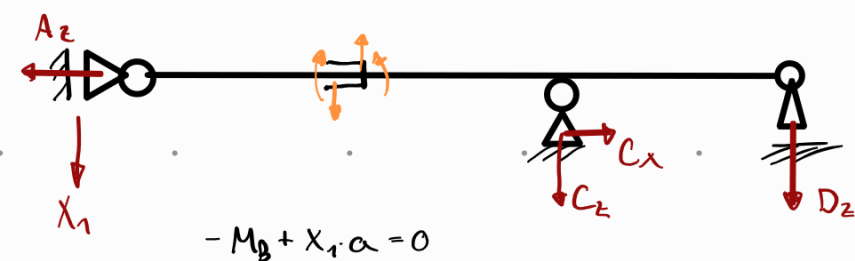
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$$I_y = \frac{(30)^4}{12} - \frac{(24)^4}{12} = 39852 \text{ cm}^4$$

$$A_x = 900 - 576 = 324 \text{ cm}^2$$



$$-M_B + x_1 \cdot a = 0$$

$$M_B = x_1 \cdot a$$

$$M_B = a$$

$$B_z + x_1 = 0$$

$$B_z = -1$$

$$-C_z \cdot a - x_1 \cdot 3a$$

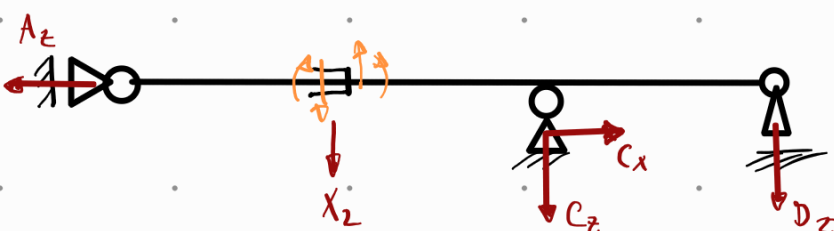
$$C_z = -3$$

$$C_z + D_z + x_1 = 0$$

$$D_z = 3 - 1 = 2$$

$$A_x = 0$$

$$C_x = 0$$



$$x_2 - B_z = 0$$

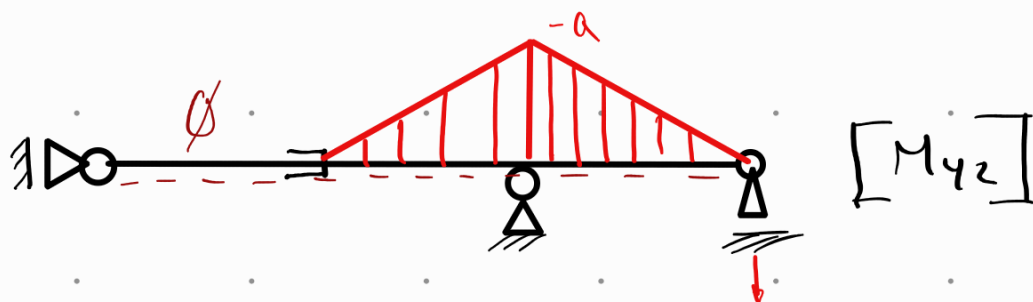
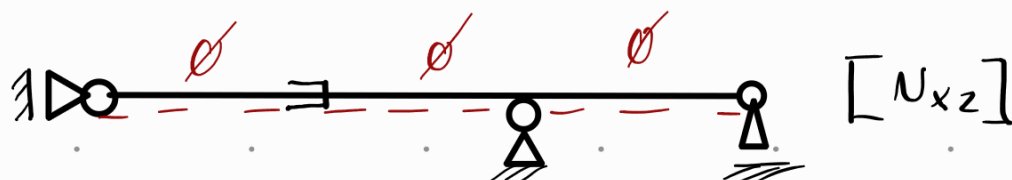
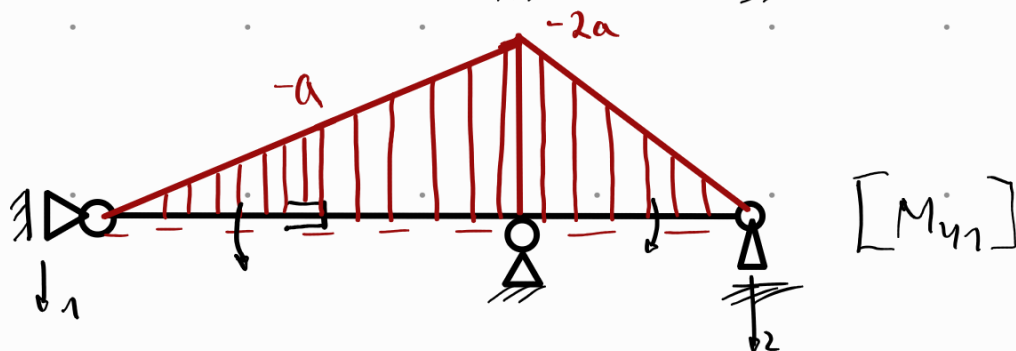
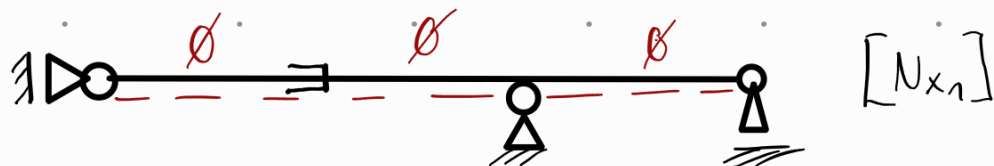
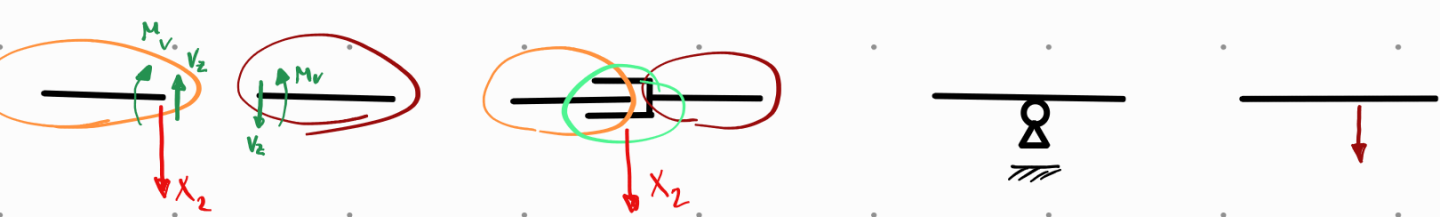
$$B_z = -1$$

$$-C_z \cdot a - x_2 \cdot 2a = 0$$

$$C_z = -2$$

$$D_z = -C_z - x_2$$

$$D_z = 2 - 1 = 1$$



$$a_{11} = \int_0^l \frac{\bar{N}_{x1} \bar{N}_{x1}}{EA_x} dx + \int_0^l \frac{\bar{M}_{y1} \bar{M}_{y1}}{I_y E} = \frac{\left(\frac{1}{3} \cdot (-2a)^2 \cdot 2a \right) + \left(\frac{1}{3} \cdot (-2a)^2 \cdot a \right)}{20000 \cdot 39852}$$

$$a_{22} = \frac{\left(\frac{1}{3} \cdot (-a)^2 \cdot a \right) \cdot 2}{20000 \cdot 39852}$$

$$a_{12} = \frac{\left(\frac{1}{3} \cdot (-2a) \cdot (-a) \cdot (a) \right) + \left(\frac{1}{6} \cdot a \cdot (2 \cdot (-2a) + (-a)) \right) \cdot (-a)}{20000 \cdot 39852}$$

$$b_1 = \int_0^l \cancel{\alpha_T \Delta T_x} \bar{N}_{x1} dx + \int_0^l \alpha_T \Delta T_z \bar{N}_{y1} dx$$

$$b_1 = \alpha_T \Delta T_z \cdot (-2a) \cdot 2a + \alpha_T \Delta T_z \cdot (-2a) a$$

$$\Delta T_z = \frac{\Delta T_1 - \Delta T_2}{h} = \frac{30}{30} = 1$$

$$10t = 30 \text{ cm}$$

$$b_2 = \alpha_T \Delta T_z \cdot (-a) a \cdot 2$$

$$a_{11} x_1 + b_1 = 0$$

$$a_{11} x_1 + a_{12} x_2 + b_1 = 0$$

$$a_{12} x_1 + a_{22} x_2 + b_2 = 0$$

$$x_1 = -737,48 \text{ cm}$$

$$x_2 = 1719,47 \text{ cm}$$

$$a_{11} = 0,2677$$

$$a_{22} = 0,0535$$

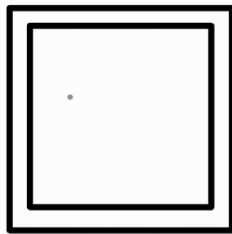
$$a_{12} = 0,1204$$

$$b_1 = -9,6$$

$$b_2 = -3,2$$

$$\Delta T_x = \frac{\Delta T_1 + \Delta T_2}{2}$$

$$\Delta T_z = \frac{\Delta T_1 - \Delta T_2}{h}$$



$$10t = h$$

