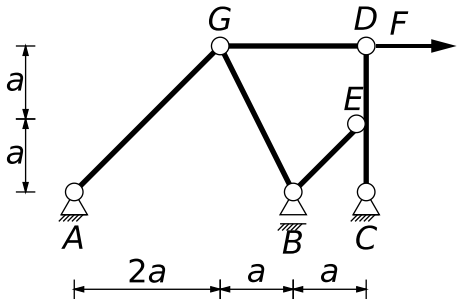
Drugi kolokvij iz TRDNOSTI (UNI-B), 15. januar 2019

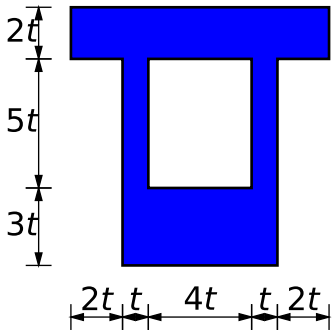
1. (30 %) Ravninski okvir na sliki je obtežen z vodoravno silo F. Vsi nosilci so iz enakega linearno elastičnega materiala.



   - Določi in skiciraj digrame notranjih sil.
   - Določi vodoravni pomik vozlišča D. Upoštevaj samo vpliv upogibnih momentov na deformiranje.

   Podatki: $a = 4$ m, $F = 10$ kN, $I_y = 20000$ cm^4, $A_x = 200$ cm^2, $E = 20\,000\ \frac{\text{kN}}{\text{cm}^2}$.

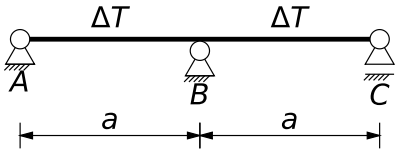
2. (15 %) Prečni prerez nosilca, prikazan na sliki desno, je obremenjen z osno silo $N_x = 10$ kN, s prečno silo $N_z = 10$ kN in z upogibnim momentom $M_y = 100$ kN m.



   - Določi po absolutni vrednosti največjo normalno napetost σ_{xx} v prerezu.
   - Določi po absolutni vrednosti največjo strižno napetost σ_{xz} v prerezu.

   Podatki: $t = 3$ cm.

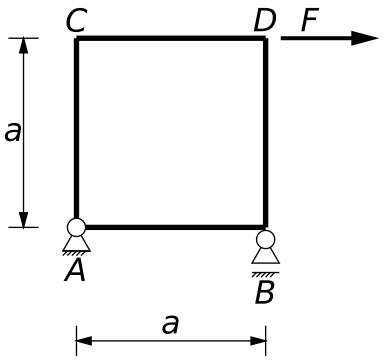
3. (20 %) Nosilec iz linearno elastičnega materiala enakomerno segrejemo za ΔT.



   - Določi notranje sile in skiciraj diagrame notranjih sil.

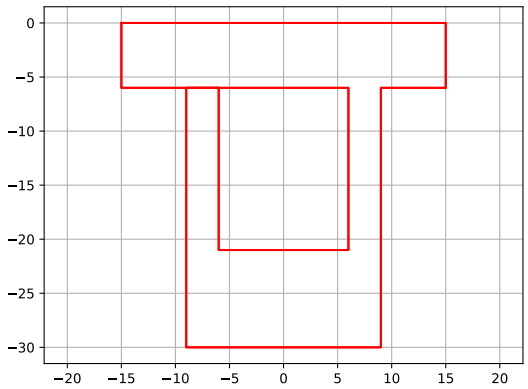
   Podatki: $a = 4$ m, $\Delta T = 20$ K, $\alpha_T = \dfrac{10^{-5}}{\text{K}}$, $A_x = 200$ cm^2, $I_y = 50\,000$ cm^4, $E = 20\,000\,000$ MPa.

4. (35 %) Ravninski okvir na sliki je obtežen z vodoravno silo F. Vsi nosilci so iz enakega linearno elastičnega materiala.



   - Določi upogibne momente in skiciraj diagrame upogibnih momentov.

   Upoštevaj samo vpliv upogibnih momentov na deformiranje.
   Podatki: $a = 4$ m, $F = 10$ kN, $I_y = 20000$ cm^4, $A_x = 200$ cm^2, $E = 20\,000\ \frac{\text{kN}}{\text{cm}^2}$.



2. df[[:Ax,:Sy,:zT]]

```
13 DataFrames.DataFrame
 Row  Ax       Sy       zT
      Float64  Float64  Float64

 1    432.0    5886.0   13.625
```

```
julia> df[[:Iy,:Iz,:Iyz]]
13 DataFrames.DataFrame
 Row  Iy        Iz       Iyz
      Float64   Float64  Float64

 1    126684.0  23004.0  -0.0
```

```
julia> df[[:IyT,:IzT,:IyzT]]
13 DataFrames.DataFrame
 Row  IyT      IzT      IyzT
      Float64  Float64  Float64

 1    46487.3  23004.0  0.0
```

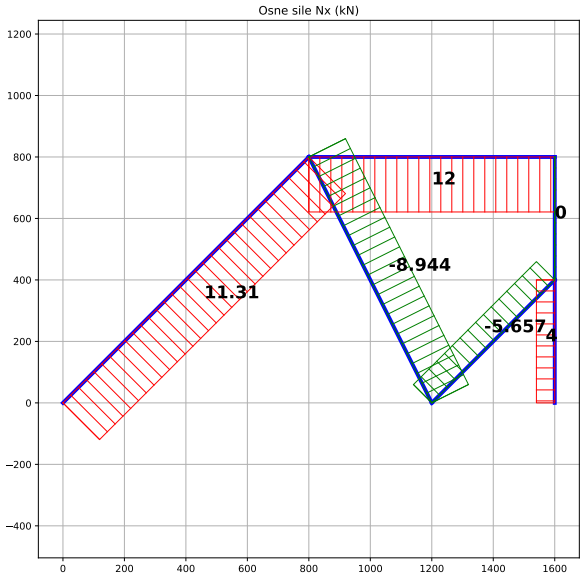
```
sxx = -My/df[:IyT]*13.625.+Nx/df[:Ax]
11 Array{Float64,2}:
 -2.9077630156655854
```

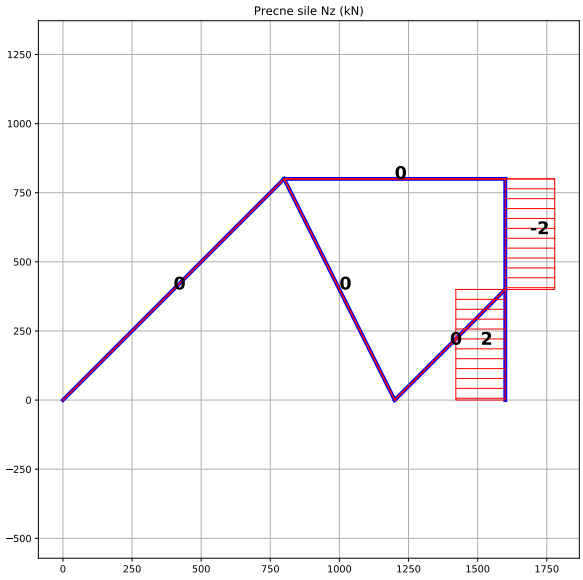
```
julia> sxx = My/df[:IyT]*16.375.+Nx/df[:Ax]
11 Array{Float64,2}:
 3.5456193633738287
```

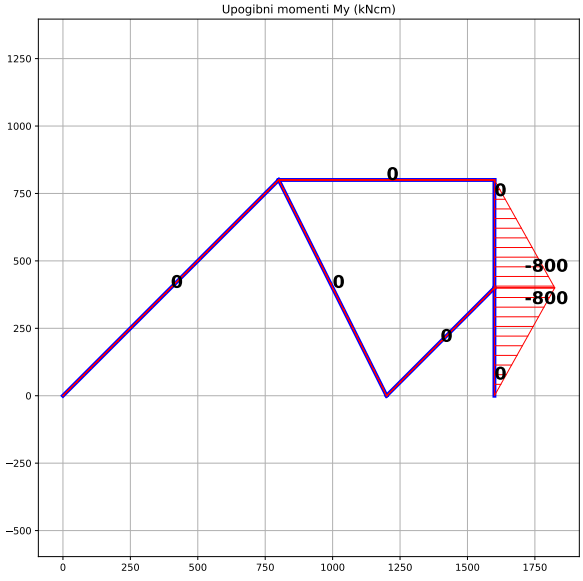
```
 SyT = 30*6*10.625 + 7.625^2/2*6
2086.921875
```

```
julia> sxz = -10*(-SyT)/df[:IyT]/6
11 LinearAlgebra.Transpose{Float64,Array{Float64,1}}:
 0.07482058252531608
```

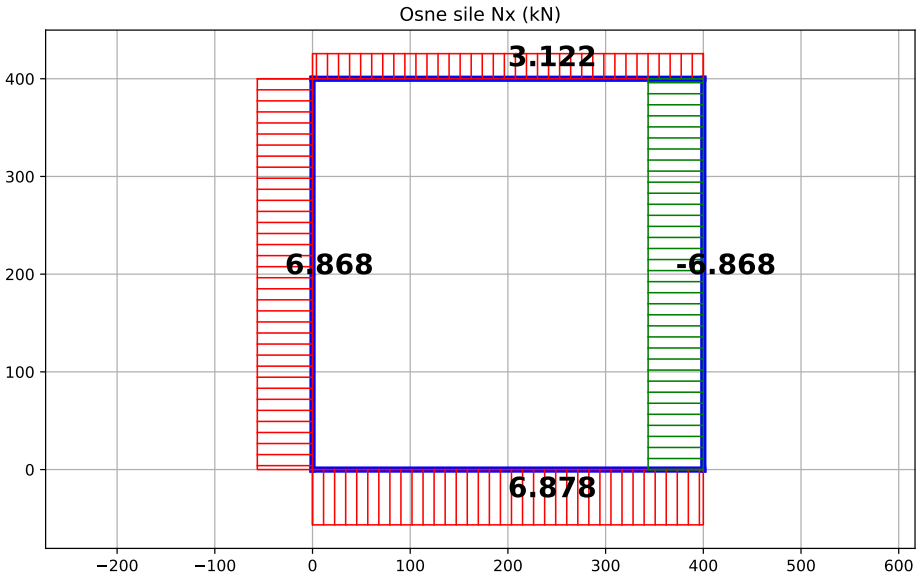
1. Diagrami notranjih sil N_x(kN), N_z(kN), M_y(kNcm), $u_{k_{\varphi=\infty}} = 12.836$ cm, $u_{k_\varphi} =$ cm.

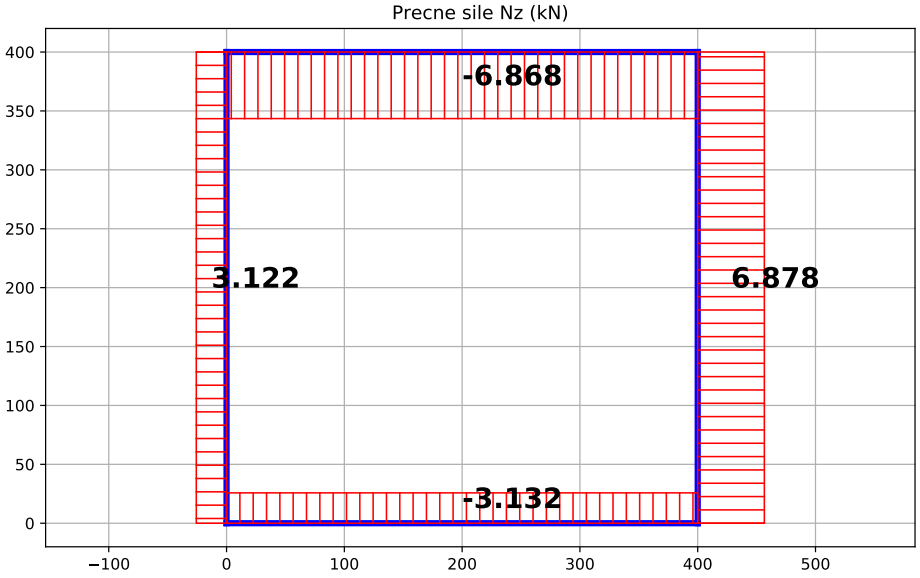


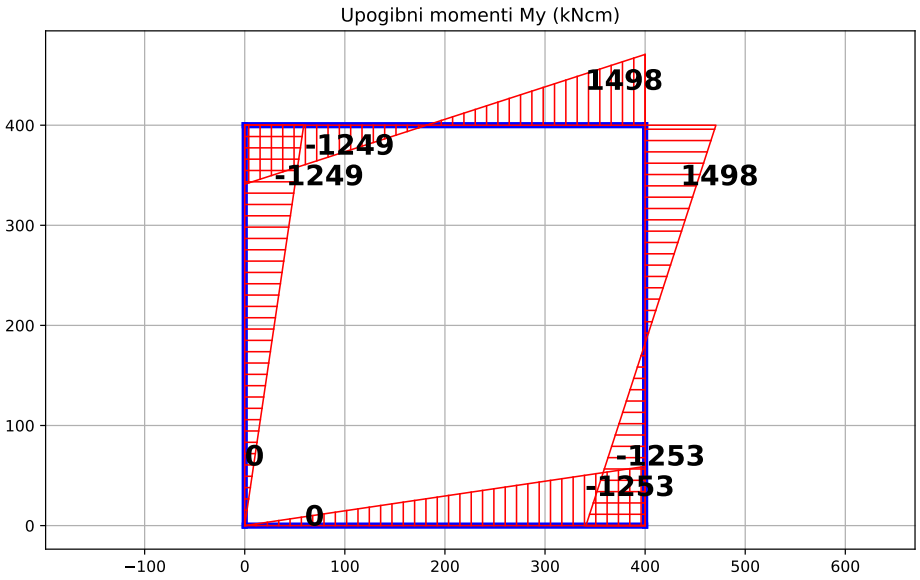




4. Diagrami notranjih sil N_x(kN), N_z(kN), M_y(kNcm):







3.

```
dT, aT, Ax, E = 20, 1e-5, 200, 20000
(20, 1.0e-5, 200, 20000)

julia> Nx = aT*dT*Ax*E
800.0
```

1. (30 %) Ravninski okvir na sliki je obtežen z vodoravno silo F. Vsi nosilci so iz enakega linearno elastičnega materiala.

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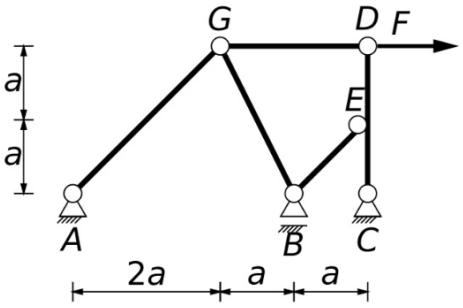
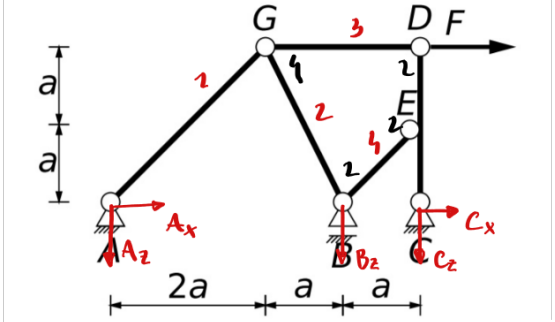
   - Določi vodoravni pomik vozlišča D. Upoštevaj samo vpliv upogibnih momentov na deformiranje.



Podatki: $a = 4$ m, $F = 10$ kN, $I_y = 20000$ cm^4, $A_x = 200$ cm^2, $E = 20000\ \frac{\text{kN}}{\text{cm}^2}$.

$n_{ps} = 15 - 5 - 10 = 0$



$\sum M_A = 0 \quad -F \cdot 2a - B_z \cdot 3a - C_z \cdot 4a = 0$

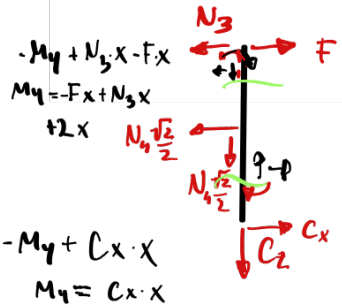
$\sum F_x = 0 \quad F + C_x + A_x = 0$

$\sum F_z = 0 \quad C_z + B_z + A_z = 0$

$A_z = -C_z - B_z$

$= N_4 \frac{\sqrt{2}}{2} + N \frac{\sqrt{2}}{2}$

$= N_4 \sqrt{2}$



$\sum F_x = 0 \quad F - N_3 - N_4 \frac{\sqrt{2}}{2} + C_x = 0$

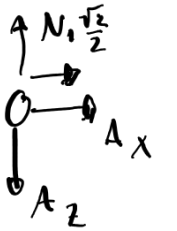
$\sum F_z = 0 \quad C_z + N_4 \frac{\sqrt{2}}{2} = 0$

$C_z = -N_4 \frac{\sqrt{2}}{2}$

$\sum M_D = 0 \quad -N_4 \frac{\sqrt{2}}{2} \cdot a + C_x \cdot 2a = 0$

$C_x = N_4 \frac{\sqrt{2}}{4}$

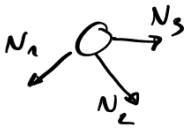
$A_x = -A_b = -N_4 \sqrt{2}$



$\sum F_x = 0 \quad A_x + N_1 \frac{\sqrt{2}}{2} = 0 \quad N_1 = -A_x \frac{2}{\sqrt{2}}$

$\sum F_z = 0 \quad A_z - N_1 \frac{\sqrt{2}}{2} = 0 \quad N_1 = A_z \frac{2}{\sqrt{2}}$

$A_z = -A_x$



$\sum F_x = 0$

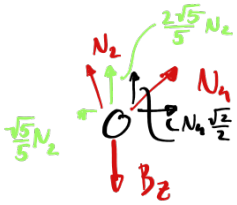
$N_3 - N_1 \frac{\sqrt{2}}{2} + N_2 \frac{\sqrt{5}}{5} = 0$

$N_3 = N_1 \frac{\sqrt{2}}{2} - N_2 \frac{\sqrt{5}}{5}$

$N_3 = -N_4 \cdot \sqrt{2} - N_4 \frac{\sqrt{2}}{2}$

$N_3 = N_4 \frac{3\sqrt{2}}{2}$

$N_3 = 12$



$\sum F_x = 0 \quad N_4 \frac{\sqrt{2}}{2} - \frac{\sqrt{5}}{5} N_2 = 0$

$N_2 = N_4 \frac{\sqrt{10}}{2}$

$\sum F_z = 0 \quad B_z - N_4 \frac{\sqrt{2}}{2} - \frac{2\sqrt{5}}{5} N_2 = 0$

$B_z = N_4 \frac{\sqrt{2}}{2} - N_4 \frac{2\sqrt{2}}{2}$

$\frac{2\sqrt{5}}{5} \cdot \frac{\sqrt{10}}{2} \sqrt{2}$

$B_z = -N_4 \frac{\sqrt{2}}{2}$

$$F = N_4\frac{\sqrt{2}}{2} - N_4\frac{\sqrt{2}}{4} - N_4\frac{3\sqrt{2}}{2}$$

$$10 = -N_4\frac{4\sqrt{2}}{4} - N_4\frac{\sqrt{2}}{4}$$

$$10 = -N_4\frac{5\sqrt{2}}{4}$$

$$N_4 = -5{,}657 \text{ kN}$$

$A_z = -8$

$A_x = 8$

$C_z = 4$

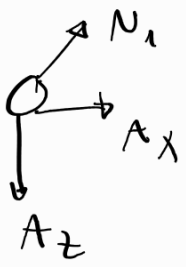
$C_x = -2$

$B_z = 4$

$N_1 = -11{,}314$

$N_2 = -8{,}945$

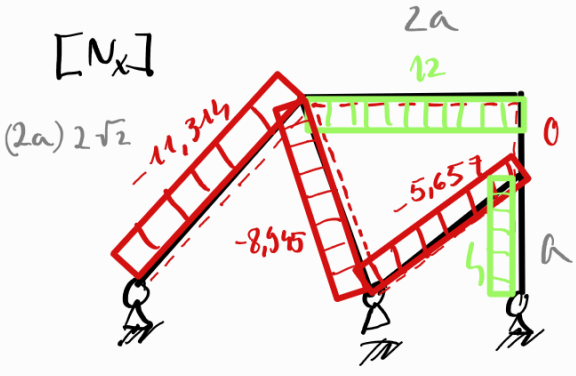
$N_3 = 12$

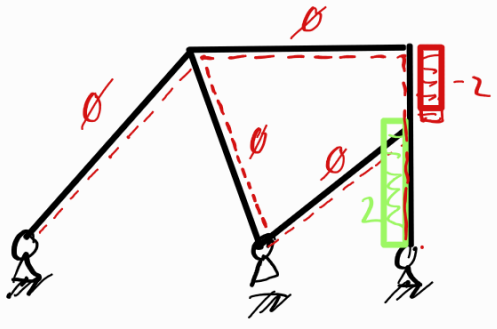


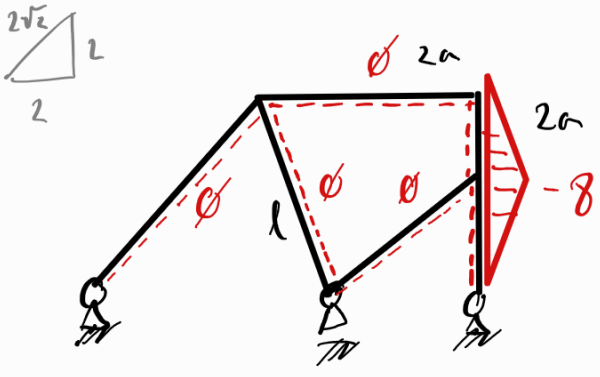
$$\Sigma F_x = 0 \qquad A_x + N_1\frac{\sqrt{2}}{2} = 0$$

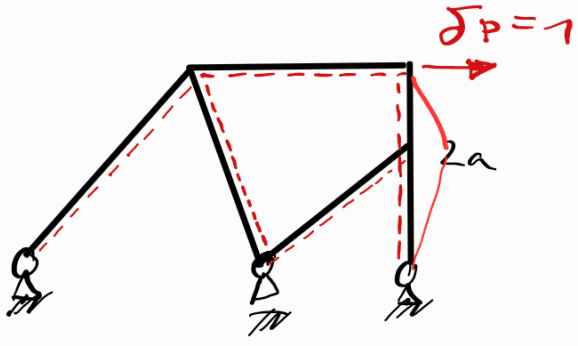
$$N_1 = -A_x\frac{2}{\sqrt{2}} = -11{,}314$$

$[N_x]$









$$u_D = \left(\frac{\frac{1}{3}\,l\cdot f\cdot g\cdot 2}{E\,A_x}\right) = \frac{\frac{1}{3}\cdot 400\cdot - \quad\cdot 8\cdot 2}{200 \quad 20000} = 4{,}2\overline{66}\cdot 10^{-4}$$

$$u_D = \sum \int_0^l \left(\frac{N_x\,N_x\,(\delta P=1)}{EA_x} + \frac{M_y\,M_y\,(\delta P=1)}{E\,I_y}\right) dx$$

$$u_D = \int_0^a \frac{-800\cdot -800\,400\cdot\frac{1}{3}\cdot 2}{E\,I_y}\,dx$$

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   - Določi upogibne momente in skiciraj diagrame upogibnih momentov.

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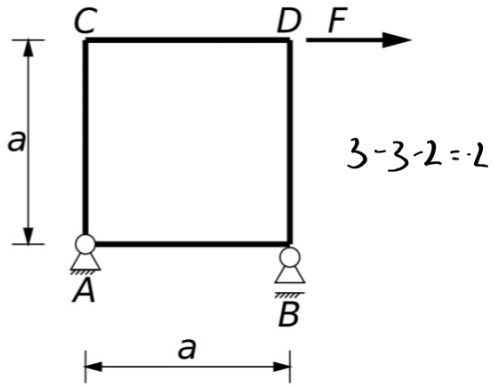
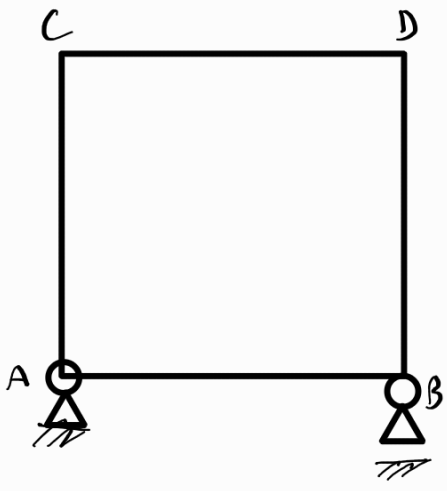
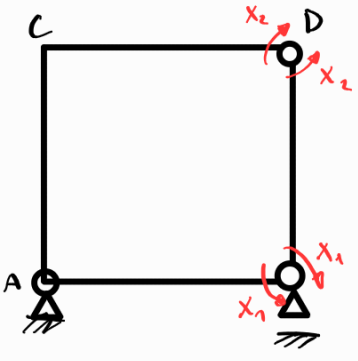
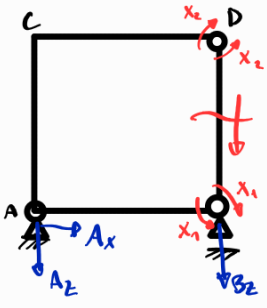


3-3-2=-2



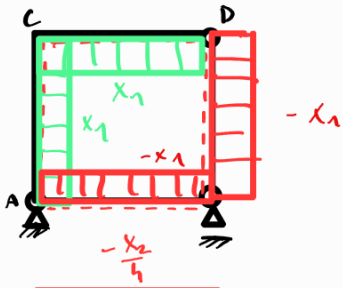
$h\tilde{p}s = 3-3-2 = -2$

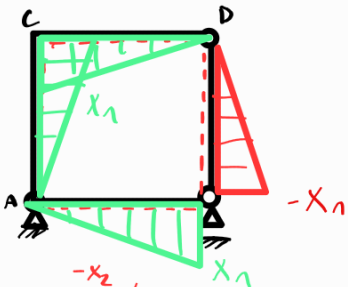


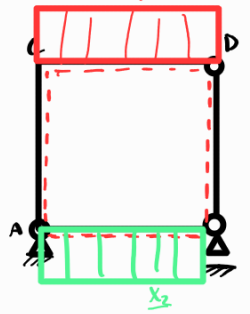


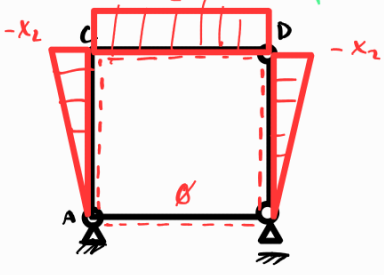
$A_x = 0$

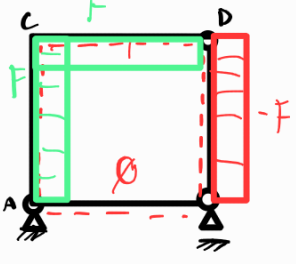
X1:

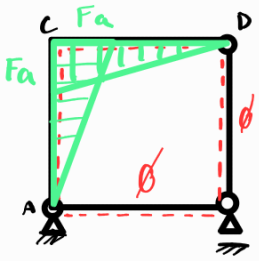












$X_1 = 0$:

D_x D_z

$\Sigma M_B\ X_1 - H_z \cdot a = 0$

$H_z = \frac{X_1}{a}$

$N_x - H_x = 0$

$N_x = H_x = -\frac{1}{4}$

$-H_z \cdot a + M_y$

$H_z = -H_x$

$H_x = -\frac{X_1}{a}$

$D_z = -H_z = -\frac{X_1}{a}$

$D_x = -H_x = \frac{X_1}{a}$

$M_y + D_x \cdot a = 0$

$M_y = -D_x \cdot a$

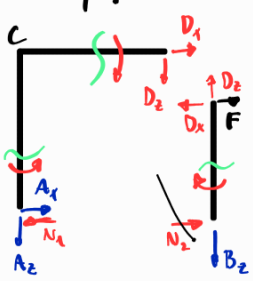
$M_y + A_x \cdot a$

$M_y = -A_x \cdot a$

$-M_1 - D_z \cdot a = 0$

$M_y = -D_z \cdot a = \frac{X_1}{a}$

F:



$\Sigma F_x \quad A_x = -F$

$\Sigma M_A = 0 \quad B_z \cdot a + F \cdot a = 0$

$B_z = -F$

$\Sigma F_z = 0 \quad A_z + B_z = 0$

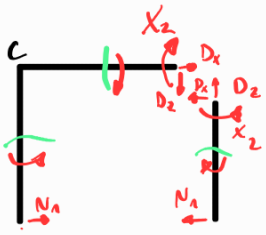
$A_z = F$

$D_z = B_z = -F \qquad N_2 = 0, \; N_1 = 0$

$D_x \cdot a - F \cdot a = 0$

$D_x = F$

X2:



$\Sigma M^B: \quad X_2 + D_x \cdot a = 0$

$D_x = \frac{X_2}{4}$

$D_x = -N_1$

$N_1 = +\frac{X_2}{4}$

$D_z = 0$

$M_y + N_1 \cdot a = 0$

$M_y = -N_1 \cdot a$

$M_y = \frac{X_2}{4} \cdot a$

$$a_{11} = \frac{1}{EA_x}\sum\int_0^l \bar{N}_{x_1}\bar{N}_{x_1}\,dx + \frac{1}{EI_y}\sum\int_0^l \bar{M}_{y_1}\bar{M}_{y_1} = \frac{(1^2 \cdot a \cdot 4)}{EA_1} + \frac{\left(\frac{1}{3}\cdot 1^2 \cdot 4a\right)}{EI_y} = 4{,}0133 \cdot 10^{-4}$$

$$a_{12} = \frac{1}{EA_x}\sum\int_0^l \bar{N}_{x_1}\bar{N}_{x_2}\,dx + \frac{1}{EI_y}\sum\int_0^l \bar{M}_{y_1}\bar{M}_{y_2}\,dx = \frac{\left(-\frac{1}{400}\cdot -\frac{1}{400}\cdot\right)}{EA_x} +$$

$$\frac{\left(\frac{1}{3}\cdot a\cdot(-1) + \frac{1}{6}\cdot a\cdot 1 + (-1)\cdot\frac{1}{2}\cdot a\right)}{I_y E} =$$

$$-1{,}25\cdot 10^{-9} - 6{,}66\cdot 10^{-7} = -6{,}654166\cdot 10^{-7}$$

$$a_{22} = \frac{1}{EA_x}\sum\int_0^l \bar{N}_{x_2}\bar{N}_{x_2}\,dx + \frac{1}{EI_y}\sum\int_0^l \bar{M}_{y_2}\bar{M}_{y_2}\,dx = \frac{\left(\frac{1}{400}\right)^2 \cdot 400 \cdot 2}{EA_x} + \frac{2\cdot\frac{1}{3}\cdot(1)^2\cdot a + (1)^2\cdot a}{EI_y}$$

$$1{,}25\cdot 10^{-9} + 1{,}66\cdot 10^{-6} = 1{,}6679\cdot 10^{-6}$$

$$b_2 = \sum\int_0^l \frac{\bar{N}_{x_2}N_x}{EA_x}\,dx + \sum\int_0^l \frac{\bar{M}_{y_2}M_{y_2}}{EI_y}\,dx = \frac{\frac{1}{3}Fa\cdot(-1)\cdot a + \frac{1}{2}\cdot Fa\cdot(-1)\,a}{E\cdot I_y} + \frac{-\frac{1}{400}\cdot F\cdot 400\cdot}{E\cdot A_x}$$

$$= -3{,}33\cdot 10^{-3} - 2{,}5\cdot 10^{-6} = -3{,}3358\cdot 10^{-3}$$

$$b_1 = \frac{3\cdot F\cdot\frac{1}{400}\cdot a}{E\cdot A_x} + \frac{Fa\cdot 2\cdot\frac{1}{3}\,a}{E\,I_y}$$

$$7{,}5\cdot 10^{-6} + 2{,}66\cdot 10^{-3} = 2{,}6741\cdot 10^{-3}$$

$a_{11}x_1 + a_{12}x_2 + b_1 = 0 \qquad x_1 = \frac{-a_{12}x_2 - b_1}{a_{11}}$

$a_{21}x_1 + a_{22}x_2 + b_2 = 0$

$$a_{21}\frac{-a_{12}x_2-b_1}{a_{11}}+a_{22}x_2+b_2=0$$

$$\frac{a_{12}^2x_2+b_1a_{12}}{a_{11}}=a_{22}x_2+b_2$$

$$a_{12}^2x_2+b_1a_{12}=a_{22}x_2a_{11}+b_2a_{11}$$

$$x_2=\frac{b_2a_{11}-b_1a_{12}}{a_{12}^2-a_{22}\cdot a_{11}}$$