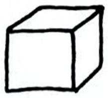
④ 3D modeli

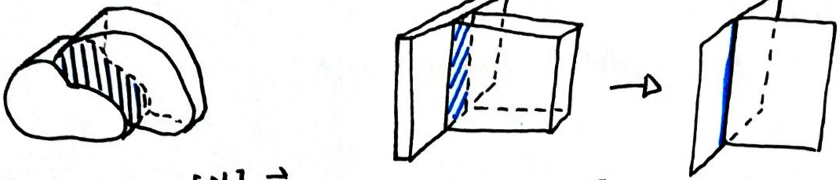


* ne moremo ničesar zanemariti *

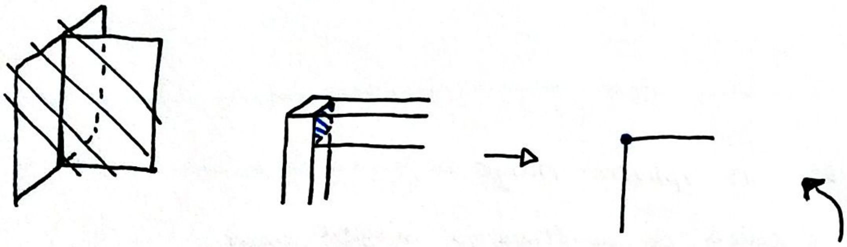
VPLIVI

1) volumska obtežba

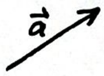
2) ploskovna obtežba $\vec{F}_s$ $\left[\frac{N}{m^2}\right]$



3) linijska obtežba $\left[\frac{N}{m}\right]$ – $\vec{f}$

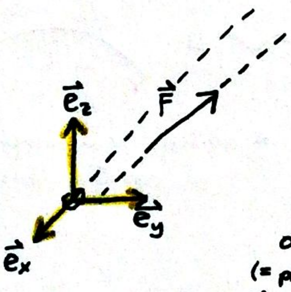


4) točkovne obtežbe = SILE = so vplivi, ki delujejo na relativno majhni površini zato jih opišemo v točki. ("sil v naravi ni. So samo poenostavljen model"). Merimo jih v N. Sila je vektor → ima velikost in smer, matematično jo opišemo z vektorji.



$\vec{F}, \vec{F}_1, \vec{G}, \vec{H}$...

enota: [N]



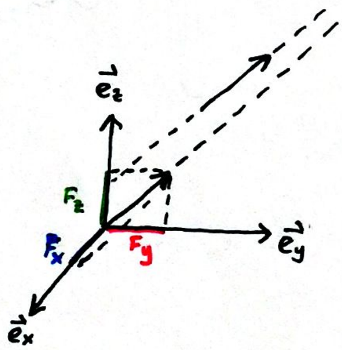
VPELJEMO BAZO PROSTORA

ONB

orto (= pravokotna): $\vec{e}_x \perp \vec{e}_y$, $\vec{e}_y \perp \vec{e}_z$, $e_x \perp e_z$

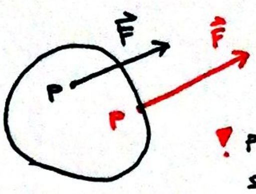
normirana (= dolžina 1): $|\vec{e}_x| = 1$, $|\vec{e}_y| = 1$, $|\vec{e}_z| = 1$

baza

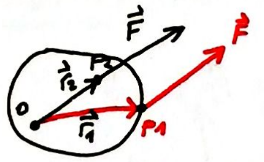


$$\vec{F} = \underbrace{F_x \vec{e}_x}_{\vec{F}_x} + \underbrace{F_y \vec{e}_y}_{\vec{F}_y} + \underbrace{F_z \vec{e}_z}_{\vec{F}_z}$$

$$\vec{F} \mapsto (F_x, F_y, F_z) \qquad \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$$



❗ PAZI: z vzporednim premikom sil se spremeni njen vpliv na telo❗



moment sile na točko

MOMENT = vektorska količina, ki je pravokotna na $\vec{F}$ in na vektor med opazovalcem in prijemališčem sile (smer)

(velikost) $|\vec{OP}| \cdot |\vec{F}| \cdot \sin\alpha$ (vektorski produkt)

$\vec{M} = \vec{OP} \times \vec{F}$

$\vec{M} = \vec{r} \times \vec{F}$ (~~navor~~ moment je ročica krat sila → pazi vrstni red besed)

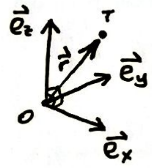
pravilo desnega vijaka - desnosučnega

1.2.

Osnovni pojmi

3-razsežni prostor

$T \rightarrow \underbrace{(x, y, z), \ (T_x, T_y, T_z)}_{\text{koordinate}}$ točka

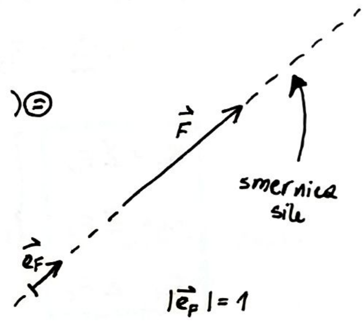


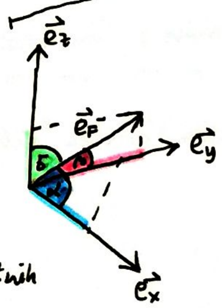
$\vec{r} = x\vec{e_x} + y\vec{e_y} + z\vec{e_z} \rightsquigarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ vektorji

sile: $\vec{F} = F \cdot \vec{e_F} = F_x \vec{e_x} + F_y \vec{e_y} + F_z \vec{e_z} = F(e_{Fx} \vec{e_x} + e_{Fy} \vec{e_y} + e_{Fz} \vec{e_z}) \ominus$

komponente sile $\vec{F}$

$\ominus \ F(\cos\alpha \vec{e_x} + \cos\beta \vec{e_y} + \cos\gamma \vec{e_z})$





$\vec{F_x} = F_x \vec{e_x}$
$\vec{F_y} = F_y \vec{e_y}$
$\vec{F_z} = F_z \vec{e_z}$

imenujemo komponentne sile, ki delujejo vzdolž koordinatnih osi

$\begin{bmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \end{bmatrix}$ smerni kosinusi

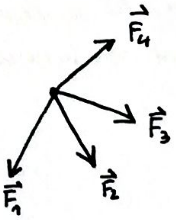
$|\vec{F_x}| = F_x$

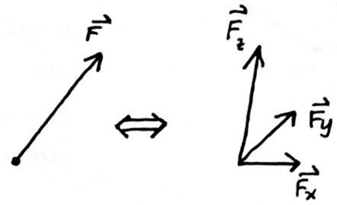
$|\vec{F_y}| = F_y$

$|\vec{F_z}| = F_z$

* sila je vektor, velikost sile pa skalar-številka *

! POMEMBNO: učinek sile $\vec{F}$, ki deluje v nekem delcu je enak kot če v istem delcu delujejo komponentne sile $\vec{F_x}$, $\vec{F_y}$ in $\vec{F_z}$ → temu rečemo razstavljanje sil (velja trikotniška neenakost), velja tudi pitagorov izrek $F^2 = F_x^2 + F_y^2 + F_z^2$





$\vec{F} = \vec{F_x} + \vec{F_y} + \vec{F_z}$

razstavljanje sil

$\vec{R} = \vec{F_1} + \vec{F_2} + \ldots + \vec{F_n} = \sum_i F_i$

rezultanta sil, postopek pa je sestavljanje sil

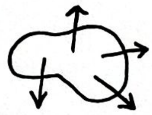
$\vec{R} \parallel \vec{a}$ (2. N. Z.) $\boxed{\vec{R} = m \cdot a}$ * velja na nivoju delca *

delec miruje (ali se giblje nepospešeno) $\Leftrightarrow$ $\boxed{\vec{R} = \vec{0}}$ (1. N. Z.) osnovna enačba statike

ENAČBA "MIROVANJA" DELCA

delec

telo – prijemališče sile!



$\vec{R} = \sum \vec{F_i}$

$\vec{R} = R_x \vec{e_x} + R_y \vec{e_y} + R_z \vec{e_z}$

$\vec{F_i} = F_{ix} \vec{e_x} + F_{iy} \vec{e_y} + F_{iz} \vec{e_z}$

$$\left.\boxed{\begin{aligned} R_x &= \sum F_{ix} \\ R_y &= \sum F_{iy} \\ R_z &= \sum F_{iz} \end{aligned}}\right\}$$ sest. sil v komponentni obliki

1. N. Z. sestavljajo 3 ravnotežne enačbe delca:

$$\left.\begin{aligned} \sum F_{ix} &= 0 \\ \sum F_{iy} &= 0 \\ \sum F_{iz} &= 0 \end{aligned}\right. \quad \begin{pmatrix} \sum x \\ \sum y \\ \sum z \end{pmatrix}$$

REZULTANTA MOMENTOV $\vec{M}_R^o$

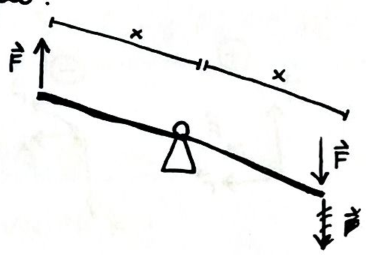
$$\vec{M}_R^o = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \dots + \vec{r}_n \times \vec{F}_n = \sum_i \vec{r}_i \times \vec{F}_i = \sum_i \vec{M}_i^o$$

je vsota ~~vseh~~ momentov vseh sil, ki delujejo na nek sistem teles glede na izbrano opazovališče.

RAVNOTEŽJE TELESA

$\vec{R} = 0$ zagotavlja mirovanje delca

kaj pa telo?



$\vec{R} = F\vec{e}_z - F\vec{e}_z = \vec{0}$

telo $\underline{\underline{\text{ne}}}$ miruje

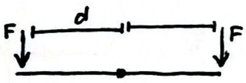
$\Sigma F = 0 \rightarrow$ vsota sil ne zadošča

rešitev zahtevamo ravnotežje momentov!

$$\boxed{\begin{array}{l} \vec{M}_R = 0 \\ \Sigma \vec{r}_i \times \vec{F}_i = \vec{0} \end{array}}$$

MOMENTNI RAVNOTEŽNI POGOJ

$\vec{M}_R = -F_x\vec{e}_z - F_x\vec{e}_z = -2F_x\,\vec{e}_z \neq \vec{0} \rightarrow$ gugalnica ni v ravnotežju



$$\left.\begin{array}{l} \vec{M}_R^o = Fd\vec{e}_z - Fd\vec{e}_z = 0 \\ \vec{R} = 2F\vec{e}_y \end{array}\right\}$$

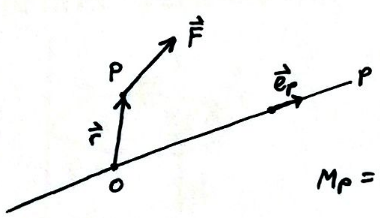
UGOTOVITEV: potrebujemo oboje

$$\boxed{\begin{array}{l} \vec{R} = \vec{0} \\ \vec{M}_R^o = \vec{0} \end{array}}$$

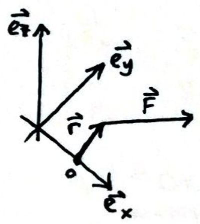
RAVNOTEŽNA POGOJA

MOMENT SILE NA OS

= število, ki ga določimo kot projekcijo momenta na poljubno točko osi na to os (oz. smer osi). Pogosto rečemo moment okrog osi p.



$$M_p = \vec{e}_p \cdot (\vec{r} \times \vec{F})$$



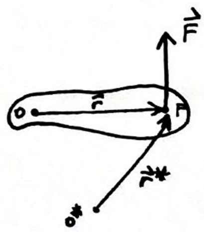
moment $\vec{F}$ okrog x

$$M_x = \vec{e}_x (\vec{r} \times \vec{F}) = \begin{vmatrix} 1 & 0 & 0 \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = yF_z - zF_y$$

$$M_y = \begin{vmatrix} 0 & 1 & 0 \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = zF_x - xF_z$$

$$M_z = \begin{vmatrix} 0 & 0 & 1 \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = xF_y - yF_x$$

Moment sile na točko

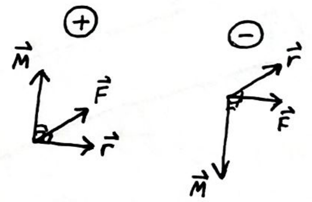


def.: $\boxed{\vec{M}^o = \vec{r} \times \vec{F}}$

$\vec{M}^{o*} = \vec{r}^* \times \vec{F}$

! PAZI: moment je odvisen od opazovališča!

$$\vec{M}^o = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$ med seboj $\perp$

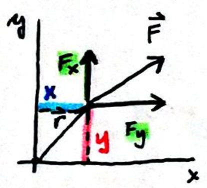


1) $\vec{M} \perp \vec{r}$, $\vec{M} \perp \vec{F}$

2) $\boxed{|\vec{M}| = |\vec{r}|\,|\vec{F}| \sin\alpha}$

3) PREDZNAK: smer desnega vijaka (palec r, kazalec F, sredinec M)

* POSEBEN PRIMER: Sile v ravnini



$$\vec{F} = F_x \vec{e}_x + F_y \vec{e}_y + \underbrace{F_z \vec{e}_z}_{=0}$$

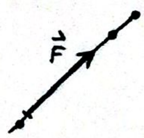
$$\vec{r} = x\vec{e}_x + y\vec{e}_y + \underbrace{z\vec{e}_z}_{=0}$$

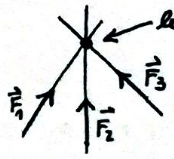
$$\vec{r} \times \vec{F} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ x & y & 0 \\ F_x & F_y & 0 \end{vmatrix} = \begin{vmatrix} x & y \\ F_x & F_y \end{vmatrix} \vec{e}_z = (xF_y - yF_x)\vec{e}_z$$

$M^o = +xF_y - yF_x$ → predznaki v prsti

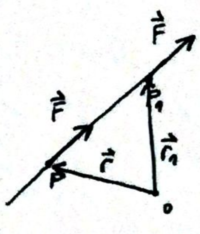
LASTNOSTI MOMENTOV

1) $\vec{r} \parallel \vec{F} \Rightarrow \vec{M} = 0 = \vec{r} \times \vec{F}$





2) premik sile vzdolž smernice



$$\vec{M}^o_{F_P} = \vec{r} \times \vec{F}$$

$$\vec{M}^o_{F_{P_1}} = \vec{r}_1 \times \vec{F} = (\vec{r} + \vec{PP_1}) \times \vec{F} = \vec{r} \times \vec{F} + \overbrace{\vec{PP_1} \times \vec{F}}^{\substack{=0 \\ \text{vzporedna}}} = \vec{M}^o_{F_P}$$

če silo premikamo vzdolž smernice povzroča enak moment

$\vec{M} = (yF_z - zF_y)\vec{e}_x + (zF_x - xF_z)\vec{e}_y + (xF_y - yF_x)\vec{e}_z = M_x\vec{e}_x + M_y\vec{e}_y + M_z\vec{e}_z$

momenti okrog pravokotnih osi so ravno komponente vektorja $\vec{r} \times \vec{F}$!

POSLEDICA: ravnotežne enačbe (2 vektorski enačbi) lahko zapišemo kot 6 skalarnih enačb

$$\boxed{\begin{array}{ll} \sum F_{ix} = 0 & (\Sigma x) \\ \sum F_{iy} = 0 & (\Sigma y) \\ \sum F_{iz} = 0 & (\Sigma z) \\ \sum y_i F_{iz} - z_i F_{iy} = \sum M^o_{ix} = 0 & \\ \sum z_i F_{ix} - x_i F_{iz} = \sum M^o_{iy} = 0 & \\ \sum x_i F_{iy} - y_i F_{ix} = \sum M^o_{iz} = 0 & \end{array}}$$

6 ravnotežnih enačb (RE)

(ΣM^o_x)

(ΣM^o_y)

(ΣM^o_z)

POSEBNI SISTEMI SIL

1 SILE V RAVNINI xy

$\vec{F}_i = F_{ix}\vec{e}_x + F_{iy}\vec{e}_y + 0$

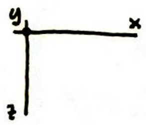
$R_z = 0$

$M_{ix} = 0 \qquad M^o_{Rx} = 0$

$M_{iy} = 0 \qquad M^o_{Ry} = 0$

3 pogoji so samodejno izpolnjeni $\longrightarrow$ $\boxed{\begin{array}{l} \Sigma x = 0 \\ \Sigma y = 0 \\ \Sigma M^o_z = 0 \end{array}}$ RE v ravnini xy

SILE V RAVNINI xz



$\vec{F}_i = (F_{ix}, 0, F_{iz})$

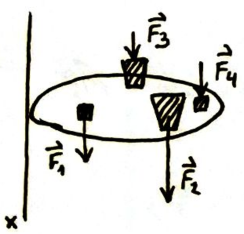
$M^o_i = (0, M^o_{iy}, 0)$

$\Sigma x = 0$

$\Sigma y = 0$

$\Sigma M^o_y = 0$

2 SISTEM VZPOREDNIH SIL

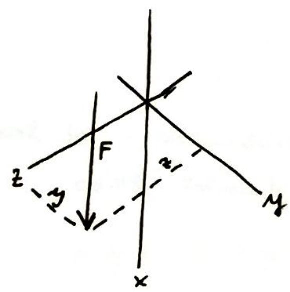


$\vec{F}_i = F_{ix}\vec{e}_x$

$\vec{R} = R_x\vec{e}_x + 0\vec{e}_y + 0\vec{e}_z$

$\Sigma x = 0 \qquad \Sigma M^o_{Ry} = 0$

$\Sigma F_{ix} = 0 \qquad \Sigma M^o_{Rz} = 0$



$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & y & z \\ F_x & 0 & 0 \end{vmatrix} = 0 \cdot \vec{e}_x + zF_x\vec{e}_y - yF_x\vec{e}_z$$

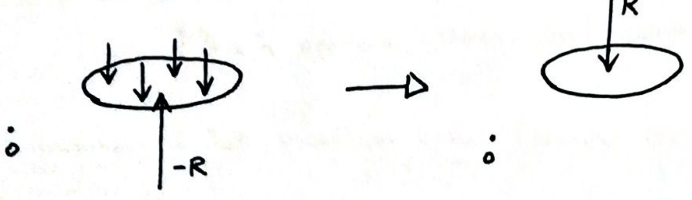
$M^o_{Rx} = 0$

$M^o_{Ry} = \sum z_i F_{ix}$

$M^o_{Rz} = -\sum y_i F_{ix}$

$$\boxed{\begin{array}{l} \sum F_{ix} = 0 \\ \sum z_i F_{ix} = 0 \\ -\sum y_i F_{ix} = 0 \end{array}}$$

$R = \Sigma F_i$

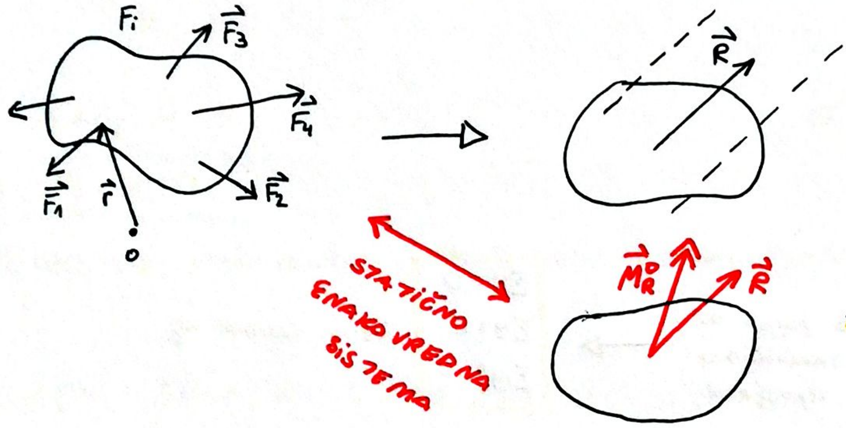


kam postaviti R_x sistema vzporednih sil, da bo imela enak učinek na togo telo?

$$R_x = \Sigma F_{ix}$$
$$z_i \cdot R_x = \sum_i z_i \cdot F_{ix} = \Sigma M^o_{iy}$$
$$-y_i R_x = -\sum_i y_i F_{ix} = \Sigma M^o_{iz}$$

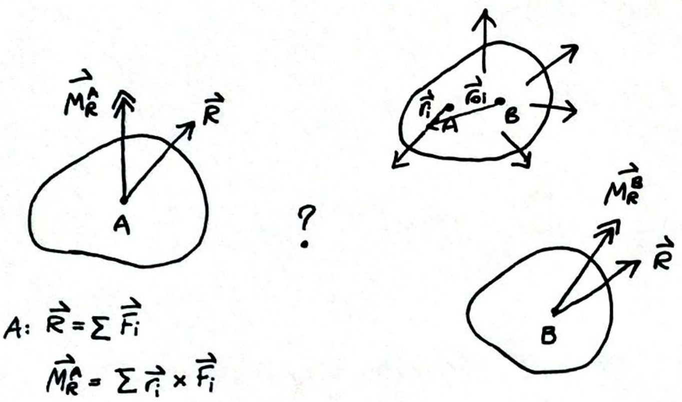
$$\boxed{z_T = \frac{\Sigma z_i F_{ix}}{\Sigma F_{ix}} \qquad y_T = \frac{\Sigma y_i F_{ix}}{\Sigma F_{ix}}}$$

KOORDINATE TEŽIŠČA



$$\vec{R} = \sum_i \vec{F_i}$$
$$\vec{M^o_R} = \sum_i \vec{r_i} \times \vec{F}$$

sistema sil sta **STATIČNO ENAKOVREDNA**, če imata enak učinek na telo. To pomeni, da imata enako rezultanto sil in momentov



A: $\vec{R} = \Sigma \vec{F_i}$

$\vec{M^A_R} = \Sigma \vec{r_i} \times \vec{F_i}$

B: $\vec{R} = \sum_i \vec{F_i}$

$\vec{M^B_R} = \Sigma \vec{\rho_i} \times \vec{F_i}$

$$\vec{\rho_i} = \vec{BA} + \vec{r_i}$$

pomembno je od kod gledamo točko

$$\vec{M^B_R} = \Sigma \vec{BA} \times \vec{F_i} + \Sigma \vec{r_i} \times \vec{F_i} = \vec{BA} \times \Sigma \vec{F_i} + \vec{M^A_R}$$

$$\boxed{\vec{M^B_R} = \vec{BA} \times \vec{R} + \vec{M^A_R}}$$

RAVNOTEŽJE

$$\left.\begin{array}{l}\vec{R} = 0 \\ \vec{M^o_R} = \vec{0}\end{array}\right\} \text{IN!}$$

to pomeni, da je rezultanta momentov 0 v **poljubni točki** idea: $\left.\begin{array}{l}\vec{R}=\vec{0}\\ \vec{M}_R^A=\vec{0}\end{array}\right\}$ bi radi nadomestili z momentnim pogojem v drugi točki

$\vec{M}_R^A=\vec{0} \longleftrightarrow \begin{array}{l}\vec{M}_R^A=0\\ \vec{M}_R^B=0\end{array}$

če velja $\vec{R}=\vec{0}$ in $\vec{M}_R^A=\vec{0}$ potem je $M_R^B=\vec{BA}\times\vec{0}+\vec{0}=\vec{0}$

$\vec{R}=0 \rightarrow \vec{M}_R^B=\vec{BA}\times\vec{0}+\vec{0}=\vec{0}$

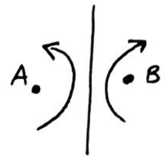
$\vec{M}_R^A=\vec{0}$

$\vec{M}_R^A=\vec{0}$ in $\vec{M}_R^B=\vec{0}$ kaj lahko povemo o $\vec{R}$?

$\cancel{\vec{M}_R^B}=\vec{BA}\times\vec{R}+\cancel{\vec{M}_R^A}$

$\vec{BA}\times\vec{R}=0$

① $\vec{R}=\vec{0}$ ② $\vec{BA}\parallel\vec{R}$



$\vec{M}_R^A=\vec{0}$ in $\vec{M}_R^B=\vec{0}$ žal ne zagotovi, da je $\vec{R}=\vec{0}$

rešitev: DODATNA ENAČBA

$\vec{e}_{BA}\circ\vec{R}=0$ (1 skalarna enačba)

$\vec{e}_\xi\cdot\vec{R}=0;\ \vec{e}_\xi \not\perp \vec{BA}$ (ta smer je že zajeta v momentnih pogojih)

! ① NADOMESTNI RAVNOTEŽNI POGOJ

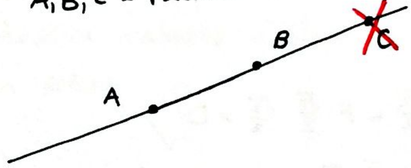
$$\boxed{\begin{array}{c}\vec{M}_R^A=\vec{0}\\ \vec{M}_R^B=\vec{0}\\ R_\xi=\vec{e}_\xi\cdot\vec{R}=0\end{array}}\quad \vec{e}_\xi \not\perp \vec{BA}\ \Bigg\}\ \text{7 skalarnih enačb} \rightarrow \text{6 jih potrebujemo, 1 pa za kontrolo}$$

projekcija rezultante sil na poljubno smer, ki ni pravokotna na zveznico skozi A in B

② NADOMESTNI RP

$$\boxed{\begin{array}{c}\vec{M}_R^A=\vec{0}\\ \vec{M}_R^B=\vec{0}\\ \vec{M}_R^C=\vec{0}\end{array}}$$

A, B, C - različne točke



A, B, C oklepajo TRIKOTNIK → A, B in C NE ležijo na isti premici!

9 enačb, 6 neodvisnih

----*

Sile v ravnini

$\vec{F}_i = (F_{ix}, F_{iy}, F_{iz})$

$\vec{M}_i^T = (0, 0, \vec{M}_z^T)$

$R_z = 0$

$\vec{M}^T_{Rx} = 0$

$\vec{M}^T_{Ry} = 0$

$R_x = 0 \qquad \Sigma X = 0$

$R_y = 0 \qquad \Sigma Y = 0$

$R_z = 0 \qquad \Sigma Z = 0$

① Nadomestni RP

$\vec{M}^A_{Rz} = 0$

$\vec{M}^B_{Rz} = 0$

$R_\xi = 0 \quad \xi \not\perp AB$

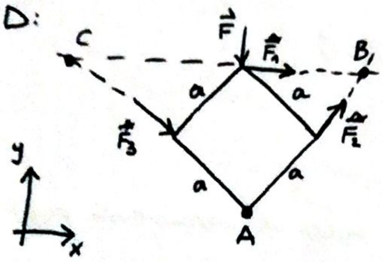
② NADOMESTNI RP

$\vec{M}^A_{Rz} = 0$

$M^B_{Rz} = 0$

$M^C_{Rz} = 0$

*** ZGLED:**



F=100N

določi ostale sile tako, da bo sistem v ravnotežju.

OSNOVNI POGOJI

$\Sigma X: F_1 + F_2 \frac{\sqrt{2}}{2} + F_3 \frac{\sqrt{2}}{2} = 0$ ← ker pod $\varphi = 45°$

$\Sigma Y: F_2 \frac{\sqrt{2}}{2} - F_3 \frac{\sqrt{2}}{2} - F = 0$

$\cancel{\Sigma Z}$

Izberemo točko A, saj nanjo ne deluje nobena sila, ki bi jo vrtela

$\Sigma M^A: -F_1 a\sqrt{2} = 0$

1. NADOMESTNI RP

$\Sigma M^A: -F_1 a\sqrt{2} = 0$

$\Sigma M^B: F\sqrt{2}a + F_3 \cdot 2a = 0$

ΣX ali ΣY

2. NADOMESTNI RP

$\Sigma M^A: -F_1 a\sqrt{2} = 0 \qquad \boxed{F_1 = 0}$

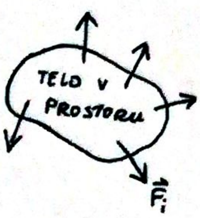
$\Sigma M^B: F a\sqrt{2} + F_3 \cdot 2a = 0 \qquad \boxed{F_3 = -F\frac{\sqrt{2}}{2}}$

$\Sigma M^C: -F a\sqrt{2} + F_2 \, 2a = 0 \qquad \boxed{F_2 = F\frac{\sqrt{2}}{2}}$

+ KONTROLA

$\Sigma X: 0 + F\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} - F\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2} = 0$ ✓

$\Sigma Y: F\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2} + F\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2} - F = 0$ ✓

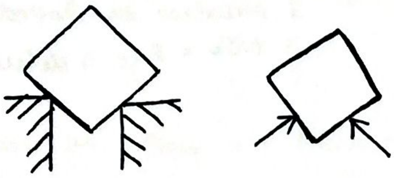


GRP v prostoru
3RP v ravnini } določa št. enačb

10.8

PODPORE IN VEZI

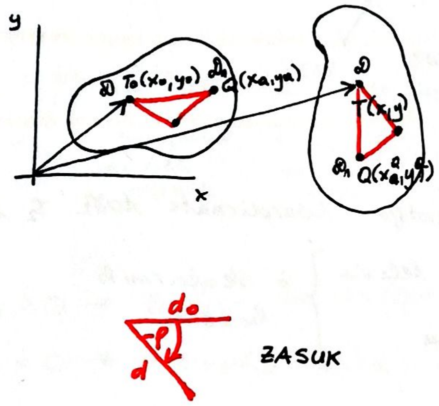
PRAVILO O SPROŠČANJU PODPOR IN VEZI



podpore in vezi bomo odstranili, če rečemo tudi da izoliramo telo in njihov vpliv nadomestili s silami in momenti.

Število neodvisnih količin, ki opiše lego togega telesa

2.1.



ugotovitev: pri togem ravninskem telesu natanko poznam lege vseh delcev telesa, če poznam legi vsaj dveh različnih delcev.

podatki: x x^Q / y y^Q } 4 podatki

dodatna informacija: razdalja med D_0 in D_1 se ne spreminja

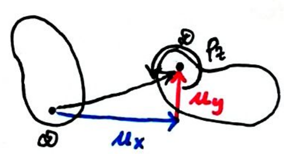
$|T_0 Q_0| = |TQ|$ (vezna enačba)

$$\sqrt{(x_0^Q - x_0)^2 + (y_0^Q - y_0)^2} = \sqrt{(x^Q - x)^2 + (y^Q - y)^2}$$

$\Rightarrow$ eno količino lahko izrazim z ostalimi 3 $\rightarrow$ TEŽKO, RAČUNSKO NEUGODNO :(

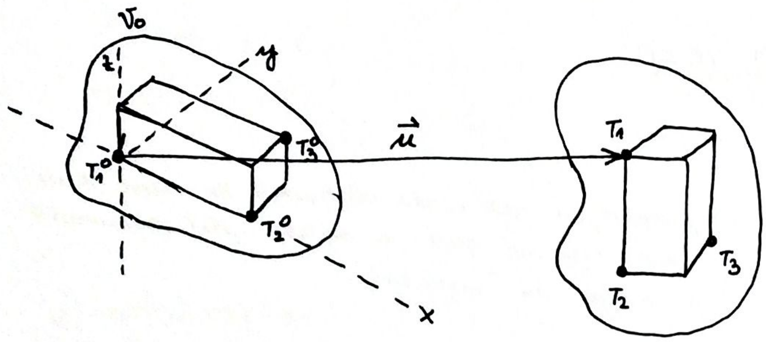
IDEJA: x^Q in y^Q nadomestimo z eno samo skalarno količino: **ZASUK** telesa v ravnini $= \varphi$

$(x, y, \varphi) \rightsquigarrow (\underbrace{u_x, u_y}_{\text{pomik v smeri } x \text{ in } y}, \varphi_z)$ ← zasuk v smeri z



3 neodvisne skalarne količine natanko določajo lego ravninskega togega telesa

TELO V PROSTORU



$T_i(x_i, y_i, z_i)$
→ 9 podatkov za koordinate 3 točk = lege 3 delcev

ti podatki niso neodvisni.

VEZI
$\overline{T_1T_2} = k_1$
$\overline{T_1T_3} = k_2$
$\overline{T_2T_3} = k_3$
} razdalje se ne spreminjajo – so konstante

$9 - 3 = 6$ neodvisnih podatkov

9 podatkov iz koordinat točk ; vezi

⇒ vpeljemo 3 nove količine: $\varphi_x, \varphi_y, \varphi_z$ ki nadomestijo koordinate točk T_2 in T_3

$\varphi_x, \varphi_y, \varphi_z$ = zasuki poljubnega delca na togem telesu
u_x, u_y, u_z = pa bodo zasuki za pomike tega delca
} 6 skalarnih količin

št. neodvisnih količin s katerimi opišemo problem imenujemo ŠTEVILO PROSTOSTNIH STOPENJ (N_{ps})
(= DOF = degrees of freedom)

SKLEP: prosto togo telo v prostoru ima 6 prostostnih stopenj. In togo telo v ravnini ima 3 prostostne stopnje.
Sistem k-teles
3k ravnina
6k prostor

Podpore

• PODPORE = telesa (običajno manjša), ki nekemu telesu preprečijo določene pomike in/ali zasuke. 2.2.

Rečemo da podpora telesu odvzame prostosne stopnje:

m_{opse} = št. odvzetih prostorskih stopenj v podpore

zato podpora deluje na telo z ustreznimi silami, momenti ali obtežbo

• dogovor: omejili se bomo na TOČKOVNE podpore, ki ustrezajo ~~dovolj~~ majhnim podporam v naravi

vezi

• vezi = telesa, ki združujejo 2 ali več teles v sistem oz. konstrukcijo.
s tem preprečijo določene pomike in/ali zasuke.
rečemo, da vez sistemu teles odvzame prostostne stopnje. oznaka: m_{OPSV} ... št. odvzetih prostostnih stopenj v vezi

* številki m_{OPSP} in m_{OPSV} sta fiksni - sta lastnosti podpor oz. vezi

$$1 \leq m_{OPSP} \leq 6$$

$$1 \leq m_{OPSV} \leq 6$$

računsko število prostostnih stopenj

$$\boxed{\tilde{m}_{PS} = 6K - \underbrace{\sum m_{OPSP}}_{\text{po vseh podporah}} - \underbrace{\sum m_{OPSV}}_{\text{po vseh vezeh}}}$$

⊕ $\tilde{m}_{ps} > 0$ → labilna konstrukcija, mehanizem → nezanimive za nas

$\tilde{m}_{ps} = 0$ → minimalna podpora

⊖ $\tilde{m}_{ps} < 0$ → ? (tipično za gradbene konstrukcije) *pentagon*

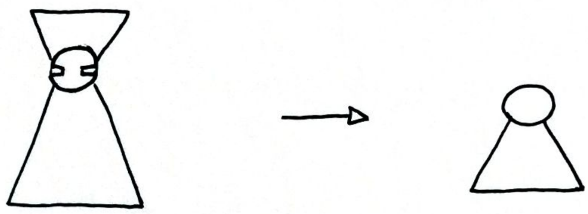
m_{ps}, $m_{ps,DEJANSKO}$ = dejansko št. prostostnih stopenj je število neodvisnih količin, ki jih potrebujemo za opis spremembe lege sistema (podprtih in povezanih) teles.

$m_{ps} \geq 0$

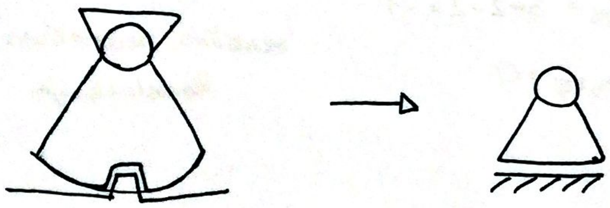
*PILON = nosilni steber

• PRIMERI

• PODPORE

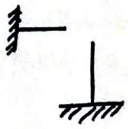


NEPOMIČNA ČLENKASTA PODPORA



POMIČNA ČLENKASTA PODPORA

PODPORE V RAVNINI

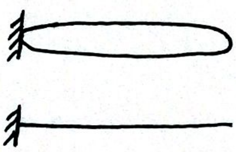
SKICA	KAJ DOVOLI?	KAJ PREPREČI?	n_{opsp}
 toga	$\emptyset$	u_x, u_y, φ	3
 členkasta podpora	zasuke	u_x, u_y	2
 pomična členkasta podpora	zasuk in premik	drugi premik	1
 "grablice"	en pomik	zasuk drugi pomik	2

20.10.2021

n_{opsp} ENO TOGO ~~ZNANO~~ TELO

prostor: $6 - \sum n_{opsp} \leq 0$ ... potrebni pogoj za mirovanje

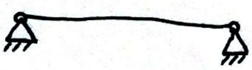
ravnina: $3 - \sum n_{opsp} \leq 0$



$\tilde{n}_{ps} = 3 - 3 = 0$

PREVISNI NOSILEC
ali
KONZOLA

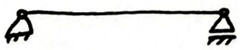
*!



$\tilde{n}_{ps} = 3 - 2 - 2 = -1$

$n_{ps,DEJ} = 0$

statično nedoločna konstrukcija



prostoležeči nosilec

$\tilde{m}_{ps} = 3-2-1=0$

$m_{ps,DEJ} = 0$

$\tilde{m}_{ps} = 3-1-1-1=0$

$m_{ps,DEJ} = 1$

labilna konstrukcija

KLASIFIKACIJA

- $\tilde{m}_{ps} > 0 \rightarrow m_{ps} > 0 \rightarrow$ DINAMIKA
  LABILNA
- $\tilde{m}_{ps} = 0 \rightarrow m_{ps} = 0$ STATIČNO DOLOČENA KONSTRUKCIJA — TAKOJ
  $\rightarrow m_{ps} > 0$ LABILNA
- $\tilde{m}_{ps} < 0 \rightarrow n_{ps} = 0$ STATIČNO NEDOLOČENA KONSTRUKCIJA — NADALJEVANJE
  $\rightarrow n_{ps} > 0$ LABILNA

- mirovanje oz. $u=0$ ali $\varphi=0$ v neki točki dosežemo s silo ali momentom s katerim podpora deluje na telo.

POSTOPEK: podporo odstranimo, njen vpliv pa nadomestimo z ustreznimi silami, ki jim rečemo reakcije.

PODPORA	KAJ PREPREČI?		REAKCIJE
A	vse	$u_x^A=0$ $u_z^A=0$ $\varphi_y^A=0$	A_x, A_z, M_A
B	premika	$u_x^B=0$ $u_z^B=0$	B_x, B_z
C	prečni premik	$u_z^C=0$	C_z
D	prečni premik zasuk	$u_z^D=0$ $\varphi^D=0$	M_D, D_z

neznane reakcije +sile v vezeh so del neznank problema, torej

- če je $\tilde{m}_{ps}=0$ je ravnotežnih enačb natanko toliko kot je neznanih reakcij in sil v vezeh 3K (6K)
- če je $\tilde{m}_{ps}<0$ potrebujemo dodatne enačbe

zaenkrat se omejimo na statično določene konstrukcije.

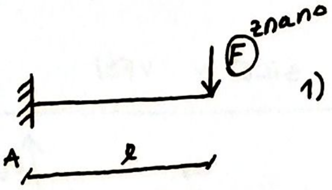
ALGORITEM

1) $\tilde{m}_{ps}=0$ • VEZI

↓ če je res - nadaljujem

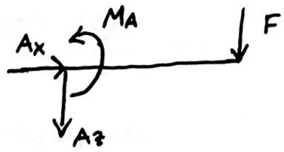
2) podpore odstranimo, vezi izrežemo in jih nadomestimo s silami
3) zapišemo ravnotežne enačbe za vsako togo telo v sistemu in jih rešimo
4) Kontrola!

PRIMER ①:



1) $\tilde{m}_{ps} = 0$

2)



glej tabelo

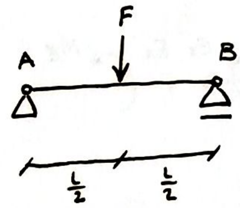
PREVISNI NOSILEC

3) $\Sigma X: \quad A_x + 0 \stackrel{!}{=} 0 \longrightarrow \boxed{A_x = 0}$

$\Sigma Y: \quad A_z + F = 0 \longrightarrow \boxed{A_z = -F}$

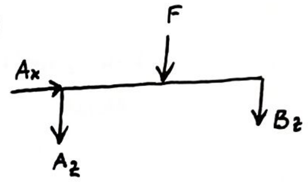
$\Sigma M^A: \quad M_A - Fl = 0 \longrightarrow \boxed{M_A = Fl}$

②: PROSTOLEŽEČI NOSILEC



1) $\tilde{m}_{ps} = 0$

2)



3) 1. nadomestni ravnotežni pogoji

$\Sigma M^A: \quad -B_z l - F\frac{L}{2} = 0 \longrightarrow \boxed{B_z = -\frac{F}{2}}$

$\Sigma M^B: \quad A_z l + F\frac{L}{2} = 0 \longrightarrow \boxed{A_z = -\frac{F}{2}}$

NAROBE!

ΣZ: pazi to je demonstracija napačne izbire!

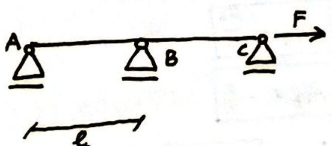
$\Sigma Z: \quad A_z + B_z + F = 0$

$-\frac{F}{2} - \frac{F}{2} + F = 0$

$0 = 0$ → gre za odvisno enačbo - to je bila kontrola ne pa rešitev

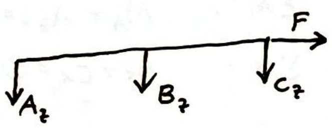
$\Sigma X: \quad \boxed{A_x = 0}$

③:



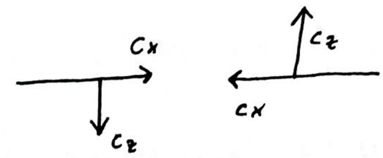
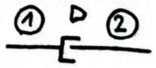
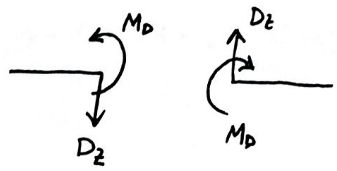
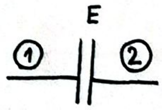
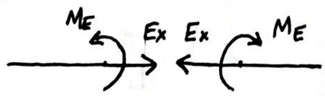
1) $\tilde{m}_{ps} = 0$ (nemo $m_{ps} = 1$)

2)

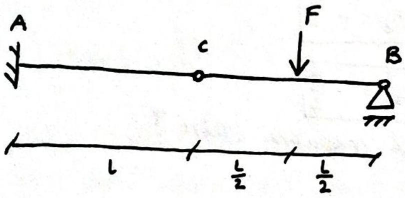


3) $F = 0$ ⟶⟵ protislovje → labilna konstrukcija

dobimo singularni sistem enačb → na izpitu samo dokažeš da gre za labilno konstrukcijo - in bo ok?

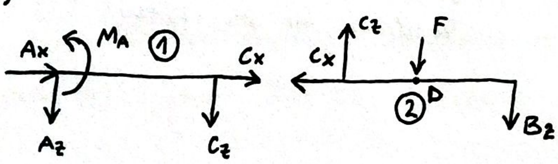
VEZ	KAJ POENOTI?	SILE V VEZI	Mopsb
 členek	oba pomika		* 3. N. Z. "akcija - reakcija" * 2
 drsma vez	prečni pomik zasuk		2
 strižni členek	osni pomik zasuk		2

* PRIMERI ①:



1) $\tilde{m}_{ps} = 2\cdot3-3-1-2=0$

2)



6 neznank

2 x 3 enačbe

3) ①: $\Sigma M^A: \quad M_A - C_z l = 0 \rightarrow \boxed{M_A = \frac{Fl}{2}}$

$\Sigma M^C: \quad M_A + A_z l = 0 \rightarrow \boxed{A_z = -\frac{F}{2}}$

$\Sigma X: \quad A_x + C_x = 0 \rightarrow \cancel{C_x}= \boxed{A_x = 0}$

②: $\Sigma X: \quad \boxed{C_x = 0}$

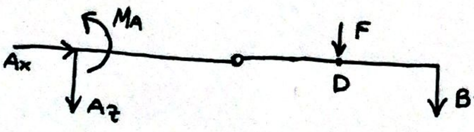
$\Sigma M^C: \quad -F\frac{L}{2} - B_z l = 0 \rightarrow \boxed{B_z = \frac{-F}{2}}$

$\Sigma M^B: \quad F\frac{l}{2} - C_z l = 0 \rightarrow \boxed{C_z = \frac{F}{2}}$

KONTROLA

1) $\Sigma Z: \quad F + B_z - C_z = F - \frac{F}{2} - \frac{F}{2} = 0$ ✓

2)

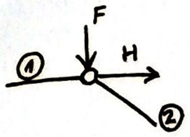


$\Sigma X: \quad A_x = 0$ ✓

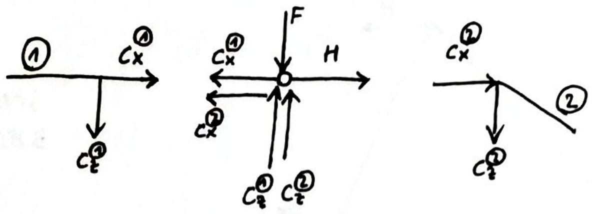
$\Sigma Z: \quad A_z + B_z + F = 0$ ✓

$\Sigma M^D: \quad A_z \cdot 3\frac{l}{2} + M_A - B_z\frac{l}{2} = 0$ ✓

VEZ



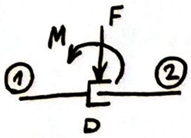
členek



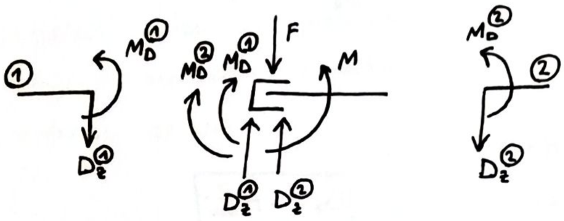
RAVNOTEŽJE V VEZI

$\Sigma X: \; C_x^{(1)} + C_x^{(2)} = H$

$\Sigma Z: \; C_z^{(1)} + C_z^{(2)} = F$



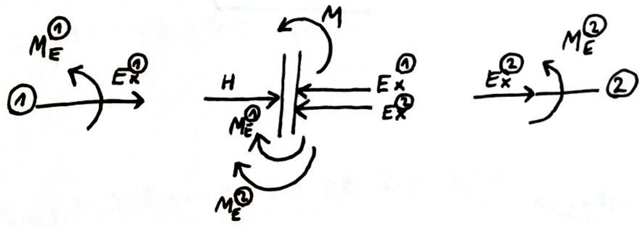
drsna vez



$D_z^{(1)} + D_z^{(2)} = F$

$M_D^{(1)} + M_D^{(2)} = M$

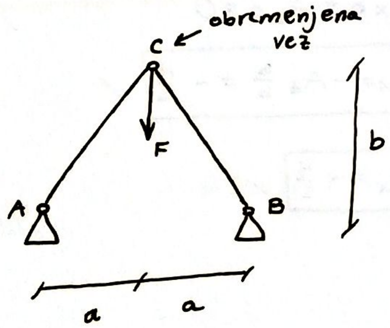
strižni členek



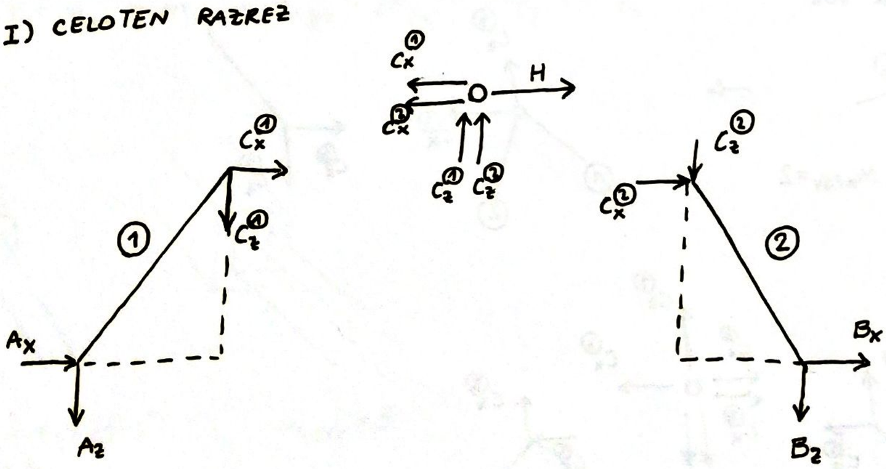
$E_x^{(1)} + E_x^{(2)} = H$

$M_E^{(1)} + M_E^{(2)} = M$

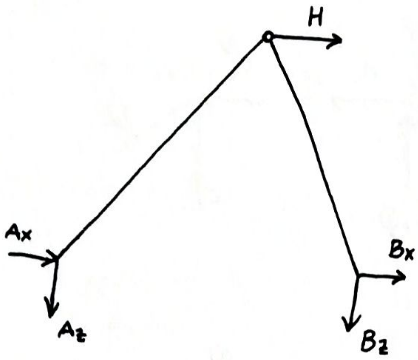
*PRIMER 2:



I) CELOTEN RAZREZ



II) POSTOPNO REŽANJE



4NN
3EN

$\Sigma x: \quad A_x + B_x + H = 0$

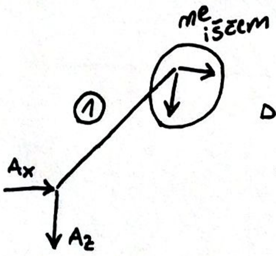
$\Sigma z: \quad A_z + B_z = 0$

$\Sigma M^B: \quad A_z \cdot 2a - Hb = 0$

$\boxed{B_z = -H\frac{b}{2a}}$

$\boxed{A_z = H\frac{b}{2a}}$

AC



DODATNA ENAČBA

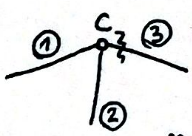
$\Sigma M^C_{AC}: \quad A_x b + A_z a = 0$

$\boxed{A_x = -A_z \frac{a}{b} = -\frac{H}{2}}$ ✓

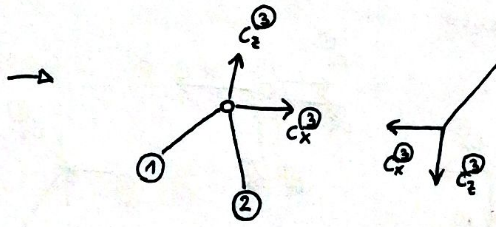
$\boxed{B_x = -\frac{H}{2}}$ ✓

⊕ DODATNO

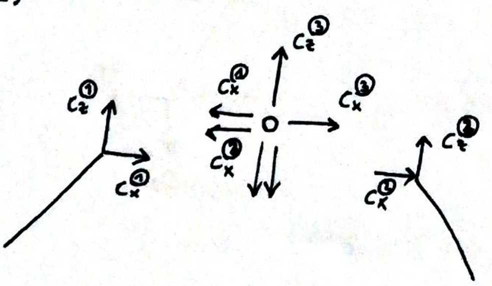
sestavljena vez



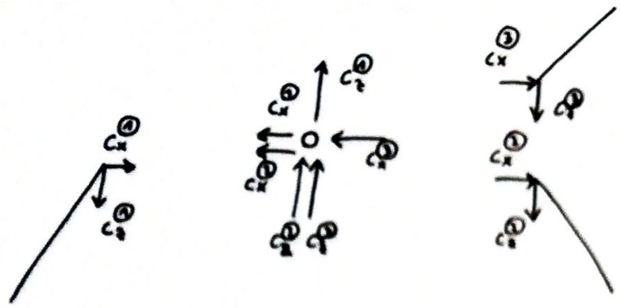
$m_{opsv} = 2$



I)



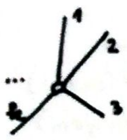
11)



$$m_{opsv} = \overset{(6K)}{3K} - \text{št. različnih vodoravnih pomikov}$$
$$- \text{št. različnih navpičnih pomikov}$$
$$- \text{št. različnih zasukov}$$

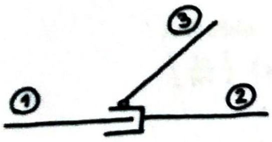
vse

$3 \cdot 3 - 1 - 1 - 3 = 4 = m_{opsv}$



$m_{opsč} = 3 \cdot k - 1 - 1 - k = 2k - 2 = 2(k-1)$

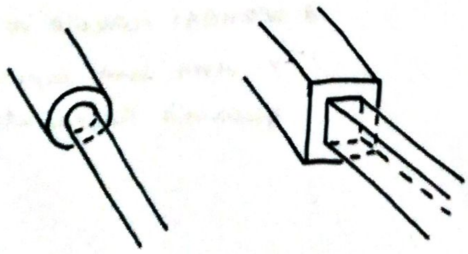
$$\boxed{m_{opsč} = 2(k-1)}$$



$3 \cdot 3 - 2 - 1 - 2 = 4$

$m_{opsv} = 6k - m_{mx} - m_{my} - m_{mgz} - m_{px} - m_{py} - m_{pz}$

prostor

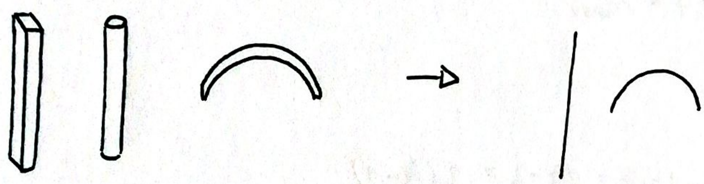


$6 \cdot 2 - 2 - 1 - 1 - 2 - 1 - 1$
$= 4$

$m_{opsv} = 5$

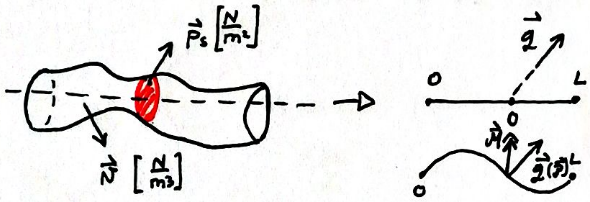
LINIJSKI NOSILCI 3.

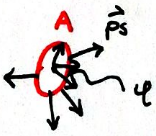
= so elementi (konstrukcijski), pri katerih je ena dimenzija pomembnejša



RAČUNSKI MODEL zato prečni prerez "skrči" v točko.

take elemente bomo opisovali z DALJICO (ravni) ali KRIVULJO (ukrivljeni)





$$\int_\ell \vec{p}_s\, ds = \vec{q} \left[\frac{N}{m}\right]$$

$$+ \int_A \vec{N}\, dA$$

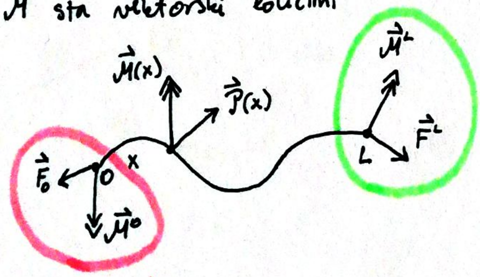
x... parameter osi

$(\vec{P})$ $\vec{q}$... linijska obtežba

$$\vec{q} = \vec{q}(x) \left[\frac{N}{m}\right]$$

$$\vec{M} = \int_\ell \vec{r} \times \vec{p}_s\, ds + \int_A \vec{r} \times \vec{N}\, dA$$

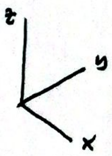
$\vec{P}$ in $\vec{M}$ sta vektorski količini



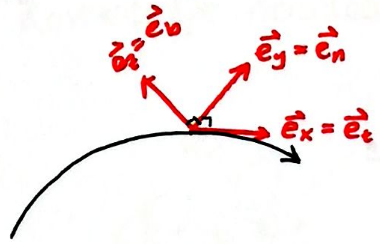
REAKCIJE in SILE V VEZEH

REAKCIJE in SILE V VEZEH

* OPOMBA: reakcije in sile v vezeh smo zapisali v globalnih koordinatah *



GLOBALNI KOORDINATNI SISTEM



$\{x, y, z\}$ bo LOKALNI KOORDINATNI SISTEM, izbran tako, da os x leži v smeri tangente, y in z pa sta nanjo pravokotne

$$\vec{P} = P_x \vec{e}_x + P_y \vec{e}_y + P_z \vec{e}_z$$

P_x ... osno porazdeljena obtežba

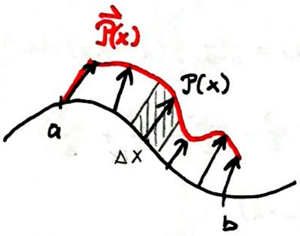
P_y, P_z ... prečno porazdeljena obtežba

$$\vec{M} = M_x \vec{e}_x + M_y \vec{e}_y + M_z \vec{e}_z$$

M_x ... torzijski porazdeljen moment $\left[\frac{Nm}{m}\right]$

M_y, M_z ... upogibna porazdeljena momenta

- upoštevanje porazdeljene obtežbe v ravnotežnih enačbah

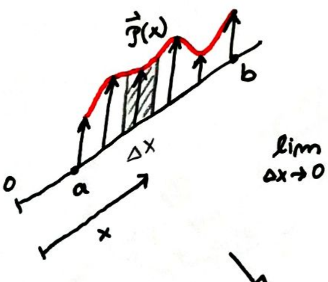


$$\vec{P}_i \approx \vec{P}(x) \cdot \Delta x$$

$$R_x = \sum \vec{P}_i = \sum_{\Delta x \in [a,b]} \vec{P}(x) \Delta x$$

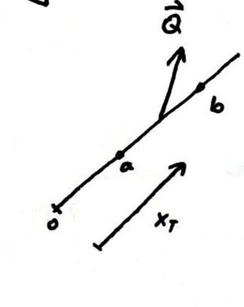
$$\lim_{\Delta x \to 0} = \boxed{\int_a^b P(x)\, dx = \vec{Q}\ [N]}$$

- naj bo os ravna



$$\vec{M}_R^0 = \sum x \vec{P}(x) \Delta x$$

$$\vec{M}_R^0 = \int_a^b x \vec{P}(x)\, dx$$

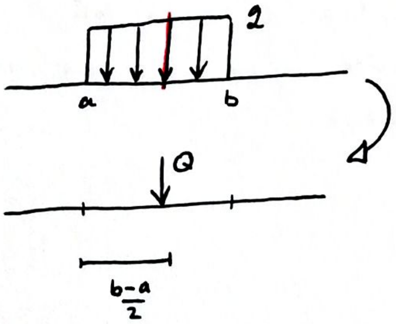


$$x_T \vec{Q} = \vec{M}_R^0$$

$$\boxed{x_T = \frac{\int_a^b x \vec{P}(x)\, dx}{\int_a^b \vec{P}(x)\, dx}}$$

- za določanje reakcij in sil v vezeh lahko porazdeljeno obtežbo nadomestimo z rezultantno silo $\vec{Q}$, ki jo postavimo na razdalji x_T od levega krajišča.

* PRIMERI : ⓐ enakomerna obtežba v prečni smeri

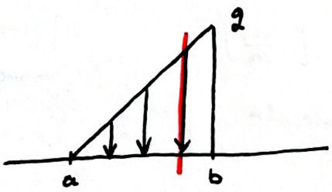


$Q = q\,(b-a)$

$x_T = a + \frac{b-a}{2}$

zunaj $[a,b]$

ⓑ trikotna obtežba



$Q = \int_a^b p(x)\,dx = q\,\frac{1}{b-a}\int_a^b (x-a)\,dx = \frac{q}{b-a}\left(\frac{x^2}{2} - ax\right)\Big|_a^b \;\ominus$

$p(x) = q\,\frac{x-a}{b-a}$

$\ominus\; \frac{q}{b-a}\left(\frac{b^2}{2} - ab - \frac{a^2}{2} + a^2\right) =$

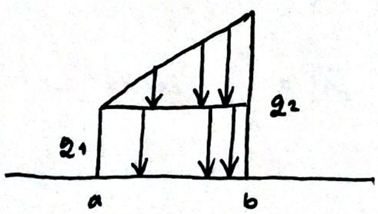
$= \frac{q}{2(b-a)}\left(b^2 - 2ab + a^2\right) = q\,\frac{b-a}{2}$

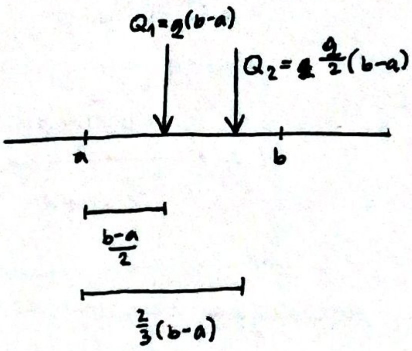
$\int_a^b x\,p(x)\,dx = \frac{q}{b-a}\int_a^b (x^2 - ax)\,dx = \frac{q}{b-a}\left(\frac{x^3}{3} - \frac{ax^2}{2}\right)\Big|_a^b =$

$= \frac{q}{b-a}\left(\frac{b^3}{3} - \frac{ab^2}{2} - \frac{a^3}{3} + \frac{a^3}{2}\right)$

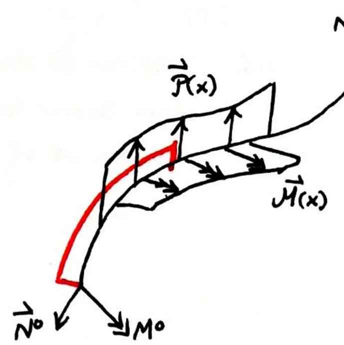
$x_T = \frac{2}{3}(b-a)$

ⓒ trapezna obtežba

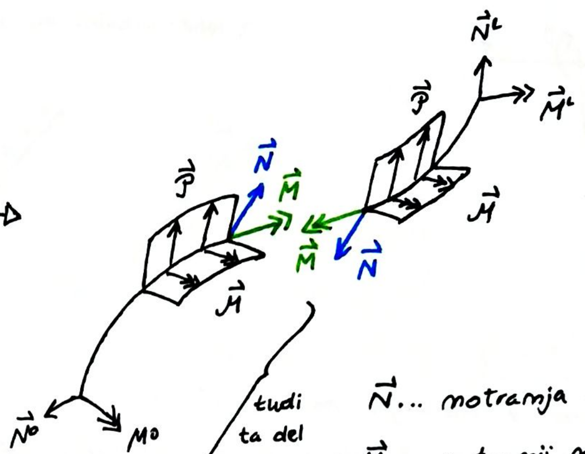




Ravnotežje nosilca



nosilec prerežemo pri x →



tudi ta del mora biti v ravnotežju

$\vec{N}$... notranja sila

$\vec{M}$... notranji moment

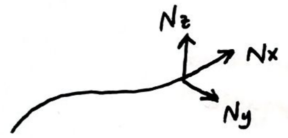
⇒ $\overset{(3)}{\vec{N}}$ in $\overset{(3)}{\vec{M}}$ morata zadoščati ravnotežnim pogojem (6)

⇒ št. neznank in enačb se ujema

DOGOVORI:

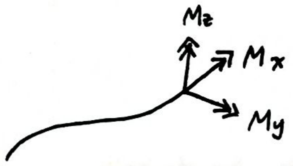
① notranje sile in momente izražamo v lokalnih koordinatah

$$\vec{N} = N_x \vec{e}_x + N_y \vec{e}_y + N_z \vec{e}_z$$



N_x... notranja osna sila

N_y, N_z... notranji prečni sili

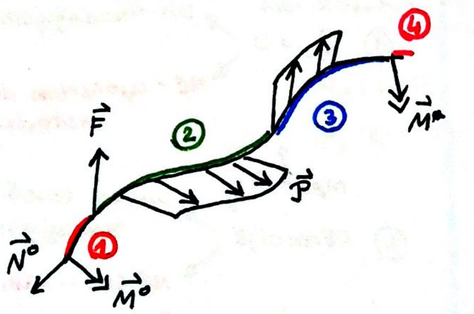


M_x... notranji torzijski moment

M_y, M_z... notranja upogibna momenta

$$\vec{M} = M_x \vec{e}_x + M_y \vec{e}_y + M_z \vec{e}_z$$

② polja



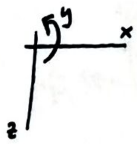
meje med polji določajo:

1) začetek / konec porazdeljene obtežbe

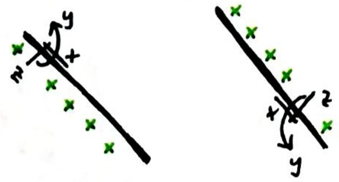
2) točkovne sile in momenti

3) lomi / kolena osi nosilca

③ definicija o pozitivni strani nosilca (za ravninske nosilce)



z izbiri lokalne osi x:



def.: pozitivna ⊕ stran je stran kamor kaže lokalna os z!

④ dogovor o risanju notranjih sil

rišemo na konstrukcijo v lokalnih koordinatah:

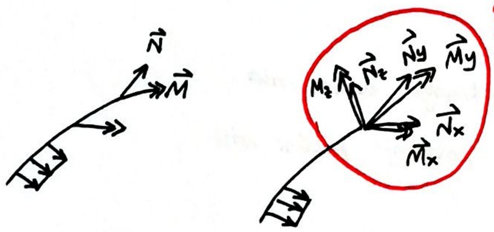
⊕ vrednosti rišemo na ⊕ stran

⊖ vrednosti rišemo na ⊖ stran

posledica: diagrami momentov so neodvisni od izbire smeri lokalne osi x

28.10.2021

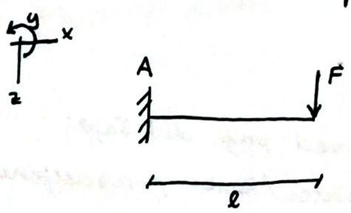
• če je $\tilde{m}_{ps} = 0$ in $m_{ps} = 0$



iščemo → uporabimo ravnotežne pogoje

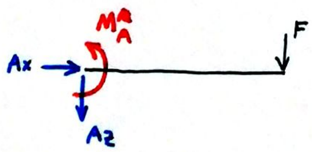
* PRIMERI: ① Določi notranje sile za naslednje preproste ravninske konstrukcije z znano obtežbo.

PREVISNI NOSILEC S SILO NA PROSTEM KONCU



1) $\tilde{m}_{ps} = 3 - 3 = 0$

2)



$\Sigma X:\quad A_x = 0$

$\Sigma Z:\quad A_z = -F$

$\Sigma M^A:\quad M_A = Fl$

\+ kontrola

ALGORITEM

① $\tilde{m}_{ps} = 0$ → DA - nadaljujem / → NE - uporabimo druge metode

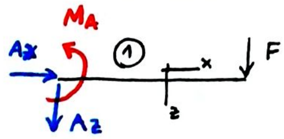
$m_{ps} \overset{?}{=} 0$

② REAKCIJE → LAHKO DOLOČIMO - nadaljujem / → NE GRE - LABILNA konstrukcija - končamo

③ POLJA IN LOKALNE KOORDINATE

④ NOTRANJE SILE PO POLJIH

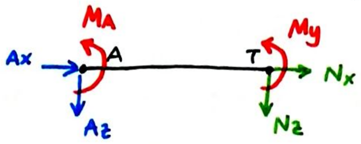
⑤ DIAGRAMI



3) 1 polje → polje 1 $x \in [0, \ell]$

4)

LEVI DEL →x DESNO



momentni pogoj vedno pišemo v prerezni točki T

$\Sigma x: \quad N_x + A_x = 0 \rightarrow \boxed{N_x = 0}$

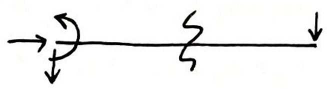
$\Sigma z: \quad N_z + A_z = 0 \rightarrow \boxed{N_z = F}$

$\Sigma M^T: \quad M_y + M_A + A_x \cdot x = 0 \rightarrow \boxed{M_y = -F\ell + Fx}$

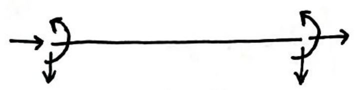
$M_y(0) = -F\ell$

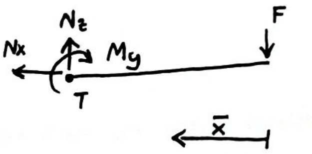
$M_y(\ell) = 0$ → momenta na prostem koncu ni

DESNI DEL



levo





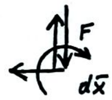
$\bar{x} = \ell - x$

3.N.Z.

$\Sigma x: \quad -N_x + 0 = 0 \quad \boxed{N_x = 0}$ ✓

$\Sigma z: \quad -N_z + F = 0 \quad \boxed{N_z = F}$ ✓

$\Sigma M^T: \quad -M_y - F \cdot \bar{x} = 0 \quad \boxed{M_y = -F\bar{x}} = -F(\ell - x)$ ✓



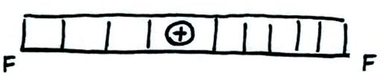
$M_y(0) = 0$

$M_y(\ell) = -F\ell$

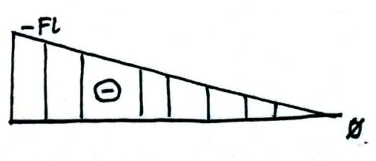
kontrola na DESNEM delu

5)

$[N_x]$



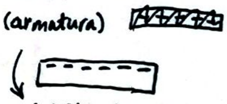
$[N_z]$



$[M_y]$

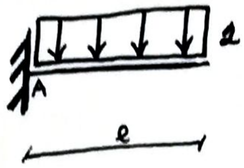
včasih

danes (armatura)



pri ~~bal~~ balkonih zgoraj, pri ostalih stvareh spodaj

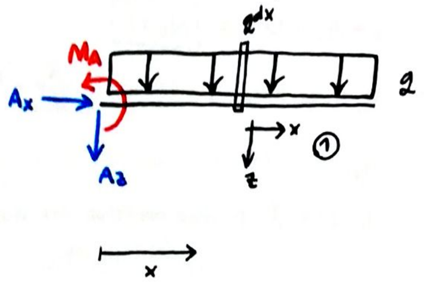
* ② PREVISNI NOSILEC Z ENAKOMERNO PREČNO PORAZDELJENO OBTEŽBO



1) $\tilde{m}_{po} = 0$

2)

$\int x q\,dx \approx q l \frac{l}{2}$



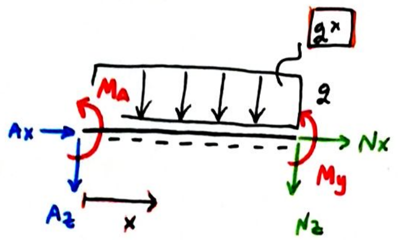
$\Sigma X: \boxed{A_x = 0}$

$\Sigma Z: \quad A_z + q \cdot l = 0 \rightarrow \boxed{A_z = -ql}$

$\Sigma M^A: \quad M_A - q l \frac{l}{2} = 0 \rightarrow \boxed{M_A = q \cdot l \frac{l}{2}}$

3) polje ①

4)



prečna sila je linearna

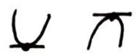
$\Sigma X: \boxed{N_x = 0}$

$\Sigma Z: \quad N_z + A_z + q \cdot x = 0 \rightarrow \boxed{N_z^{(x)} = ql - qx}$

$\Sigma M^T: \quad M_y + M_A + A_z x + q x \frac{x}{2} = 0$

$$\boxed{M_y^{(x)} = -q\frac{l^2}{2} + qlx - q\frac{x^2}{2}}$$

↳ KVADRATNA ENAČB



$\frac{dM_y}{dx} = q \cdot l - qx$

$= q(l - x)$

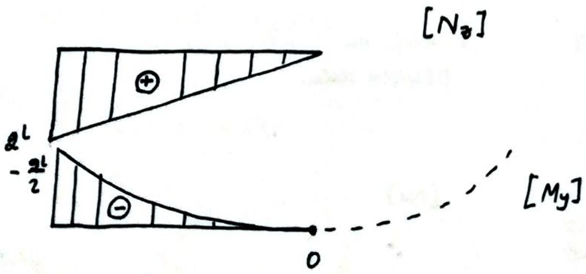
$x = l \ldots$ TEME

$M_y(0) = 0$

$M_y(0) = -q\frac{l^2}{2}$

dvojna mitla

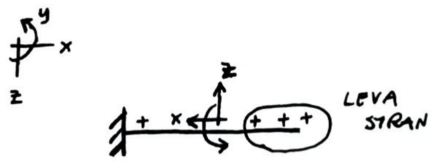
5)



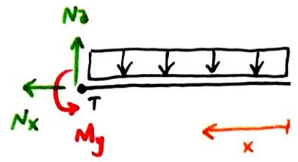
Zato nosilci

* Lahko bi tudi

Vpliv nasprotne izbire lokalnih koordinat (isti primer kot PRIMER ②)



leva:



$N_x = 0$

$N_z - qx = 0 \rightarrow \boxed{N_z = qx}$

$N_z(0) = 0$

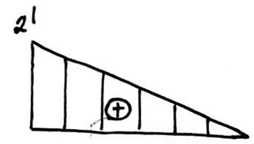
$N_z(l) = ql$

$M_y - qx\frac{x}{2} = 0 \rightarrow \boxed{M_y = q\frac{x^2}{2}}$

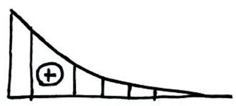
razlika samo v predznaku

splošno velja: če izberemo nasproten lokalni koordinatni sistem, se osne in prečne sile ne spremenijo. Momenti spremenijo predznak.

[Nz]

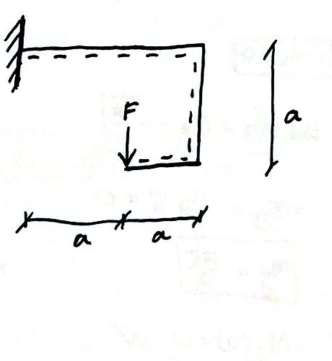


[My]

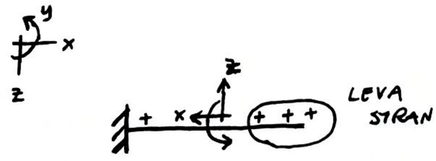


grafi za momente so vedno enaki (spremeni se samo predznak)

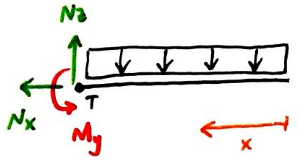
pri nekaterih ni logično "na prvi uč" kako obrniti lokalni



Vpliv nasprotne izbire lokalnih koordinat (isti primer kot PRIMER ②)



leva:



$N_x = 0$

$N_z - qx = 0 \rightarrow \boxed{N_z = qx}$

$N_z(0) = 0$

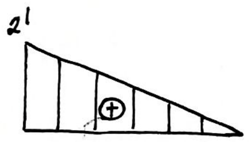
$N_z(l) = ql$

$M_y - qx\frac{x}{2} = 0 \rightarrow \boxed{M_y = q\frac{x^2}{2}}$

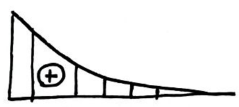
razlika samo s predznaku

splošno velja: če izberemo nasprotem lokalni koordinatni sistem, se osne im prečne sile ne spremenijo. Momenti spremenijo predznak.

[N_z]

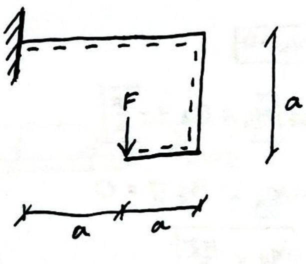


[M_y]

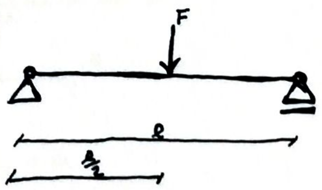


grafi za momente so vedno enaki (spremeni se samo predznak)

pri nekaterih ni logično "na prvi uč" kako obrniti lokalni

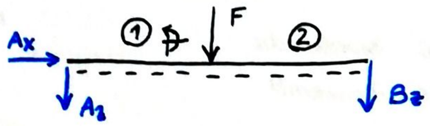


*③ PROSTOLEŽEČI NOSILEC S SILO NA SREDI RAZPONA



1) $m_{pz} = 3-2-1=0$

2) REAKCIJE



$\Sigma X: \ \boxed{A_x = 0}$

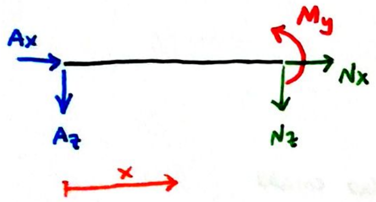
$\Sigma Z: \ A_z + B_z = -F \rightarrow \boxed{A_z = -\frac{F}{2}}$

$\Sigma M^A: \ -B_z \cdot l - F \cdot \frac{l}{2} = 0 \rightarrow \boxed{B_z = -\frac{F}{2}}$

3) POLJA ↗

4) NOTRANJE SILE

POLJE ①



$x \in [0, \frac{l}{2}]$

$\Sigma X: \ \boxed{N_x = -A_x = 0}$

$\Sigma Z: \ \boxed{N_z = -A_z = \frac{F}{2}}$

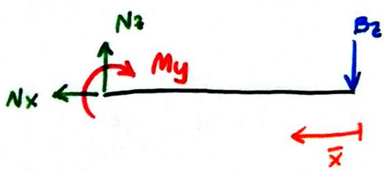
$\Sigma M^T: \ M_y + A_z \cdot x = 0$

$\boxed{M_y = \frac{Fx}{2}}$

$M_y(0) = 0$ ✓ → V ČLENKU NI MOMENTA !!

$M_y(\frac{l}{2}) = \frac{Fl}{4}$

POLJE ② → z desne



$\bar{x} = \frac{l}{2} - x \qquad \bar{x} \in [0, \frac{l}{2}]$

$\Sigma X: \ \boxed{-N_x = 0}$

$\Sigma Z: \ \boxed{N_z = B_z = -\frac{F}{2}}$

$\Sigma M^T: \ -M_y - B_z \bar{x} = 0$

$\boxed{M_y = \frac{F\bar{x}}{2}}$

$M_y(0) = 0$ ✓

$M_y(\frac{l}{2}) = \frac{Fl}{4}$

$M_y(\bar{x}) = \frac{F}{2}\bar{x}$

$\bar{x} = \frac{l}{2} - x$

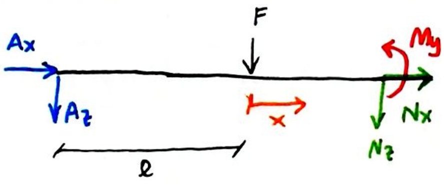
$M_y(x) = \frac{F}{2}(\frac{l}{2} - x)$

$M_y(x) = \frac{Fl}{4} - \frac{Fl}{2}$

$M_y(x) =$

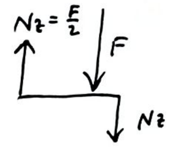
$M_y|_{x=0} = \frac{Fl}{4} \quad M_y|_{\frac{l}{2}} = 0$

kontrola polja 2 z leve



$\Sigma x: \; N_x = 0$

$\Sigma z: \; N_z + A_z + F = 0 \rightarrow N_z = -\frac{F}{2}$ ✓



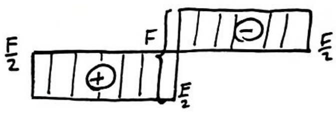
$\Sigma M_y: \; M_y + Fx + A_z\left(\frac{l}{2} + x\right) = 0$

$M_y = \frac{F}{2}x - \frac{Fl}{4} = 0$

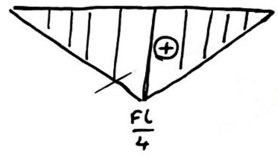
$\boxed{M_y = \frac{Fl}{4} - \frac{Fx}{2}}$

5) [Nx]

[Nz]

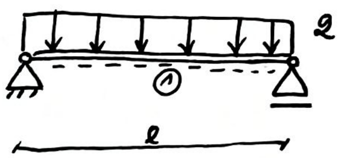


[My]



JANEŽKOVE ZAPOVEDI

* 4

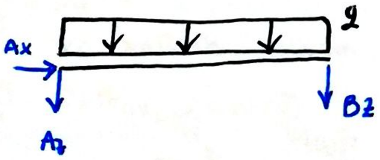


PROSTO LEŽEČ NOSILEC Z ~~BA~~ PREČNO PORAZDELJENO OBTEŽBO

najpogostejša formula $\frac{ql^2}{8} \rightarrow$ moment na sredi razpona prosto ležečega nosilca

1) $\tilde{m}_{ps} = 0$

2)



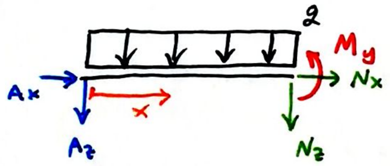
$\Sigma x: \; \boxed{A_x = 0}$

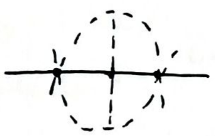
$\Sigma z: \; A_z + B_z = -ql \rightarrow \boxed{A_z = -\frac{ql}{2}}$

$\Sigma M^A: \; B_z \cdot l - ql\frac{l}{2} = 0 \rightarrow \boxed{B_z = -\frac{ql}{2}}$

3) polja → samo 1

4)





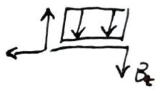
$x \in [0, l]$

$\Sigma X: \quad N_x = 0$

$\Sigma Z: \quad N_z + A_z + qx = 0 \rightarrow \boxed{N_z = \frac{ql}{2} - qx}$

$N_z(0) = \frac{ql}{2}$

$N_z(l) = -\frac{ql}{2}$



$\Sigma M^T: \quad M_y + A_z x + qx \frac{x}{2} = 0$

$M_y(x) = -q\frac{x^2}{2} + \frac{ql}{2}x$

$M_y(0) = 0$ ✓
$M_y(l) = 0$ ✓ } super, ker je členek

$\frac{dM_y}{dx} = -qx + \frac{ql}{2} = 0$

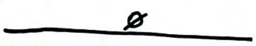
$\boxed{x_E = \frac{l}{2}}$ → kvadratna enačba ima ekstrem na sredini, je simetrična

$M_y(\frac{x}{2}) = -q\frac{l^2}{8} + \frac{ql^2}{4} = \frac{ql^2}{8}$

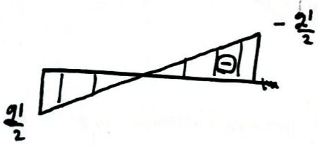
$\boxed{M_y(\frac{l}{2}) = \frac{ql^2}{8}}$

5) DIAGRAMI

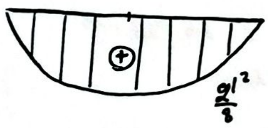
$[N_x]$



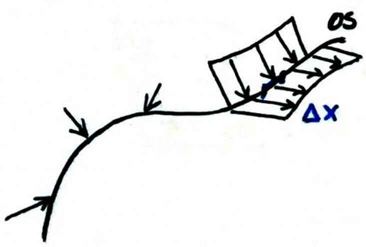
$[N_z]$



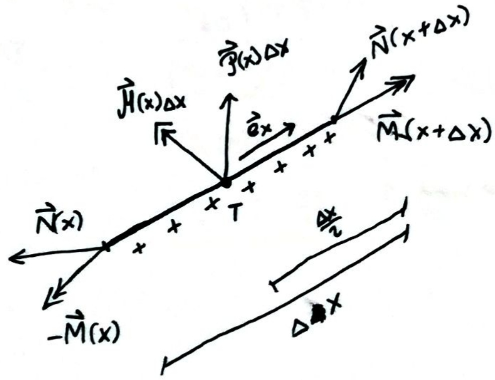
$[M_y]$



Ravnotežje infinitezimalnega dela nosilca 3.2.



opazujemo del nosilca dolžine Δx

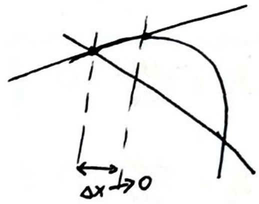


VSOTA SIL

$$\Sigma \vec{F} = \vec{0}$$

$$\vec{N}(x+\Delta x) - \vec{N}(x) + \vec{P}(x) \cdot \Delta x = \vec{0}$$

$$\frac{\vec{N}(x+\Delta x) - \vec{N}(x)}{\Delta x} + \vec{P}(x) = \vec{0}$$



$$\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} : \boxed{\frac{d\vec{N}}{dx} + \vec{P}(x) = \vec{0}}$$

VSOTA MOMENTOV

$$\Sigma \vec{M} = \vec{0}$$

$$\vec{M}(x+\Delta x) - \vec{M}(x) + \frac{\Delta x}{2} \vec{e}_x \times \vec{N}(x+\Delta x) - \frac{\Delta x}{2} \vec{e}_x \times (-\vec{N}(x)) + \vec{\mathcal{M}}(x) \cdot \Delta x = \vec{0}$$

$$\frac{\vec{M}(x+\Delta x) - \vec{M}(x)}{\Delta x} + \frac{1}{2} \vec{e}_x \times \vec{N}(x+\Delta x) + \frac{1}{2} \vec{e}_x \times \vec{N}(x) + \vec{\mathcal{M}}(x) = \vec{0}$$

$$\lim_{\Delta x \to 0} \quad \frac{d\vec{M}}{dx} + \vec{e}_x \times \vec{N} + \vec{\mathcal{M}} = \vec{0}$$

$$\lim_{\Delta x \to 0} \quad \boxed{\frac{d\vec{M}}{dx} + \vec{e}_x \times \vec{N} + \vec{\mathcal{M}} = \vec{0}}$$

RAVNOTEŽNA POGOJA V DIFERENCIALNI OBLIKI

KOMPONENTNA OBLIKA

$$\vec{N} = N_x \vec{e}_x + N_y \vec{e}_y + N_z \vec{e}_z$$

$$\vec{P} = P_x \vec{e}_x + P_y \vec{e}_y + P_z \vec{e}_z$$

$$\vec{M} = M_x \vec{e}_x + M_y \vec{e}_y + M_z \vec{e}_z$$

$$\vec{\mathcal{M}} = \mathcal{M}_x \vec{e}_x + \mathcal{M}_y \vec{e}_y + \mathcal{M}_z \vec{e}_z$$

$$\vec{e}_x \times \vec{N} = \begin{vmatrix} e_x & e_y & e_z \\ 1 & 0 & 0 \\ N_x & N_y & N_z \end{vmatrix} = 0\vec{e}_x - N_z \vec{e}_y + N_y \vec{e}_z$$

RAVNOTEŽJE SIL

$$\frac{dN_x}{dx} + P_x = 0$$

$$\frac{dN_y}{dy} + P_y = 0$$

$$\frac{dN_z}{dz} + P_z = 0 \rightarrow (*)$$

RAVNOTEŽJE MOMENTOV

$$\frac{dM_x}{dx} + \mathcal{M}_x = 0$$

$$\frac{dM_y}{dx} - N_z + \mathcal{M}_y = 0 \rightarrow \frac{d}{dx} (*)$$

$$\frac{dM_z}{dx} + N_y + \mathcal{M}_z = 0 \rightarrow \frac{d}{dx} (*)$$

UGOTOVITEV: kadar na nosilec ne deluje porazdeljena momentna obtežba, je odvod momenta (do predznaka natančno) enak prečni sili.

$$\vec{\mathcal{M}} = \vec{0} \Rightarrow \frac{dM_y}{dx} = N_z$$

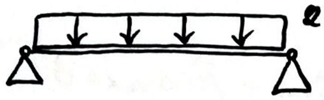
$$\Rightarrow \frac{dM_z}{dx} = -N_y$$

(*) $\frac{d^2M_y}{dx^2} - \frac{dN_z}{dx} + \frac{dm_y}{dx} = 0$

(A) $$\boxed{\begin{aligned} &\frac{d^2M_y}{dx^2} + p_z + \frac{dm_y}{dx} = 0 \\ &\frac{d^2M_z}{dx^2} - p_y + \frac{dm_z}{dx} = 0 \end{aligned}}$$

ALTERNATIVNA OBLIKA MOMENTNIH POGOJEV, V KATERIH NE NASTOPAJO PREČNE SILE

* PRIMER



$p_x = 0$

$p_z = q$

$m_y = 0$

$\frac{dN_x}{dx} = 0 \qquad N_x = \text{konst.} = C_1$

$\frac{dN_z}{dx} = -q \qquad N_z = -qx + C_2$

$\frac{d^2M_y}{dx^2} + q = 0 \qquad \frac{dM_y}{dx} = -qx + C_3 \qquad \boxed{M_y(x) = -q\frac{x^2}{2} + C_3 x + C_4}$

ROBNI POGOJI

$M_y(0) = 0$ (členek) $\rightarrow \boxed{C_4 = 0}$

$M_y(l) = 0$ (členek)

$-\frac{ql^2}{2} + C_3 l = 0$

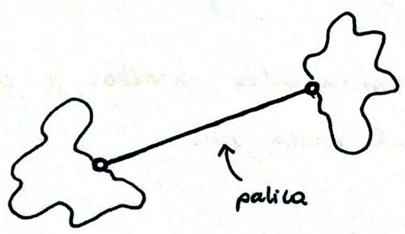
$C_3 = \frac{ql}{2}$

$\boxed{M_y(x) = -\frac{qx^2}{2} + \frac{qlx}{2}}$

PALICA

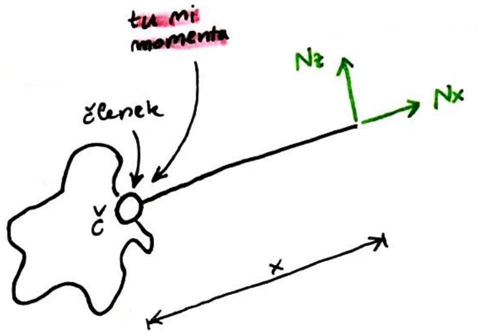
(= poseben primer nosilca, ki je

1. raven
2. v krajiščih podprt ali povezan s členki
3. vzdolž dolžine ne deluje nobena obtežba in ni nobenih dodatnih vezi ali podpor)



$$\frac{d\vec N}{dx} + \cancel{\vec p} = \vec 0 \Rightarrow \frac{d\vec N}{dx} = \vec 0 \Rightarrow \text{notranja sila je kvečjemu konstantna}$$

$$\frac{d\vec M}{dx} + \vec e_x \times \vec N + \cancel{\vec m} = \vec 0$$



$\Sigma M^{č}$: levi del palice

$\Sigma M_z: \; x N_y = 0 \Rightarrow N_y = 0$

$\Sigma M_y: \; -x N_z = 0 \Rightarrow N_z = 0$

• $\Rightarrow \vec e_x \times \vec N = \vec 0 \Rightarrow \frac{d\vec M}{dx} = \vec 0 \Rightarrow \vec M$ je največ konstanten

ker je moment v členku (to je v krajiščih enak 0) so notranji momenti v palici enaki nič.

- v palici so zato notranje prečne sile enake 0
- v palici so zato notranji momenti enaki nič
- v palici so neničelne le osne sile, ki pa so konstantne po dolžini palice

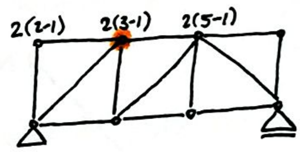
- konstrukcijam, ki so sestavljene iz samih palic rečemo PALIČJA !

za paličja lahko izpeljemo posebne računske postopke !

$\tilde m_{ps,\, paličja} = ?$

$\tilde m_{ps} = 3K - \Sigma m_{opsp} - \Sigma m_{opsv}$

$m_{opsč} = 2(k-1)$



$$\tilde m_{ps} = 3K - r_1 - 2r_2 - \underbrace{\sum_{\text{po vseh vozliščih}} 2(k-1)}_{4K - 2V}$$

$= 3K - r_1 - 2r_2 - 4K + 2V$

$= 2(V - r_2) - r_1 - K$

$= 2V_p - K + r_1$

$$\boxed{\tilde m_{ps,\, paličja} = 2V_p + r_1 - K}$$

OZNAKE

V ... št. vozlišč ($V = V_p + r_1 + r_2$)

$p = K$ ... št. palic

r_1 ... št. drsnih podpor

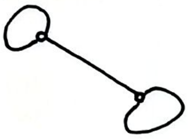
r_2 ... št. nepomičnih podpor

V_p ... št. nepodprtih vozlišč

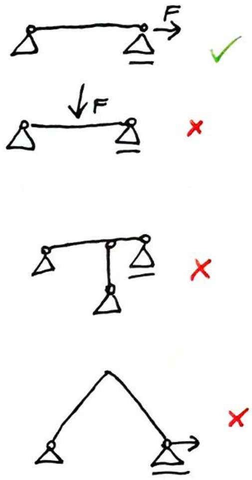
PALIČJA

= so konstrukcije iz samih palic

PALICA



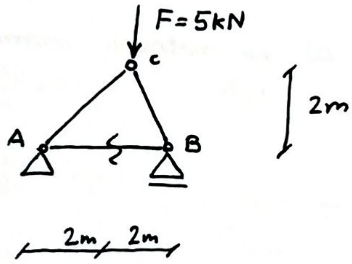
✓ ravna
✓ členki na koncih
✓ vmes ničesar (ne obtežbo, ne vezi)



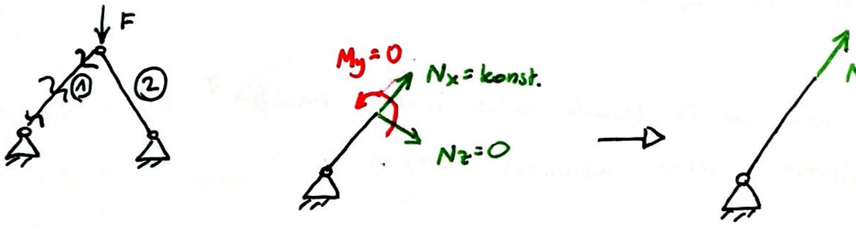
N_x = konst.

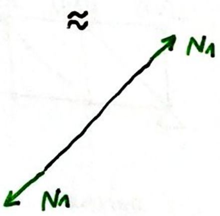
ostale rot. sile so zanemarljive

METODE DOLOČANJA OSNIH SIL V PALIČJIH

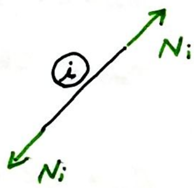


ideja



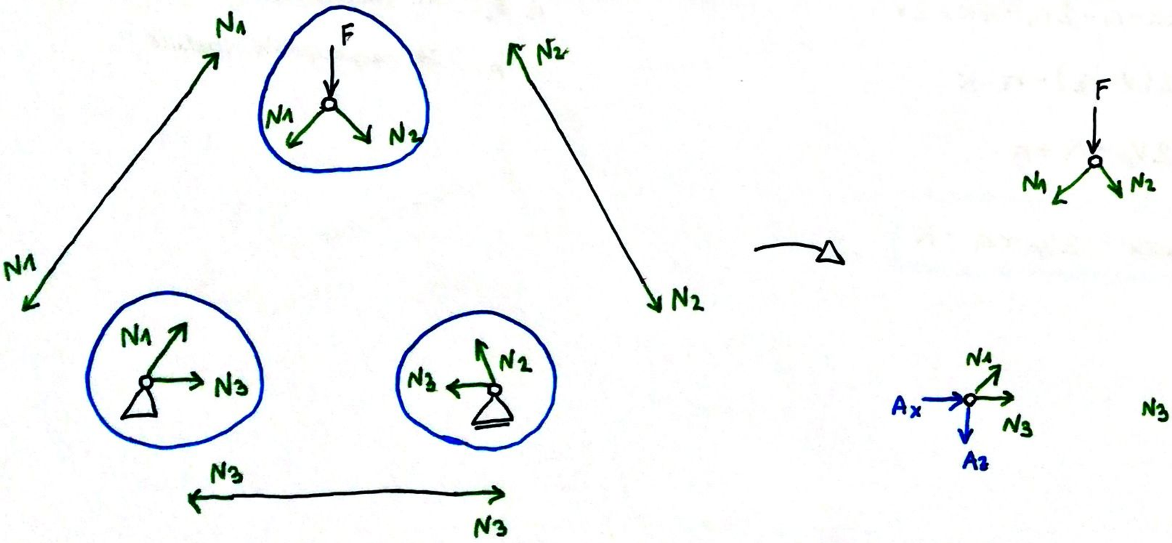


DOGOVOR: palico izrežemo, njen vpliv nadomestimo z osno silo usmerjeno iz palice - navzven



$N_i > 0$ → NATEZNO OBREMENJENA PALICA ⊕

$N_i < 0$ → TLAČNO OBREMENJENA PALICA ⊖



3·2 enačbe
6 neznank
3 osne sile
3 reakcije

$\tilde{m}_{ps,\text{paličja}} = 2V_p + r_1 - K$

- V_p … št. nepodprtih vozlišč
- r_1 … št. drsnih podpor
- K … št. palic

$V = V_p + r_1 + r_2$

enačb: $2V$: ΣX, ΣZ

($2V$ … št. enačb)

neznank: $K + r_1 + 2r_2$

št. enačb: $2(V_p + r_1 + r_2) = 2r_2 + r_1 + \underbrace{(r_1 + 2V_p)}_{=K,\ \text{če je } \tilde{m}_{ps}=0}$

① IZREZOVANJE VOZLIŠČ

Algoritem:

1) $\tilde{m}_{ps} = 0$ → DA - nadaljujem; → NE - končam

2) izrežemo vsa vozlišča
   - obtežbo prenišemo
   - vpliv palic nadomestimo s silami v palicah •
   - vpliv podpor nadomestimo z reakcijami •

3) za vsako vozlišče zapišemo 2 ravnotežna pogoja (ΣX, ΣZ)

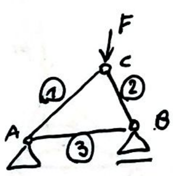
   * ker je $\tilde{m}_{ps} = 0$ je enačb toliko kot neznank *

4) rešimo sistem enačb

   $2V \times 2V$

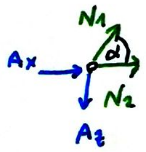
   * če je enolično rešljiv, je paličje statično določeno *

*PRIMER:



① $\tilde{m}_{ps} = 2\cdot 1 + 1 - 3 = 0$

②



③ A: $\Sigma x: A_x + N_1 \cos\alpha + N_3 = 0$
$\Sigma z: A_z - N_1 \sin\alpha = 0$ } KONTROLA

6 enačb
6 neznank
↓
matrika

B: $\Sigma x: -N_3 - N_2 \cos\alpha = 0 \rightarrow N_3 = \frac{F}{2} \cot tg\alpha = \frac{F}{2}$

$\boxed{N_3 = \frac{F}{2}}$

$\Sigma z: B_z - N_2 \sin\alpha = 0$ }

C: $\Sigma x: -N_1 \cos\alpha + N_2 \cos\alpha = 0$
$\Sigma z: F + N_1 \sin\alpha + N_2 \sin\alpha = 0 \rightarrow \boxed{N_1 = N_2 = -\frac{F}{2\sin\alpha}}$

↳ tlačno obremenjeni palici

$$
\underbrace{
\begin{array}{cccccc}
N_1 & N_2 & N_3 & A_x & A_z & B_z \\
\end{array}
}
$$

$$
\underbrace{\begin{bmatrix}
\cos\alpha & 0 & 1 & 1 & 0 & 0 \\
-\sin\alpha & 0 & 0 & 0 & 1 & 0 \\
0 & -\cos\alpha & -1 & 0 & 0 & 0 \\
0 & -\sin\alpha & 0 & 0 & 0 & 1 \\
-\cos\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\
\sin\alpha & \sin\alpha & 0 & 0 & 0 & 0
\end{bmatrix}}_{K}
\begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ A_x \\ A_z \\ B_z \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -F \end{bmatrix}
$$

$\det(K) \neq 0$

1) POSTOPNO IZREZOVANJE VOZLIŠČ

motivacija: pogosto imamo le 2 neznani sili

① $\tilde{m}_{ps} = 0$

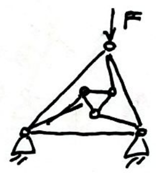
② REAKCIJE

③ PONAVLJAJ

- poišči vozlišče z največ 2 neznanima silama
- reši sistem 2x2

DOKLER JE ŠE KAJ TAKŠNIH VOZLIŠČ

④ OSTANEJO MI 3 ENAČBE → KONTROLA



algoritem ne deluje

$\Sigma X: A_x = 0$

$\Sigma z: A_z + B_z = -F$

$\Sigma M^A: -B_z \cdot 4 - F \cdot 2$

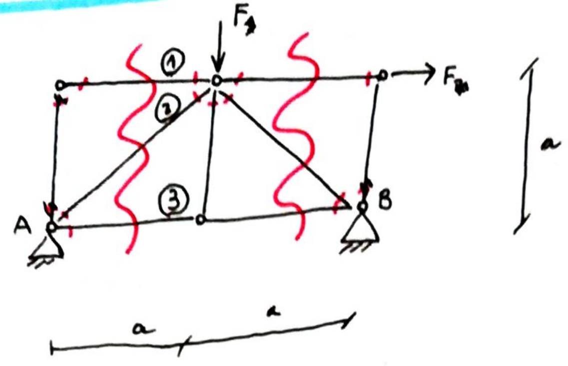
$\boxed{A_z = B_z = -\frac{F}{2}}$

$tg\alpha = 1$

④ vstavimo št. notr. oz. člk ✓

* PRIMER:

② RAZREZ NA 2 DELA



① $\tilde{m}_{ps}=0$

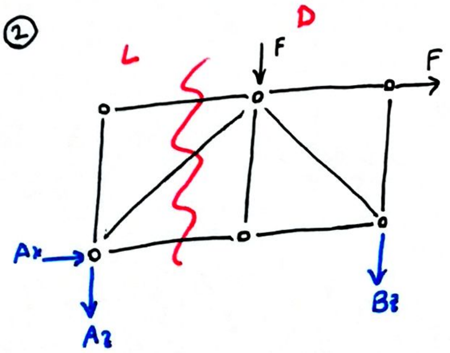
② REAKCIJE

③ REŽEMO PREKO VSAJ 3 PALIC (če je le mogoče režemo preko natanko 3 palic, ki se ne sekajo v isti točki)

④ ZA 1 DEL PALIČJA ZAPIŠEMO 3 RAVNOTEŽNE ENAČBE

① $\tilde{m}_{ps}=4\cdot2+1-9=0$

②

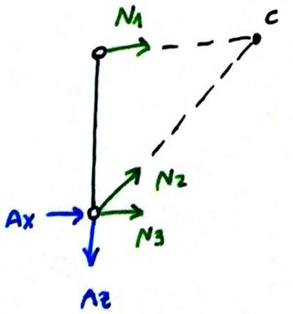


$\Sigma X: \quad A_x=-F$

$\Sigma Z: \quad A_z+B_z=-F$

$\Sigma M^A: \quad A_z=B_z=-\frac{F}{2}$

③



$\Sigma M^A: \quad -N_1 a=0 \rightarrow \boxed{N_1=0}$

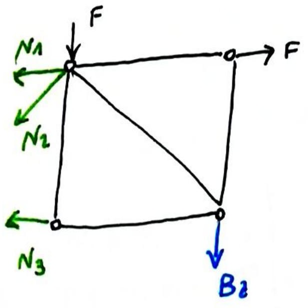
$\Sigma M^C: \quad N_3 \cancel{a} + A_x \cancel{a} + A_z \cdot \cancel{a} = 0$

$\boxed{N_3=+3\frac{F}{2}}$ ali je smiselno, da je natezno obremenjena? Da

$\Sigma Z: \quad A_z - N_2 \sin\alpha = 0 \rightarrow \boxed{N_2=-\frac{F}{\sqrt{2}}\sqrt{2}}$

* manjši kos vzamem za izračunat, večji del za kontrolo *

④ KONTROLA → DESNI DEL



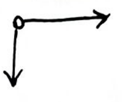
$$\Sigma x: \; -N_1 - N_3 - N_2 \frac{\sqrt{2}}{2} + F \overset{?}{=} 0$$

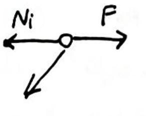
$$-\frac{3F}{2} + \frac{F}{4} + F = 0 \checkmark$$

$$\Sigma z: \; F + B_z + N_2 \cdot \frac{\sqrt{2}}{2} \overset{?}{=} 0$$

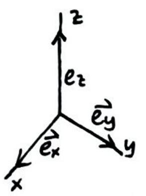
$$F - \frac{F}{2} - \frac{F}{\sqrt{2}} = 0 \checkmark$$

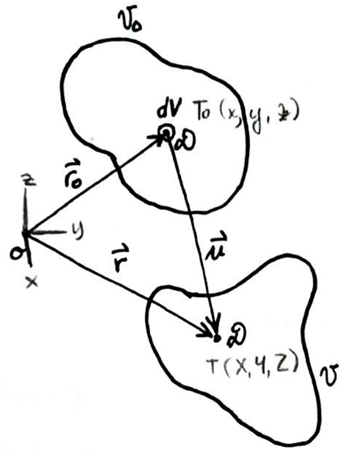
* kjerkoli v paličju najdemo vozlišča kjer se stikata 2 neobremenjeni palici ali 3, 2 vzporedni je sila nič.





DEFORMACIJE TELESA





$\mathcal{V}$... območje v prostoru

V... prostornina tega območja

$\mathcal{S}$... površina

s... ploščina

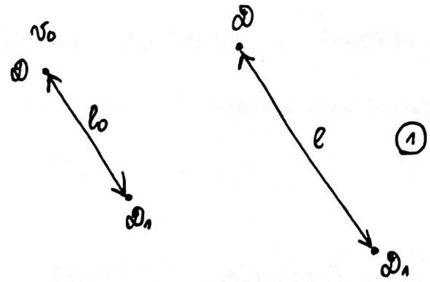
$\mathcal{A}$... območje v ravnini

A... ploščina

$\vec{u}$... pomik delca (= sprememba lege): merimo, opazujemo

· naloga: določiti želimo spremembo oblike – DEFORMACIJE.

kaj znamo izmeriti?



$\Delta L = L - L_0$

① $\boxed{D_L = \frac{\Delta L}{L_0}}$ specifična sprememba dolžine [/ ali %]

⊕ raztezek

⊖ skrček

② $\boxed{D_{ab} = \frac{\pi}{2} - \alpha_{ab}}$ sprememba pravega kota [v RADIANIH]

* Deformacije nimajo enot! *

$\alpha = \frac{L}{2\pi r}$ → radiani

vpeljemo koordinatni sistem in enotske vektorje

$|\vec{e}_i| = 1$

$\vec{e}_i \cdot \vec{e}_j = 0 \quad i \neq j$

$\vec{u} = \vec{u}(x, y, z) = u_x(x, y, z)\vec{e}_x + u_y(x, y, z)\vec{e}_y + u_z(x, y, z)\vec{e}_z$

$\vec{r}_0 = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z$

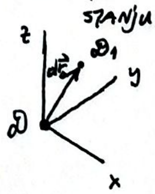
$\vec{r} = X\vec{e}_x + Y\vec{e}_y + Z\vec{e}_z$

$$\vec{u}(x,y,z) \Rightarrow \boxed{\begin{aligned} X &= x + u_x(x,y,z) \\ Y &= y + u_y(x,y,z) \\ Z &= z + u_z(x,y,z) \end{aligned}}$$

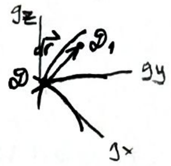
LAGRAGEV OPIS → tipičen za trdnine
→ izberemo

$$\vec{u}(X,Y,Z) \Rightarrow \left.\begin{aligned} X &= x + u_x(X,Y,Z) \\ Y &= y + u_y(X,Y,Z) \\ Z &= \cancel{y}z + u_z(X,Y,Z) \end{aligned}\right\} \text{EULERJEV OPIS}$$

KOORDINATE V ZAČETNEM NEDEFORMIRANEM STANJU



KRIVOČRTNE KOORDINATE V TRENUTNEM STANJU



$$d\vec{\varrho} = dx\,\vec{e}_x + dy\,\vec{e}_y + dz\,\vec{e}_z$$
$$d\vec{r} = dX\,\vec{e}_x + dY\,\vec{e}_y + dZ\,\vec{e}_z$$

IDEJA: obdržimo koordinate, spremenimo pa bazo

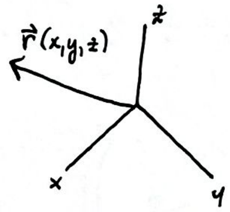
$$\boxed{d\vec{r} = dx\,\vec{g}_x + dy\,\vec{g}_y + dz\,\vec{g}_z}$$

g_x, g_y, g_z ... so BAZNI VEKTORJI v krivočrtnih koordinatah

* OPOMBA: ker je vse deformiranje opisano z novo bazo $\vec{g}_i$ niso niti enotski niti ortogonalni

* SPOMNIMO SE: TOTALNI (POPOLNI) DIFERENCIAL

$$d\vec{r} = dx\,\frac{\partial \vec{r}}{\partial x} + dy\,\frac{\partial \vec{r}}{\partial y} + dz\,\frac{\partial \vec{r}}{\partial z}$$

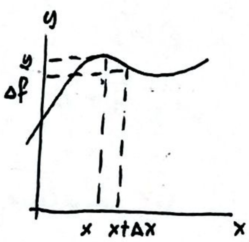


$\frac{\partial \vec{r}}{\partial x}$ parcialni/delni odvod

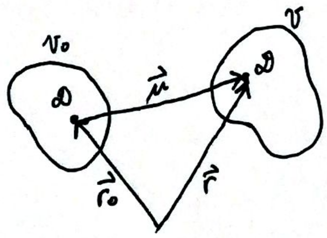
$$\frac{\partial \vec{r}}{\partial x} = \lim_{\Delta x \to 0} \frac{\vec{r}(x+\Delta x, y, z) - \vec{r}(x,y,z)}{\Delta x}$$

$$\vec{r} = r_x\vec{e}_x + r_y\vec{e}_y + r_z\vec{e}_z$$

$$\Rightarrow \boxed{\begin{aligned} \vec{g}_x &= \frac{\partial \vec{r}}{\partial x} \\ \vec{g}_y &= \frac{\partial \vec{r}}{\partial y} \\ \vec{g}_z &= \frac{\partial \vec{r}}{\partial z} \end{aligned}}$$



$$\frac{\Delta f}{\Delta x}$$

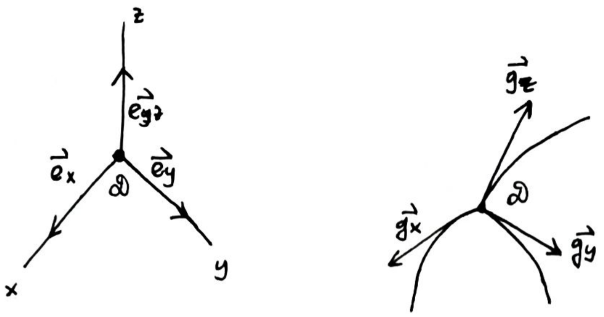


$$\vec{r} = \vec{r}_0 + \vec{u}_0 = (x+u_x)\vec{e}_x + (y+u_y)\vec{e}_y + (z+u_z)\vec{e}_z$$

$$\boxed{\frac{\partial \vec{r}}{\partial x} = \vec{e}_x + \frac{\partial \vec{u}}{\partial x} = \vec{g}_x}$$

$$\boxed{\frac{\partial \vec{r}}{\partial y} = \vec{e}_y + \frac{\partial \vec{u}}{\partial y} = \vec{g}_y}$$

$$\boxed{\frac{\partial \vec{r}}{\partial z} = \vec{e}_z + \frac{\partial \vec{u}}{\partial z} = \vec{g}_z}$$



$$D_{xx} = |\vec{g}_x| - 1 = \sqrt{\left(\vec{e}_x + \frac{\partial \vec{u}}{\partial x}\right)\left(\vec{e}_x + \frac{\partial \vec{u}}{\partial x}\right)} - 1 \;=$$

$$\vec{u} = u_x \vec{e}_x + u_y \vec{e}_y + u_z \vec{e}_z$$

$$\frac{\partial \vec{u}}{\partial x} = \frac{\partial u_x}{\partial x}\vec{e}_x + \frac{\partial u_y}{\partial x}\vec{e}_y + \frac{\partial u_z}{\partial x}\vec{e}_z$$

$$= \sqrt{1 + 2\frac{\partial u_x}{\partial x} + \frac{\partial \vec{u}}{\partial x}\cdot\frac{\partial \vec{u}}{\partial x}} - 1 \;\approx$$

$$\frac{\partial \vec{u}}{\partial x}\cdot\frac{\partial \vec{u}}{\partial x} = \left(\frac{\partial u_x}{\partial x}\right)^2 + \cancel{\left(\frac{\partial u_y}{\partial x}\right)^2} + \cancel{\left(\frac{\partial u_z}{\partial x}\right)^2}$$

ZANEMARIMO, če sta $\frac{\partial u_y}{\partial x}$ in $\frac{\partial u_z}{\partial x}$ MAJHNA

$$\approx \sqrt{1 + 2\frac{\partial u_x}{\partial x} + \left(\frac{\partial u_x}{\partial x}\right)^2} - 1 = \sqrt{\left(1 + \frac{\partial u_x}{\partial x}\right)^2} \rightarrow \boxed{D_{xx} \approx \frac{\partial u_x}{\partial x}} \equiv \varepsilon_{xx}$$

deformacija je trend – sprememba premika (v smeri)

$$\boxed{D_{yy} \approx \frac{\partial u_y}{\partial y}} \equiv \varepsilon_{yy}$$

$$\boxed{D_{zz} \approx \frac{\partial u_z}{\partial z}} \equiv \varepsilon_{zz}$$

kaj pa spremembe pravih kotov?

$\vec{e}_x \perp \vec{e}_y$ $\qquad g_x \not\perp g_y$ $\qquad \vec{g}_x \cdot \vec{g}_y = |\vec{g}_x| \cdot |\vec{g}_y| \cdot \cos \vartheta_{xy}$

$$\vartheta_{xy} = \frac{\pi}{2} - \varphi_{xy} = \frac{\pi}{2} - D_{xy}$$

↖ sprememba pravega kota (v radianih)

$$\cos\left(\frac{\pi}{2} - D_{xy}\right) = \frac{\vec{g}_x \cdot \vec{g}_y}{|\vec{g}_x| \cdot |\vec{g}_y|}$$

$$=$$

$$\sin(D_{xy}) = \frac{\vec{g}_x \cdot \vec{g}_y}{|\vec{g}_x| \cdot |\vec{g}_y|} \approx \vec{g}_x \cdot \vec{g}_y = \left(\vec{e}_x + \frac{\partial \vec{u}}{\partial x}\right) \circ \left(\vec{e}_y + \frac{\partial \vec{u}}{\partial y}\right) =$$

MAJHNOST ?? $\qquad$ SS MAJHNOST

D_{xy} $\qquad$ $\vec{g}_x \cdot \vec{g}_y$

$$\sin x = x + \frac{x^3}{3!} + \frac{x^5}{5!} \dots$$

$$\Rightarrow \sin x \approx x$$

$$\left(\vec{e}_x + \frac{\partial u_x}{\partial x}\vec{e}_x + \frac{\partial u_y}{\partial y}\vec{e}_y + \frac{\partial u_z}{\partial z}\right)\left(\vec{e}_y + \frac{\partial u_x}{\partial y}\vec{e}_x + \frac{\partial u_y}{\partial y}\vec{e}_y + \frac{\partial u_z}{\partial y}\right) = \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} + \underbrace{\cancel{\frac{\partial u_x}{\partial x}\cdot\frac{\partial u_x}{\partial y}}}_{} +$$

$$+ \underbrace{\cancel{\frac{\partial u_y}{\partial x}\frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial x}\cdot\frac{\partial u_z}{\partial z}}}_{\text{členi višjega reda}}$$

ZANEMARIMO, kadar so deformacije majhne

$$\Rightarrow \boxed{D_{xy} \approx \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} = 2\varepsilon_{xy} \qquad D_{xz} \approx \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} = 2\varepsilon_{xz} \qquad D_{yz} \approx \frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z} = 2\varepsilon_{yz}}$$

$$u = u_x\vec{e}_x + u_y\vec{e}_y + u_z\vec{e}_z \Rightarrow \underline{\underline{J}} = \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{bmatrix}$$

jakobijeva matrika

diagonalne komponente prepišem, izvendiagonalne povprečimo

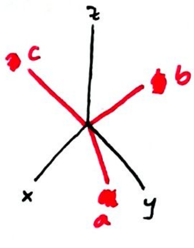
$$\Downarrow$$

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix} = \boxed{\underline{\underline{\varepsilon}} = \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{1}{2}\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right) & \frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right) \\ \frac{1}{2}\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right) & \frac{\partial u_y}{\partial y} & \frac{1}{2}\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right) \\ \frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right) & \frac{\partial u_z}{\partial x} \end{bmatrix}}$$

TENZOR MAJHNIH DEFORMACIJ

STRIŽNE DEFORMACIJE

NORMALNE DEFORMACIJE



enačb, ki povezuje defomacije in pomike

$$\boxed{\begin{aligned} \varepsilon_{xx} &= \frac{\partial u_x}{\partial x} & \varepsilon_{xy} &= \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) \\ \varepsilon_{yy} &= \frac{\partial u_y}{\partial y} & \varepsilon_{xz} &= \frac{1}{2}\left(\frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z}\right) \\ \varepsilon_{zz} &= \frac{\partial u_z}{\partial z} & \varepsilon_{yz} &= \frac{1}{2}\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right) \end{aligned}}$$

KINEMATIČNE ENAČBE

9 neznank

* NALOGA: IŠČEMO EKSTREME

Določili bi radi tako smer $\vec{e}_a$, da bo ε_{aa} največja možna

$$\text{MAX}\ (\varepsilon_{aa}) \quad \text{ob pogoju} \quad |\vec{e}_a| = 1$$

= vezani ekstremi (metoda Lagrangevih množiteljev)

$$\text{MAX}\ (\varepsilon_{aa} + \lambda(1 - a_x^2 - a_y^2 - a_z^2))$$

$$F = \begin{bmatrix} a_x & a_y & a_z \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} a_x + \varepsilon_{xy} a_y + \varepsilon_{xz} a_z \\ \varepsilon_{xy} a_x + \varepsilon_{yy} a_y + \varepsilon_{yz} a_z \\ \varepsilon_{xz} a_x + \varepsilon_{zy} a_y + \varepsilon_{zz} a_z \end{bmatrix} + \lambda(1 - a_x^2 - a_y^2 - a_z^2)$$

$$F = \varepsilon_{xx} a_x^2 + \varepsilon_{yy} a_y^2 + \varepsilon_{zz} a_z^2 + 2\varepsilon_{xy} a_x a_y + 2\varepsilon_{xz} a_x a_z + 2\varepsilon_{yz} a_y a_z + \lambda(1 - a_x^2 - a_y^2 - a_z^2)$$

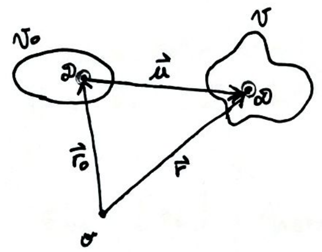
$$\frac{\partial F}{\partial a_x} = 2\varepsilon_{xx} a_x + 2\varepsilon_{xy} a_y + 2\varepsilon_{xz} a_z - \lambda 2 a_x = 0$$

$$\frac{\partial F}{\partial a_y} = 2\varepsilon_{yy} a_y + 2\varepsilon_{xy} a_x + 2\varepsilon_{yz} a_y - \lambda 2 a_y = 0$$

$$\frac{\partial F}{\partial a_z} = 2\varepsilon_{zz} a_z + 2\varepsilon_{xz} a_x + 2\varepsilon_{yz} a_y - \lambda 2 a_z = 0$$

$$\underbrace{\begin{bmatrix} \varepsilon_{xx} - \lambda & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} - \lambda & \varepsilon_{zy} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} - \lambda \end{bmatrix}}_{\text{to je problem lastnih vrednosti}} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

8.11.2021



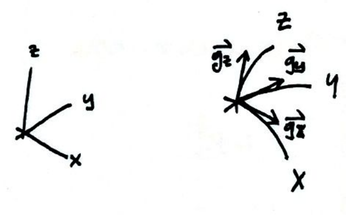
$$d\vec{r}_0 = dx\,\vec{e}_x + dy\,\vec{e}_y + dz\,\vec{e}_z$$

$$d\vec{r} = dx\,\vec{g}_x + dy\,\vec{g}_y + dz\,\vec{g}_z = dX\,\vec{e}_x + dY\,\vec{e}_y + dZ\,\vec{e}_z$$

$$\vec{g}_x = \vec{e}_x + \frac{\partial \vec{u}}{\partial x}$$

$$\vec{g}_y = \vec{e}_y + \frac{\partial \vec{u}}{\partial y}$$

$$\vec{g}_z = \vec{e}_z + \frac{\partial \vec{u}}{\partial z}$$

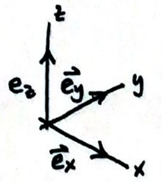


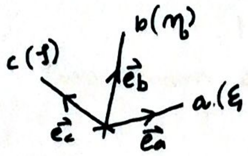
krivočrtna baza

$$\begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix} = \underbrace{\begin{bmatrix} 1 + \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & 1 + \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & 1 + \frac{\partial u_z}{\partial x} \end{bmatrix}}_{= F \text{ deformacijski gradient}} \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}$$

$$\rightarrow \boxed{\underline{\underline{F}} = \underline{\underline{I}} + \underline{\underline{J}}}$$

KOORDINATNA TRANSFORMACIJA





$\underline{\underline{\varepsilon}}\{a,b,c\}$ želimo povezati z $\underline{\underline{\varepsilon}}(x,y,z)$

$$\underline{\underline{\varepsilon}}\{a,b,c\} = \begin{bmatrix} \varepsilon_{aa} & \varepsilon_{ab} & \varepsilon_{ac} \\ & \varepsilon_{bb} & \varepsilon_{bc} \\ S/M & & \varepsilon_{cc} \end{bmatrix} \qquad \underline{\underline{\varepsilon}}\{x,y,z\} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ & \varepsilon_{yy} & \varepsilon_{yz} \\ S/M & & \varepsilon_{zz} \end{bmatrix}$$

$\vec{e}_a = a_x \vec{e}_x + a_y \vec{e}_y + a_z \vec{e}_z$

$\vec{e}_b = b_x \vec{e}_x + b_y \vec{e}_y + b_z \vec{e}_z$

$e_c = c_x \vec{e}_x + c_y \vec{e}_y + c_z \vec{e}_z$

$a_x^2 + a_y^2 + a_z^2 = 1 \rightarrow$ VEZ (norma 1)

$a_x b_x + a_y b_y + a_z b_z = 0 \rightarrow$ VEZ (ortogonalnost)

- KOORDINATNA TRANSFORMACIJA

$$\begin{bmatrix} \varepsilon_{aa} & \varepsilon_{ab} & \varepsilon_{ac} \\ & \varepsilon_{bb} & \varepsilon_{bc} \\ & & \varepsilon_{cc} \end{bmatrix} = \underbrace{\boxed{\begin{bmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{bmatrix}}}_{P^T} \underbrace{\boxed{\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ & \varepsilon_{yy} & \varepsilon_{yz} \\ S/M & & \varepsilon_{zz} \end{bmatrix}}}_{\varepsilon\{x,y,z\}} \underbrace{\boxed{\begin{bmatrix} a_x & b_x & c_x \\ a_y & b_y & c_y \\ a_z & b_z & c_z \end{bmatrix}}}_{P \text{ (prehodna matrika)}}$$

UGOTOVITVE:

- V $\underline{\underline{\varepsilon}}\{x,y,z\}$ so zapisane vse deformacije v vseh smereh, zato rečemo, da je ε TENZOR. $D_{aa} \approx \varepsilon_{aa} = ?$, $D_{ab} \approx \varepsilon_{ab} = ?$

$$\varepsilon_{aa} = \begin{bmatrix} a_x & a_y & a_z \end{bmatrix} \begin{bmatrix} \varepsilon\{x,y,z\} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

$$\varepsilon_{ab} = \begin{bmatrix} b_x & b_y & b_z \end{bmatrix} \begin{bmatrix} \varepsilon\{x,y,z\} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{1}{2}\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right) & \frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right) \\ & \frac{\partial u_y}{\partial y} & \frac{1}{2}\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right) \\ \text{SIM} & & \frac{\partial u_z}{\partial z} \end{bmatrix}$$

TENZOR

STRIŽNE DEFORMACIJE (Δ kotov)

NORMALNE DEFORMACIJE

SIM... simetrično

ε_{aa}, ε_{ab} v poljubni smeri

$$\varepsilon_{aa} = \boxed{e_a} \, \boxed{\varepsilon\{x,y,z\}} \, \boxed{e_a}$$

MAX?

$$(\underline{\underline{\varepsilon}} - \lambda \underline{\underline{I}})\underline{e} = \underline{0}$$

$$\left.\begin{aligned} \varepsilon_{11} &= \lambda_1 \\ \varepsilon_{22} &= \lambda_2 \\ \varepsilon_{33} &= \lambda_3 \end{aligned}\right\} \varepsilon_{11} \geq \varepsilon_{22} \geq \varepsilon_{33}$$

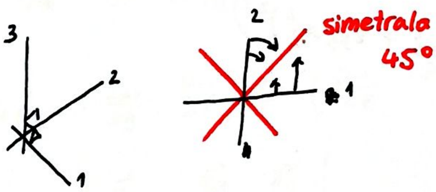
$$\underline{\underline{\varepsilon}}_{\{1,2,3\}} = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix}$$

"Kadar se indeks ponovi, gre za normalne deformacije"

$\vec{e_1}, \vec{e_2}, \vec{e_3}$ imenujemo GLAVNE SMERI

ugotovitev: spremembe pravih kotov med glavnimi smermi so enake 0!

MAX $\varepsilon_{ab} = ?$



→ "bolj kot grem stran od idealnih smeri (1,2) večji bo strig. → MAX strig bo na diagonali pod kotom 45°"

* NALOGA: iščemo tako smer $\vec{e_a}$, da bo ε_{ab} največji možen za neko pravokotno smer $\vec{e_b}$ $(\vec{e_b} \cdot \vec{e_a} = 0)$

$$\varepsilon_{ab} = [e_b] \begin{bmatrix} \varepsilon_{\{xyz\}} \end{bmatrix} \begin{bmatrix} e_a \end{bmatrix}$$

IDEJA: nalogo rešujemo v bazi $\{1,2,3\}$

$$\underline{\underline{\varepsilon}}_{\{1,2,3\}} = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix}$$

$$\vec{e_a} = a_1\vec{e_1} + a_2\vec{e_2} + a_3\vec{e_3}$$

$$\vec{\varepsilon}_x = \varepsilon_{xx}\vec{e}_x + \underbrace{\varepsilon_{xy}\vec{e}_y + \varepsilon_{xz}\vec{e}_z}_{\text{STRIŽNI DEL}} \rightarrow \text{DEFORMACIJSKI VEKTOR}$$

"(živi v pravokotni ravnini na os x)"

$$\vec{\varepsilon}_a = \vec{\varepsilon}_{aa}\vec{e}_a + \vec{\varepsilon}_{ab}\vec{e}_{ab}$$

$$\varepsilon_{ab}^2 = \vec{\varepsilon}_a \cdot \vec{\varepsilon}_a - \vec{\varepsilon}_{aa}^{\,2}$$

iščemo MAX

$$\varepsilon_{aa} = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} \varepsilon_{11} & & \\ & \varepsilon_{22} & \\ & & \varepsilon_{33} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} =$$

$$= \varepsilon_{11}a_1^2 + \varepsilon_{22}a_2^2 + \varepsilon_{33}a_3^2$$

$$\varepsilon_x = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} \vec{e}_x \\ \vec{e}_y \\ \vec{e}_z \end{bmatrix}$$

$$\vec{\varepsilon}_1 = \varepsilon_{11}\vec{e}_1$$

$$\vec{e}_a = a_1\vec{e}_1 + a_2\vec{e}_2 + a_3\vec{e}_3$$

$$\varepsilon_a = \varepsilon_{11}a_1\vec{e}_1 + \varepsilon_{22}a_2\vec{e}_2 + \varepsilon_3 a_3\vec{e}_3$$

$$\text{MAX}\left(\varepsilon_{11}^2 a_1^2 + \varepsilon_{22}^2 a_2^2 + \varepsilon_{33}^2 a_3^2 - (\varepsilon_{11}a_1^2 + \varepsilon_{22}a_2^2 + \varepsilon_{33}a_3^2)^2\right)$$

ob pogoju $a_1^2 + a_2^2 + a_3^2 = 1$

$$a_3^2 = 1 - a_1^2 - a_2^2$$

$$\left.\begin{aligned} \frac{\partial F}{\partial a_1} &= \dots \\ \frac{\partial F}{\partial a_2} &= \dots \end{aligned}\right\} \rightarrow$$

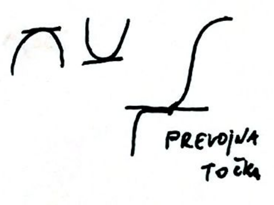
$a_1 = 0$	$a_2 = \pm\frac{\sqrt{2}}{2}$	$a_3 = \pm\frac{\sqrt{2}}{2}$	$\vec{e}_I = \pm\frac{\sqrt{2}}{2}\vec{e}_2 \pm \frac{\sqrt{2}}{2}\vec{e}_3$
$a_2 = 0$	$a_1 = \pm\frac{\sqrt{2}}{2}$	$a_3 = \pm\frac{\sqrt{2}}{2}$	$\boxed{\vec{e}_{II} = \pm\frac{\sqrt{2}}{2}\vec{e}_1 \pm \frac{\sqrt{2}}{2}\vec{e}_3}$
$a_3 = 0$	$a_1 \pm\frac{\sqrt{2}}{2}$	$a_2 = \pm\frac{\sqrt{2}}{2}$	$\vec{e}_{III} = \pm\frac{\sqrt{2}}{2}e_1 \pm \frac{\sqrt{2}}{2}\vec{e}_2$

$$\gamma_I = \pm\frac{\varepsilon_{22} - \varepsilon_{33}}{2}$$

$$\boxed{\gamma_{II} = \pm\frac{\varepsilon_{11} - \varepsilon_{33}}{2}}$$

$$\gamma_{III} = \pm\frac{\varepsilon_{11} - \varepsilon_{22}}{2}$$

EKSTREMNA STRIŽNA DEFORMACIJA



za $\varepsilon_{11} \geq \varepsilon_{22} \geq \varepsilon_{33}$

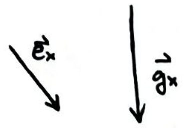
"na simetrali se zgodijo kandidati za ekstrem - izkaže se da je to tista diagonala, ki leži med 1 in 3"

ostale mere za deformiranje

- vemo že $D_{xx} \approx \varepsilon_{xx}$ $\quad D_{yy} \approx \varepsilon_{yy}$ $\quad D_{zz} \approx \varepsilon_{zz}$

  $2\varepsilon_{xy} \approx D_{xy}$ $\quad 2\varepsilon_{xz} \approx D_{xz}$ $\quad 2\varepsilon_{yz} \approx D_{yz}$

(3) sprememba smeri



$|\vec{e}_x \times \vec{g}_x| = |\vec{e}_x||\vec{g}_x| \sin\alpha$

$\alpha \approx$ zasuk koordinatnega sistema

↳ za majhne zasuke

$\vec{R}_x = \vec{e}_x \times \vec{g}_x$

$\vec{R}_y = \vec{e}_y \times \vec{g}_y$

$\vec{R}_z = \vec{e}_z \times \vec{g}_z$

$$R_x = \begin{vmatrix} e_x & e_y & e_z \\ 1 & 0 & 0 \\ 1+\frac{\partial u_x}{\partial x} & \frac{\partial u_y}{\partial x} & \frac{\partial u_z}{\partial x} \end{vmatrix} = \frac{\partial u_y}{\partial x}\vec{e}_z - \frac{\partial u_z}{\partial x}\vec{e}_y$$

«nikjer se ne smejo indeksi ponovit»

$$R_y = \begin{vmatrix} e_x & e_y & e_z \\ 0 & 1 & 0 \\ \frac{\partial u_x}{\partial y} & 1+\frac{\partial u_y}{\partial y} & \frac{\partial u_z}{\partial y} \end{vmatrix} = \frac{\partial u_z}{\partial y}\cdot\vec{e}_x - \frac{\partial u_x}{\partial y}\vec{e}_z$$

$$R_z = \begin{vmatrix} e_x & e_y & e_z \\ 0 & 0 & 1 \\ \frac{\partial u_x}{\partial z} & \frac{\partial u_y}{\partial z} & 1+\frac{\partial u_z}{\partial z} \end{vmatrix} = \frac{\partial u_x}{\partial z}\vec{e}_y - \frac{\partial u_y}{e_z}\vec{e}_x$$

$$\vec{R}_x + \vec{R}_y + \vec{R}_z = \left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right)\vec{e}_x + \left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right)\vec{e}_y + \left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)\vec{e}_z$$

→ 3 spremembe kotov (ki so majhni

$$= \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u_x & u_y & u_z \end{vmatrix}$$ = ROTOR VEKTORJA $\vec{u} = rot(\vec{u})$

$\omega_{yz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right) \equiv \omega_x$ rotacija vrtenje okoli osi x

$\omega_{zx} = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right) \equiv -\omega_{xz} = \omega_y$

$\omega_{xz} \equiv -\omega_y$

«Če zamenjamo indeksa pride $-$»

$\omega_{xy} = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) \equiv \omega_z$

$$\boxed{\begin{gathered} \underline{\underline{\omega}} = \begin{bmatrix} 0 & \omega_{xy} & \omega_{xz} \\ -\omega_{xy} & 0 & \omega_{yz} \\ -\omega_{xz} & -\omega_{yz} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \omega_z & -\omega_y \\ -\omega_z & 0 & \omega_x \\ \omega_y & -\omega_x & 0 \end{bmatrix} \\ \vec{\omega} = \omega_x\vec{e}_x + \omega_y\vec{e}_y + \omega_z\vec{e}_z \end{gathered}}$$

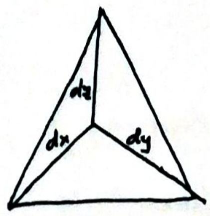
TENZOR MAJHNIH ROTACIJ (antisimetričen)

→ POENOSTAVLJENA MATRIKA VRTENJA

→ ROTACIJSKI VEKTOR

(4) Sprememba PROSTORNINE

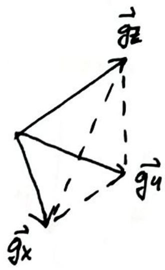
$$\varepsilon_v = \frac{V - V_0}{V_0} = \frac{V}{V_0} - 1$$



$$V_0 = \frac{1}{6}\, dx\, dy\, dz$$

$$V_0 = \frac{1}{6}\, dx\, dy\, dz\, \vec{e}_x \cdot (\vec{e}_y \times \vec{e}_z)$$

$$= \frac{1}{6}\, dx\, dy\, dz\, \vec{g}_x (\vec{g}_y \times \vec{g}_z)$$



$$= \frac{1}{6}\, \underbrace{\begin{vmatrix} 1 + \frac{\partial u_x}{\partial x} & \frac{\partial u_y}{\partial x} & \frac{\partial u_z}{\partial x} \\ \frac{\partial u_x}{\partial y} & 1 + \frac{\partial u_y}{\partial y} & \frac{\partial u_z}{\partial y} \\ \frac{\partial u_x}{\partial z} & \frac{\partial u_y}{\partial z} & 1 + \frac{\partial u_z}{\partial z} \end{vmatrix}}_{= \underline{\underline{F}}}$$

dxdydz

$$\varepsilon_v = \frac{\frac{1}{6}\, \cancel{dx\, dy\, dz}\, \det F}{\frac{1}{6}\, \cancel{dx\, dy\, dz}} - 1$$

$$\varepsilon_v = \det F - 1$$

0 jih zanemarimo

$$\det F \approx 1 + \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} + \text{členi višjih redov}$$

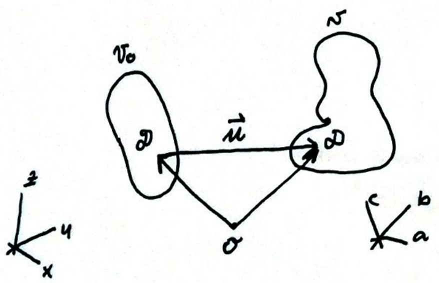
$\Rightarrow \det F = 1 +$ vsota diagonalcev $= 1 +$ sled $(\underline{\underline{\varepsilon}})$

$$\Rightarrow \boxed{\varepsilon_v \approx \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}} = \frac{\partial u_a}{\partial a} + \frac{\partial u_b}{\partial b} + \frac{\partial u_c}{\partial c} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$$

I. INVARIANTA

10. 11. 2021

DEFORMACIJE



$\vec{u}$... podatek o spremembi lege delca

u_x, u_y, u_z → 3 neznanke

$$\left.\begin{aligned} &\varepsilon_{xx} = \frac{\partial u_x}{\partial x} \quad \varepsilon_{yy} = \frac{\partial u_y}{\partial y} \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z} \\ &\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right) \\ &\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right) \\ &\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right) \end{aligned}\right\}$$

6 neznank

KINEMATIČNE ENAČBE

= 9 neznank

6 enačb

$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ & \varepsilon_{yy} & \varepsilon_{yz} \\ S/M & & \varepsilon_{zz} \end{bmatrix} = [\varepsilon_{ij}]_{ij}$$

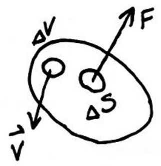
$$\varepsilon_{aa} = \boxed{e_a^T} \boxed{\underline{\underline{\varepsilon}}\{x,y,z\}} \boxed{e_a} \qquad \varepsilon_{ab} = \boxed{e_b^T} \boxed{\underline{\underline{\varepsilon}}\{x,y,z\}} \boxed{e_a}$$

Napetosti in ravnotežne enačbe

$$\Sigma \vec{F}_i = \vec{0}$$
$$\Sigma \vec{M}_i = \vec{0}$$

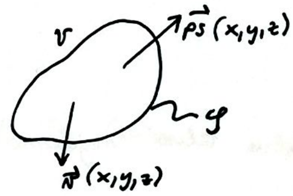
$$\frac{\vec{F}}{\Delta S} = \vec{p}$$

$$\lim_{\Delta S \to 0} \frac{\Delta \vec{F}}{\Delta S} = \vec{p}_s \quad \left[\frac{N}{m^2}\right] \quad \left(\frac{kN}{cm^2}\right)$$



$\rightarrow$

$$\lim_{\Delta V} \frac{\vec{V}}{\Delta V} = \vec{v} \left[\frac{N}{m^3}\right]$$



RAVNOTEŽJE TELESA

$$\lim_{\Delta S \to 0} \frac{\Delta F}{\Delta S} = \frac{d\vec{F}}{dS} = \vec{p}_s \rightarrow d\vec{F} = \vec{p}_s \cdot dS$$

$$\vec{F}_s = \int_{\mathcal{S}} d\vec{F} = \int_{\mathcal{S}} \vec{p}_s \, dS$$

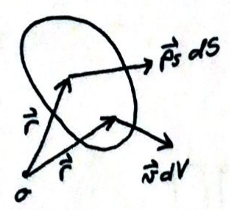
$$\vec{V} = \int_{v} \vec{v} \, dV$$

RAVNOTEŽJE SIL

$$\boxed{\int_{\mathcal{S}} \vec{p}_s \, dS + \int_{v} \vec{v} \, dV = \vec{0}}$$

Splošen izrek o gibalni količini

$$\int_{\mathcal{S}} \vec{p}_s \, dS + \int_{v} \vec{v} \, dV = \frac{d}{dt} \int_{v} \rho \, \vec{r} \, dV$$

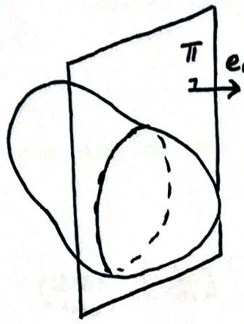


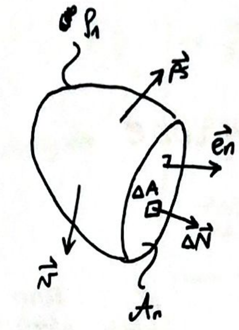
$$\boxed{\int_S \vec r \times \vec p_s\, dS + \int_V \vec r \times \vec N\, dV = 0}$$

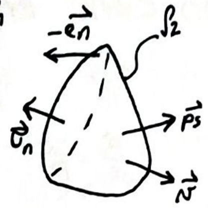
RAVNOTEŽJE MOMENTOV

izrek o vrtilni količini

$$\int_S \vec r \times \vec p_s\, dS + \int_V \vec r \times \vec N\, dV = \frac{d}{dt}\int_V \vec r \times \rho\, \dot{\vec r}\, dV \quad \Big\}\ \dot{\vec r} \approx 0$$







π… prerezna ravnina
e_n… normala na prerezno ravnino

$$\boxed{\lim_{\Delta A \to 0} \frac{\Delta \vec N_n}{\Delta A} = \frac{d\vec N_n}{dA_n} = \vec\sigma_n}$$

NAPETOST / NAPETOSTNI VEKTOR

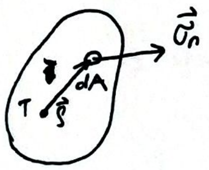
"v delu telesa najdemo napetosti, ne pa sil"

* ! POZOR: napetostni vektor je odvisen od izbire prerezne ravnine ! *

$$dN_n = \vec\sigma_n\, dA$$

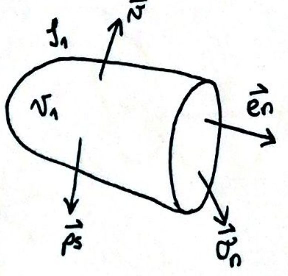
$$\boxed{\vec N_n = \int_{A_n} \vec\sigma_n\, dA_n}$$

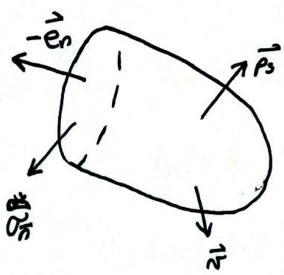
REZULTANTNA NOTRANJA SILA PREREZA



$$\boxed{\vec M_n = \int_{A_n} \vec\rho \times \vec\sigma_n\, dA_n}$$

REZULTANTNI MOMENT





o

LEVI DEL

DESNI DEL

LEVI DEL | **DESNI DEL**

$$\int_{s_1} \vec{p}_s\, ds + \int_A \vec{\sigma}_n\, dA + \int_{V_1} \vec{v}\, dV = \vec{0} \quad \oplus \quad \int_{s_2} \vec{p}_s\, ds + \int_{V_2} \vec{v}\, dV + \int_A \vec{\sigma}_{\bar{n}}\, dA = \vec{0}$$

$$\int_{s_1} \vec{r} \times \vec{p}_s\, ds + \int_A \vec{r} \times \vec{\sigma}_n\, dA + \int_{V_1} \vec{r} \times \vec{v}\, dV = 0 \qquad \int_{s_2} \vec{r} \times \vec{p}_s\, ds + \int_{V_2} \vec{r} \times \vec{v}\, dV + \int_A \vec{r} \times \vec{\sigma}_{\bar{n}}\, dA = 0$$

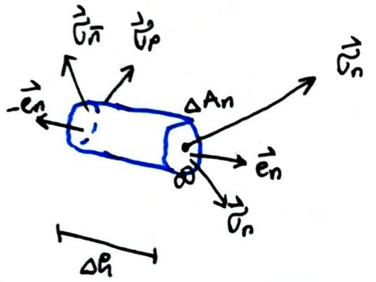
$$\underbrace{\int_s \vec{p}_s\, ds + \int_V \vec{v}\, dV}_{=0} + \boxed{\int_A (\vec{\sigma}_n + \vec{\sigma}_{\bar{n}})\, dA = \vec{0}} \rightarrow \vec{N}_n = -\vec{N}_{\bar{n}}$$

$$\boxed{\int_A \vec{r} \times (\vec{\sigma}_n + \vec{\sigma}_{\bar{n}})\, dA = \vec{0}} \rightarrow \vec{M}_n = -\vec{M}_{\bar{n}}$$

$$\int_{s_1} + \int_{s_2} = \int_s$$

$$\int_{V_1} + \int_{V_2} = \int_V$$

* !POZOR: $\vec{\sigma}_n$ in $\vec{e}_n$ OBIČAJNO <u>NISTA</u> VZPOREDNI!*



izrežem majhen valj v okolici delca

$$\vec{\sigma}_n \Delta A_n + \vec{\sigma}_{\bar{n}} \Delta A_n + \vec{\sigma}_p \cdot o \Delta h + \vec{v} \cdot \Delta A_n \Delta h = \vec{0} \quad / : \Delta A_n$$

$$\vec{\sigma}_n + \vec{\sigma}_{\bar{n}} + \vec{\sigma}_p \cancel{\frac{o \Delta h}{\Delta A_n}}_{\to 0} + \cancel{\vec{v} \Delta h}_{\to 0} = 0$$

$$\lim_{\Delta h \to 0}$$

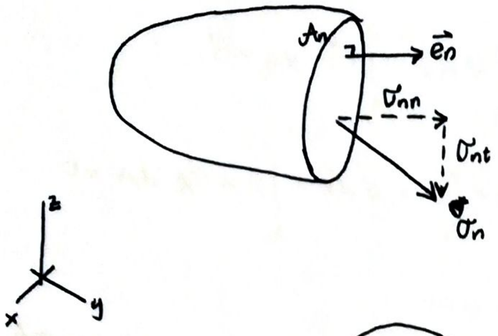
$$\vec{\sigma}_n + \vec{\sigma}_{\bar{n}} = 0$$

$$\boxed{\vec{\sigma}_n = -\vec{\sigma}_{\bar{n}}}$$

napetostni $\vec{N}$ vektor na prerezu z ⊖ normalo je nasprotno enak vektorju v prerezu

~~napetostni vektor~~ v njem

napetosti v nekem delcu v prereznih ravninah z različno predznačeno normalo so nasprotno enake.

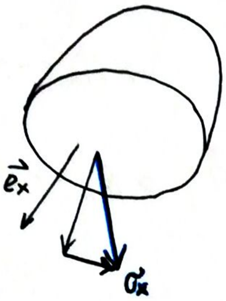


$$\boxed{\vec\sigma_n = \sigma_{nn}\vec e_n + \sigma_{nt}\vec e_t}$$

NORMALNA NAPETOST — STRIŽNA NAPETOST

$\vec e_t \perp \vec e_n$

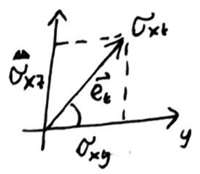
"t kot tangenta
n kot normala"

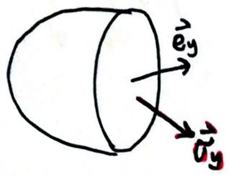


- $\vec\sigma_x = \sigma_x \vec e_x + \sigma_{xt}\vec e_t$

$\sigma_{xt}\vec e_t = \sigma_{xy}\vec e_y + \sigma_{xz}\vec e_z$

$$\boxed{\vec\sigma_x = \sigma_{xx}\vec e_x + \sigma_{xy}\vec e_y + \sigma_{xz}\vec e_z}$$

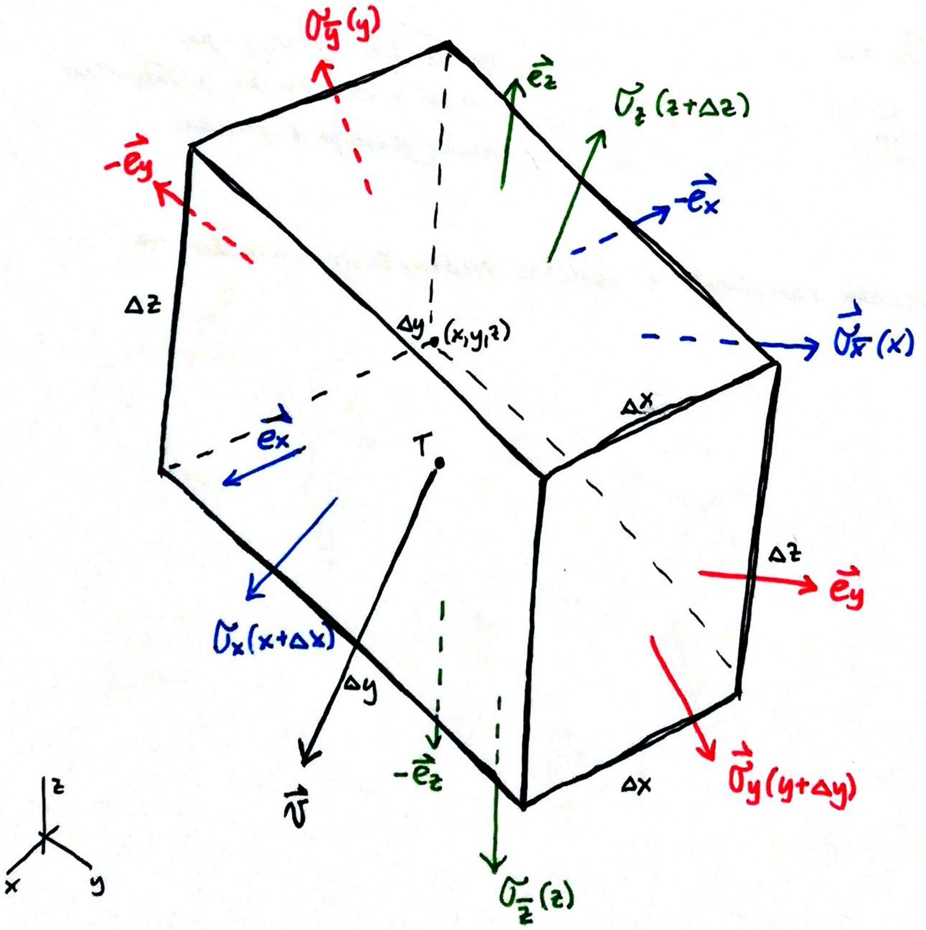




$$\boxed{\vec\sigma_y = \sigma_{yy}\vec e_y + \sigma_{yx}\vec e_x + \sigma_{yz}\vec e_z}$$

$$\boxed{\vec\sigma_z = \sigma_{zz}\vec e_z + \sigma_{zx}\vec e_x + \sigma_{zy}\vec e_y}$$

9 NOVIH NEZNANK



$\sum \vec F_i = \vec 0$

$\sum \vec M_i = \vec 0$

$$\vec{\sigma}_x(x+\Delta x)\Delta y\Delta z + \vec{\sigma}_y(y+\Delta y)\Delta x\Delta z + \vec{\sigma}_z(z+\Delta z)\Delta x\Delta y + \underbrace{\vec{\sigma}_{\bar{x}}(x)}_{-\vec{\sigma}_x}\Delta y\Delta z + \underbrace{\vec{\sigma}_{\bar{y}}(y)}_{-\vec{\sigma}_y}\Delta x\Delta z + \underbrace{\vec{\sigma}_{\bar{z}}(z)}_{-\vec{\sigma}_z}\Delta x\Delta y \oplus$$

$$\oplus\ \vec{v}\,\Delta x\Delta y\Delta z = \vec{0} \quad / : \Delta x\Delta y\Delta z$$

$$\frac{\vec{\sigma}_x(x+\Delta x)-\vec{\sigma}_x(x)}{\Delta x} + \frac{\vec{\sigma}_y(y+\Delta y)-\vec{\sigma}_y(y)}{\Delta y} + \frac{\vec{\sigma}_z(z+\Delta z)-\vec{\sigma}_z(z)}{\Delta z} + \vec{v} = \vec{0}$$

$$\lim_{\Delta x\to 0,\ \Delta y\to 0,\ \Delta z\to 0}$$

$$\boxed{\frac{\partial\vec{\sigma}_x}{\partial x} + \frac{\partial\vec{\sigma}_y}{\partial y} + \frac{\partial\vec{\sigma}_z}{\partial z} + \vec{v} = \vec{0}}$$

enačba za ravnotežje sil v diferencialni (infinitenzimalni) obliki na nivoju delca

težišče

$$\sum M^T \quad \frac{\Delta x}{2}\vec{e}_x \times \vec{\sigma}_x(x+\Delta x)\Delta y\Delta z + \frac{\Delta y}{2}\vec{e}_y \times \vec{\sigma}_y(y+\Delta y)\Delta x\Delta z + \frac{\Delta z}{2}\vec{e}_z \times \vec{\sigma}_z(z+\Delta z)\Delta x\Delta y +$$

$$-\frac{\Delta x}{2}\vec{e}_x \times \vec{\sigma}_{\bar{x}}(x)\Delta y\Delta z - \frac{\Delta y}{2}\vec{e}_y \times \vec{\sigma}_{\bar{y}}(y)\Delta x\Delta z + \frac{\Delta z}{2}\vec{e}_z \times \vec{\sigma}_{\bar{z}}(z)\Delta x\Delta y = \vec{0} \quad / : \Delta x\Delta y\Delta z$$

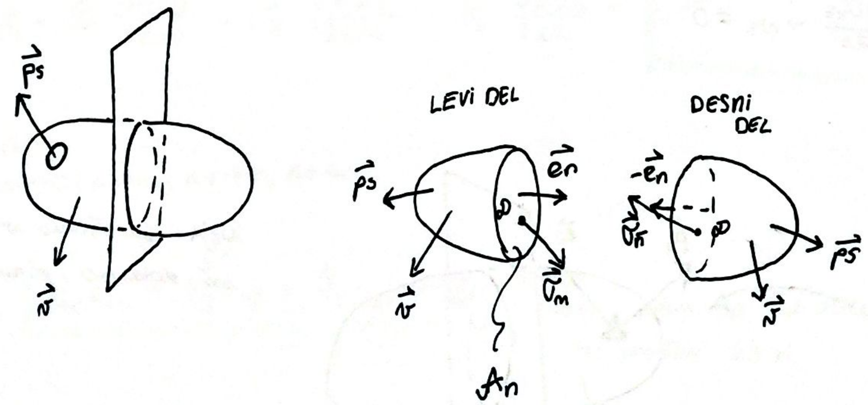
$$\frac{1}{2}\vec{e}_x \times \left(\vec{\sigma}_x(x+\Delta x)+\vec{\sigma}_x(x)\right) + \frac{1}{2}\vec{e}_y \times \left(\vec{\sigma}_y(y+\Delta y)+\vec{\sigma}_y(y)\right) + \frac{1}{2}\vec{e}_z \times \left(\vec{\sigma}_z(z+\Delta z)+\vec{\sigma}_z(z)\right) = \vec{0}$$

$$\lim_{x\to 0,\ y\to 0,\ z\to 0}$$

$$\boxed{\vec{e}_x \times \vec{\sigma}_x + \vec{e}_y \times \vec{\sigma}_y + \vec{e}_z \times \vec{\sigma}_z = \vec{0}}$$

momentni ravnotežni pogoj

11. 11. 2021



$\vec{\sigma}_n \nparallel \vec{e}_n$

$\vec{\sigma}_n = -\vec{\sigma}_{\bar{n}}$

$\vec{\sigma}_n$ ... napetostni vektor

→ koordinatni napetostni vektorji $\vec{\sigma}_x, \vec{\sigma}_y, \vec{\sigma}_z$ $\overset{?}{\text{vs}}$ $\vec{\sigma}_n$

$$\begin{cases} \dfrac{\partial\vec{\sigma}_x}{\partial x} + \dfrac{\partial\vec{\sigma}_y}{\partial y} + \dfrac{\partial\vec{\sigma}_z}{\partial z} + \vec{v} = \vec{0} \\[2ex] \vec{e}_x \times \vec{\sigma}_x + \vec{e}_y \times \vec{\sigma}_y + \vec{e}_z \times \vec{\sigma}_z = \vec{0} \end{cases}$$

ravnotežje sil v infinitenzimalnem (= neskončno majhnem) delcu telesa

*OPOMBA: enačbi veljata le za majhne deformacije

$$\vec{e}_x \times \vec{\sigma}_x = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 1 & 0 & 0 \\ \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \end{vmatrix} = \sigma_{xy}\vec{e}_z - \sigma_{xz}\vec{e}_y$$

$$\vec{e}_y \times \vec{\sigma}_y = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 1 & 0 \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \end{vmatrix} = \sigma_{yz}\vec{e}_x - \sigma_{yx}\vec{e}_z$$

$$\vec{e}_z \times \vec{\sigma}_z = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & 1 \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{vmatrix} = \sigma_{zx}\vec{e}_y - \sigma_{zy}\vec{e}_x$$

$$\underbrace{(\sigma_{yz} - \sigma_{zy})}_{=0}\vec{e}_x + \underbrace{(\sigma_{zx} - \sigma_{xz})}_{=0}\vec{e}_y + \underbrace{(\sigma_{xy} - \sigma_{yx})}_{=0}\vec{e}_z = \vec{0}$$

$$\sigma_{xy} - \sigma_{yx} = 0 \rightarrow \boxed{\sigma_{xy} = \sigma_{yx}}$$
$$\sigma_{zx} - \sigma_{xz} = 0 \rightarrow \boxed{\sigma_{zx} = \sigma_{xz}}$$
$$\sigma_{yz} - \sigma_{zy} = 0 \rightarrow \boxed{\sigma_{yz} = \sigma_{zy}}$$

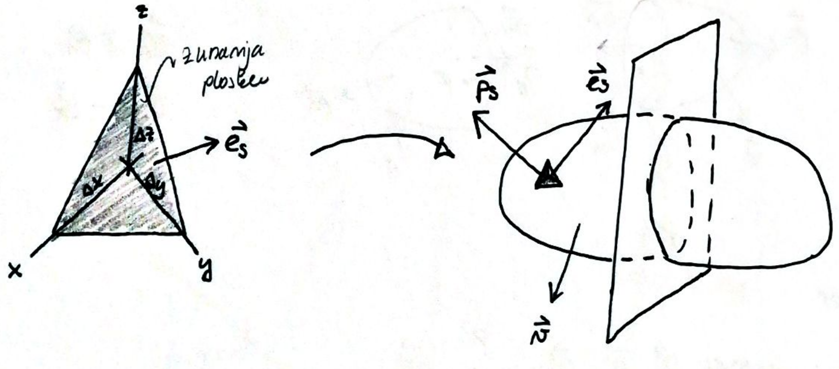
REDUKCIJA ŠTEVILA NEZNANK

$$\frac{\partial \vec{\sigma}_x}{\partial x} + \frac{\partial \vec{\sigma}_y}{\partial y} + \frac{\partial \vec{\sigma}_z}{\partial z} + \vec{\nu} = 0$$

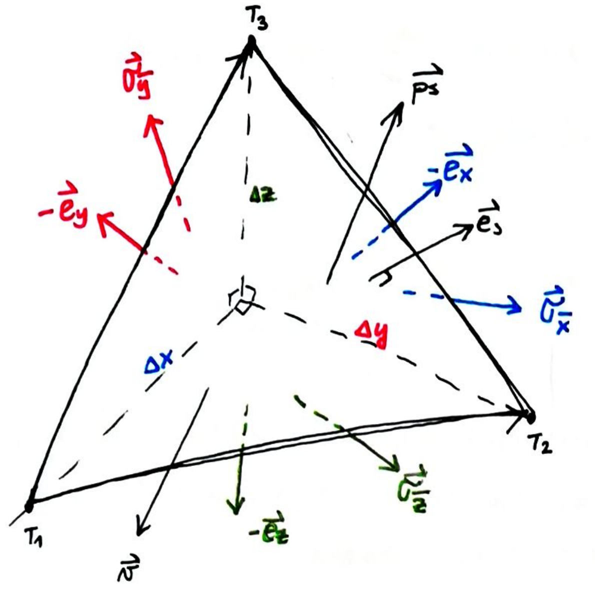
$$\boxed{\begin{aligned} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + \nu_x &= 0 \\ \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + \nu_y &= 0 \\ \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \nu_z &= 0 \end{aligned}}$$

3 skalarne ravnotežne enačbe (v notranjosti telesa)

+ ROBNI POGOJI



iDEJA: zarežemo v telo in dobimo piramidico



(1) $\vec{p_s}\cdot\Delta A_s + \vec{\sigma_x}\frac{\Delta y\Delta z}{2} + \vec{\sigma_y}\frac{\Delta x\Delta z}{2} + \vec{\sigma_z}\frac{\Delta x\Delta y}{2} + \vec{f}\,\Delta A_s\frac{\Delta h}{3} = \vec{0}$

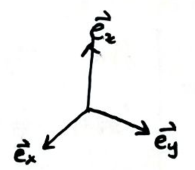
$\Delta A_s = ?$

$\Delta\vec{A_s} = \frac{1}{2}\,\vec{T_1T_2}\times\vec{T_1T_3} = \Delta A_s\,\vec{e_s}$

$\hookrightarrow$ $\Delta\vec{A_s}$ pravokoten na ravnino ki jo oklepata vektorja $\vec{T_1T_2}$ in $\vec{T_1T_3}$

$\vec{T_1T_2} = -\Delta x\,\vec{e_x} + \Delta y\,\vec{e_y}$

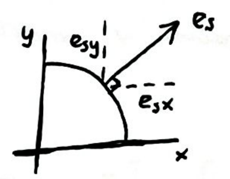
$\vec{T_1T_3} = -\Delta x\,\vec{e_x} + \Delta z\,\vec{e_z}$



$$\vec{T_1T_2}\times\vec{T_1T_3} = -\Delta x\Delta y\cdot\vec{e_y}\times\vec{e_x} - \Delta y\Delta z\cdot\vec{e_y}\times\vec{e_z} + \\ -\Delta x\Delta z\cdot\vec{e_x}\times\vec{e_z}$$

$$= \Delta x\Delta y\,\vec{e_z} + \Delta x\Delta z\,\vec{e_y} + \Delta y\Delta z\,\vec{e_x}$$

$$\vec{e_s} = \underbrace{\frac{\Delta y\Delta z}{2\Delta A_s}}_{= e_{sx}}\vec{e_x} + \underbrace{\frac{\Delta x\Delta z}{2\Delta A_s}}_{= e_{sy}}\vec{e_y} + \underbrace{\frac{\Delta x\Delta y}{2\Delta A_s}}_{= e_{sz}}\vec{e_z}$$



če (1) enačbo $/\Delta A_s$

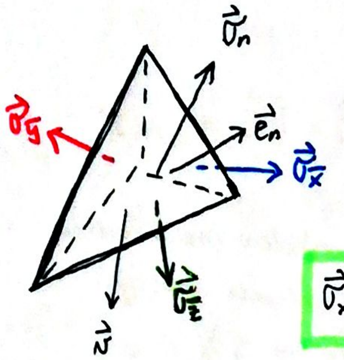
$$\vec{p_s} - \vec{\sigma_x}\underbrace{\frac{\Delta y\Delta z}{2\Delta A_s}}_{e_{sx}} - \vec{\sigma_y}\underbrace{\frac{\Delta x\Delta z}{2\Delta A_s}}_{e_{sy}} - \vec{\sigma_z}\underbrace{\frac{\Delta x\Delta y}{2\Delta A_s}}_{e_{sz}} + \underbrace{\cancel{\vec{f}\frac{\Delta h}{3}}}_{\to 0} = \vec{0}$$

lim
$\Delta h\to 0,\ \Delta x\to 0,\ \Delta y\to 0,\ \Delta z\to 0$

$$\boxed{\vec{\sigma_x}\,e_{sx} + \vec{\sigma_y}\,e_{sy} + \vec{\sigma_z}\,e_{sz} = \vec{p_s}}$$

ROBNI pogoj (deluje za vse delce na površini telesa)

KAJ PA ČE TAKO PIRAMIDO IZREŽEM ZNOTRAJ TELESA?



vlogo $\vec{p}_s$ prevzame $\vec{\sigma}_n$, vlogo $\vec{e}_s$ pa $\vec{e}_n$

$$\vec{e}_n = e_{nx}\vec{e}_x + e_{ny}\vec{e}_y + e_{nz}\vec{e}_z$$

$$\boxed{\vec{\sigma}_x e_{nx} + \vec{\sigma}_y e_{ny} + \vec{\sigma}_z e_{nz} = \vec{\sigma}_n}$$

CAUCHY-eva ENAČBA

Cauchy-eva enačba pove, da lahko iz koordinatnih napetosti vektorjev $\vec{\sigma}_x$, $\vec{\sigma}_y$ in $\vec{\sigma}_z$ določimo napetostni vektor v poljubni prerezni ravnini z normalo $\vec{e}_n$ (skozi isti delec). Zato rečemo, da napetosti tvorijo TENZOR:

$$\underline{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix}$$

MATRIKA, ki PRIPADA TENZORJU NAPETOSTI V BAZI $\{x, y, z\}$

Cauchy-eva enačba v matrični obliki

$$\underbrace{\begin{bmatrix} \sigma_{nx} \\ \sigma_{ny} \\ \sigma_{nz} \end{bmatrix}}_{\text{napetostni vektor}} = \underbrace{\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} e_{nx} \\ e_{ny} \\ e_{nz} \end{bmatrix}}_{\text{projekcija tenzorja napetosti na smer } \vec{e}_n}$$

interpretacija robne enačbe: projekcija tenzorja napetosti na ~~normalo~~ smer normale na površino mora uravnotežiti površinsko obtežbo. $\vec{\sigma}_s = \vec{p}_s$

KOORDINATNA TRANSFORMACIJA

$$\sigma_{nn} = [e_n^T]\left[\sigma_{\{x,y,z\}}\right][e_n]$$

$$\vec{\sigma}_n = \vec{\sigma}_x e_{nx} + \vec{\sigma}_y e_{ny} + \vec{\sigma}_z e_{nz}$$

$$\vec{\sigma}_n = \sigma_{nn}\vec{e}_n + \sigma_{nt}\vec{e}_t \quad /\cdot\vec{e}_n \qquad \vec{e}_n \perp \vec{e}_t$$

$$\vec{\sigma}_n \vec{e}_n = \sigma_{nn}$$

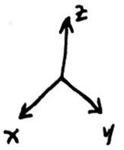
$$\sigma_{nt} = [e_t^T]\left[\sigma_{\{x,y,z\}}\right][e_n]$$

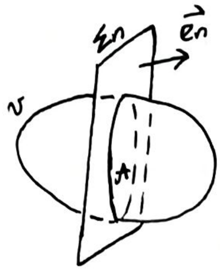
$$\vec{\sigma}_n \cdot \vec{e}_t = \sigma_{nt}$$

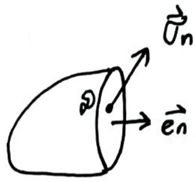
$$\vec{\sigma}_n\vec{e}_t = [e_{tx} \ e_{ty} \ e_{tz}]\begin{bmatrix} \sigma_{nx} \\ \sigma_{ny} \\ \sigma_{nz} \end{bmatrix}$$

$$\vec{\sigma}_n\vec{e}_n = [e_{nx} \ e_{ny} \ e_{nz}]\begin{bmatrix} \sigma_{nx} \\ \sigma_{ny} \\ \sigma_{nz} \end{bmatrix}$$

12. 11. 2021







$\vec{\sigma}_x, \vec{\sigma}_y, \vec{\sigma}_z$

napetostno stanje v telesu

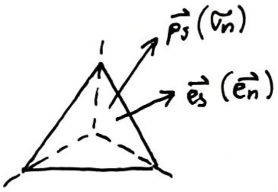
$$\frac{\partial \vec{\sigma}_x}{\partial x} + \frac{\partial \vec{\sigma}_y}{\partial y} + \frac{\partial \vec{\sigma}_z}{\partial z} + \vec{v} = \vec{0}$$ RAVNOTEŽNA enačba – SILE

$$\frac{dN_x}{dx} + p_x = 0$$ enak način enačba

$$\left.\begin{aligned} \sigma_{xy} &= \sigma_{yx} \\ \sigma_{xz} &= \sigma_{zx} \\ \sigma_{yz} &= \sigma_{zy} \end{aligned}\right\} \text{samo za male deformacije}$$

$p_s \to$ površina telesa

$e_s \to$ notranjost telesa



$[\sigma][e_s] = [p_s]$

$[\sigma][e_n] = [\sigma_n]$

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix}$$

σ_{nn} σ_{nt}

- ① MAX (σ_{nn}, ob pogoju $e_{nx}^2 + e_{ny}^2 + e_{nz}^2 = 1$)

  vemo $(\underline{\underline{\sigma}} - \lambda \underline{\underline{I}})\underline{e} = \underline{0}$ (problem lastnih vrednosti)

② alternativna naloga: iščemo tako smer $\vec{e}_n$, da bo strižna komponenta enaka 0!

~~ETAN~~ $\vec{e}_1$: $\vec{\sigma}_1 = \sigma_{11}\vec{e}_1 + \vec{0}$

$$\begin{bmatrix} & & \\ & \sigma & \\ & & \end{bmatrix}\begin{bmatrix} e_{1x} \\ e_{1y} \\ e_{1z} \end{bmatrix} = \sigma_{11}\begin{bmatrix} e_{1x} \\ e_{1y} \\ e_{1z} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{xx}-\sigma_{11} & \sigma_{xy} & \sigma_{xz} \\ & \sigma_{yy}-\sigma_{11} & \sigma_{yz} \\ s/m & & \sigma_{zz}-\sigma_{11} \end{bmatrix}\begin{bmatrix} e_{1x} \\ e_{1y} \\ e_{1z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ nalogi ① in ② sta enakovredni!

$\sigma_{11}, \sigma_{22}, \sigma_{33}$ $\qquad \sigma_{11} \geq \sigma_{22} \geq \sigma_{33}$