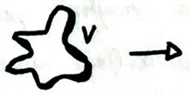
OSNOVE MEHANIKE

1. Modeli teles in vplivi med njimi in nanje
2. Toga telesa
   - analiza nosilcev
3. Ravnotežni pogoji
   * literatura: Stanek, Turk: Statika I
   - statično določene konstrukcije
4. Deformabilna telesa
   * literatura: Stanek, Turk: Osnove mehanike trdnih teles
5. Enačbe linijskih nosilcev
   - statično nedoločene konstrukcije (realne situacije)
   * literatura: Srpčič: Trdnost II
     spletna učilnica - spletna verzija knjige

UVOD — 1.

Modeliranje teles in vplivov nanje — 1.1.

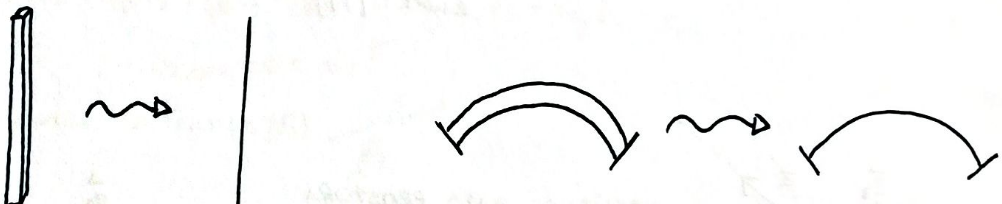
① delec = model majhnega telesa ob upoštevanju njegove mase



D (zaseda 1 točko prostora)

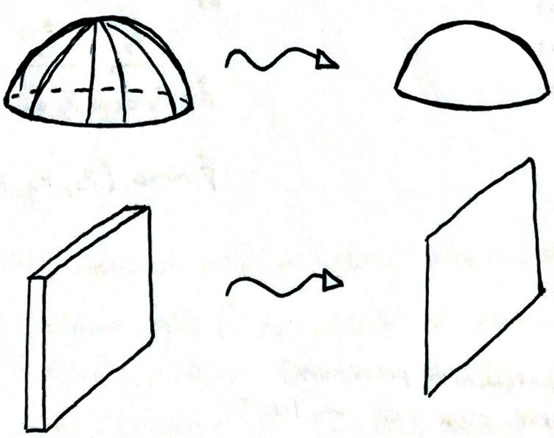
* mala telesa *

② nosilec = model telesa, kjer je 1 dimenzija pomembnejša



* dolžina pomembnejša od ostalih dimenzij *

③ lupine (plošče, stene)



* debelina manjša v primerjavi z dolžino? *

• MAX σ_{nt} CAUCHY KOORDINATNA TRANSFORMACIJA

$$\vec{\sigma}_n = \sigma_{nn}\vec{e}_n + \sigma_{nt}\vec{e}_t$$

iščemo smer $\vec{e}_I$ in pravokotno smer $\vec{e}_{It}$, da bo σ_{nt} največji

$$\underline{\underline{\sigma}}_{\{1,2,3\}} = \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix}$$

napetosti so diagonalne v glavnih koordinatah

$$\vec{e}_I = e_{I1}\vec{e}_1 + e_{I2}\vec{e}_2 + e_{I3}\vec{e}_3$$

$$\vec{e}_I = \pm\frac{1}{\sqrt{2}}\vec{e}_2 \pm \frac{1}{\sqrt{2}}\vec{e}_3$$

→ simetrale glavnih smeri $\varphi = 45°$

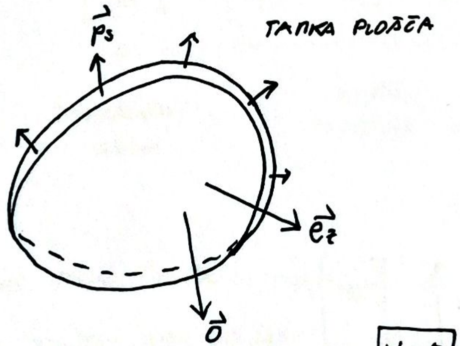
$$\boxed{\vec{e}_{II} = \pm\frac{1}{\sqrt{2}}\vec{e}_1 \pm \frac{1}{\sqrt{2}}\vec{e}_3}$$

$$\vec{e}_{III} = \pm\frac{1}{\sqrt{2}}\vec{e}_1 \pm \frac{1}{\sqrt{2}}\vec{e}_2$$

$$\sigma_{It} = \pm\frac{\sigma_{22} - \sigma_{33}}{2} = \tau_I$$

$$\boxed{\sigma_{IIt} = \tau_{II} = \pm\frac{\sigma_{11} - \sigma_{33}}{2}}$$

Ravninsko napetostno stanje



$$\vec{p}_s = p_{sx}\vec{e}_x + p_{sy}\vec{e}_y$$

stranski ploskvi z normalo $\vec{e}_z$ nista obteženi.

$\boxed{V_z = 0}$

$$\begin{bmatrix} & & \\ & \sigma & \\ & & \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \pm 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

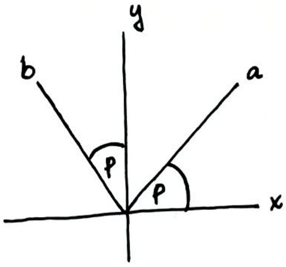
$\pm\sigma_{xz} = 0$

$\pm\sigma_{yz} = 0$

$\pm\sigma_{zz} = 0$

$$\boxed{\begin{aligned} \frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xy}}{\partial y} + v_x &= 0 \\ \frac{\partial\sigma_{xy}}{\partial x} + \frac{\partial\sigma_{yy}}{\partial y} + v_y &= 0 \end{aligned}}$$

$$\Rightarrow \quad \underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix}$$



$\vec{e}_a = \cos\varphi\, \vec{e}_x + \sin\varphi\, \vec{e}_y$

$\vec{e}_b = -\sin\varphi\, \vec{e}_x + \cos\varphi\, \vec{e}_y$

transponiramo

$[\sigma_{\{x,y,z\}}]$

$$\begin{bmatrix}\sigma_{aa} & \sigma_{ab}\\ \sigma_{ab} & \sigma_{bb}\end{bmatrix} = \begin{bmatrix}\cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi\end{bmatrix}\begin{bmatrix}\sigma_{xx} & \sigma_{xy}\\ \sigma_{xy} & \sigma_{yy}\end{bmatrix}\begin{bmatrix}\cos\varphi & -\sin\varphi\\ \sin\varphi & \cos\varphi\end{bmatrix}$$

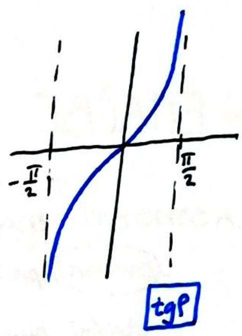
$\| \; e_b^T \qquad \| \; \sigma_{\{x,y,z\}} \qquad \| \; e_a$

$$\sigma_{ab} = \underset{e_b^T}{\begin{bmatrix}-\sin\varphi & \cos\varphi\end{bmatrix}}\begin{bmatrix}\sigma_{xx}\cos\varphi + \sigma_{xy}\sin\varphi\\ \sigma_{xy}\cos\varphi + \sigma_{yy}\sin\varphi\end{bmatrix} = -\sigma_{xx}\sin\varphi\cos\varphi + \sigma_{yy}\sin\varphi\cos\varphi - \sigma_{xy}\sin^2\varphi + \sigma_{xy}\cos^2\varphi =$$

$$= \sigma_{xy}(\cos^2\varphi - \sin^2\varphi) + (\sigma_{yy} - \sigma_{xx})(\sin\varphi\cos\varphi) = \sigma_{xy}\cdot\cos 2\varphi + \frac{1}{2}(\sigma_{yy} - \sigma_{xx})\sin 2\varphi$$

pri kotu φ_G naj bo $\sigma_{ab} = 0 \rightarrow \sin 2\varphi_G = \frac{2\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}} \cdot \cos 2\varphi_G$

$$\boxed{\operatorname{tg} 2\varphi_G = \frac{2\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}}}$$

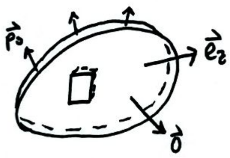


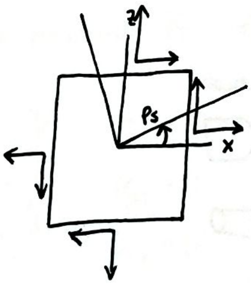
$$\boxed{\sigma_{11,22} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\sigma_{xy}^2 + \frac{1}{4}\underbrace{(\sigma_{xx} - \sigma_{yy})^2}}}$$

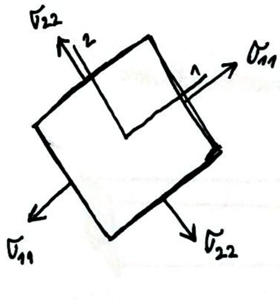
strig

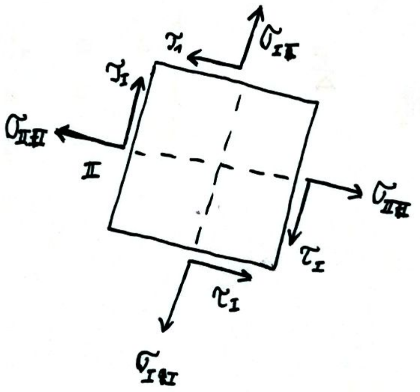
diagonalna člena → razlika normalnih členov

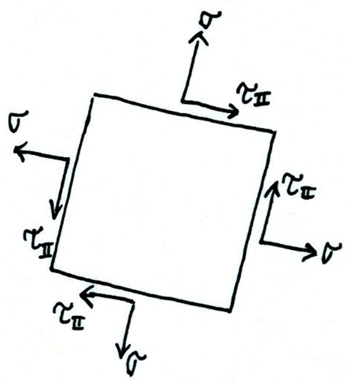
$$\tau_{I,II} = \pm\sqrt{\sigma_{xy}^2 + \frac{1}{4}(\sigma_{xx} - \sigma_{yy})^2}$$











Povzetek

- NEZNANKE: $\vec{u}$, $\underline{\underline{\varepsilon}}$, $\underline{\underline{\sigma}}$ = 15 neznank
  - $\vec{u}$ → 3, $\underline{\underline{\varepsilon}}$ → 6, $\underline{\underline{\sigma}}$ → 6

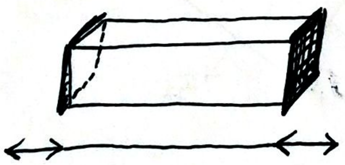
- ENAČBE: $\left.\begin{array}{l}\text{6 kinematične} \\ \text{3 ravnotežne}\end{array}\right\}$ = 9 enačb (6 premalo)
  - ⊕ neodvisne
  - ⊖ vse nove bodo odvisne

ENAČBE SNOVI (MATERIALNE ENAČBE) 5.

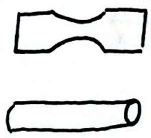
- LASTNOSTI MATERIALOV → preskusi
  - enoosni preiskus (za tlačne obremenitve) 
  - strižni preiskusi 
  - standardni upogibni preiskusi 
  - triosni preiskusi
- Ločimo
  - homogene materiale / nehomogene materiale
    - homogene → se enako obnašajo v vseh delcih telesa
  - izotropne / anizotropne (= smer – npr. les = anizotropen : v prečni smeri se bo zelo drugače obnašal, kot v smeri vlaken)
    - izotropne → se enako obnašajo v vseh smereh (jeklo, beton)

- ENOOSNI PREISKUS

PREISKUŠANEC



lahko tudi

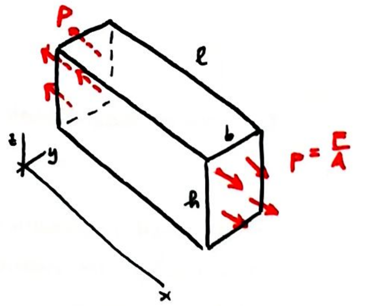


$\frac{F}{A}$

ENOOSNI PREISKUS

18. 11. 2021

PREIZSKUŠANEC: prizmatičen



sila čim bolj enakomerno razporejena

$\varepsilon_{xy} \approx 0$

$\varepsilon_{xz} \approx 0$

$\varepsilon_{yz} \approx 0$

DEFORMACIJSKO STANJE

$$\varepsilon_{xx} \approx \frac{l - l_0}{l_0}$$

$$\varepsilon_{yy} \approx \frac{b - b_0}{b_0}$$

$$\varepsilon_{zz} \approx \frac{h - h_0}{h_0}$$

$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}$$

NAPETOSTNO STANJE

Robni pogoji:

ploskev z normalo $\vec{e}_x$ $(-\vec{e}_x)$

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} (-)1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (-)p \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{matrix} \sigma_{xx} = p \ \ ((-p) = -\sigma_{xx}) \\ \sigma_{xy} = 0 \\ \sigma_{xz} = 0 \end{matrix}$$

ploskev z normalo $\vec{e}_y$ $(-\vec{e}_y)$

$$\begin{bmatrix} & & \\ & \sigma & \\ & & \end{bmatrix} \begin{bmatrix} 0 \\ \pm 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{matrix} \sigma_{xy} = 0 \\ \sigma_{yy} = 0 \\ \sigma_{yz} = 0 \end{matrix}$$

ploskev z normalo $\vec{e}_z$ $(-\vec{e}_z)$

$$\cancel{\begin{bmatrix} \sigma \end{bmatrix}} \quad \begin{matrix} \sigma_{xz} = 0 \\ \sigma_{yz} = 0 \\ \sigma_{zz} = 0 \end{matrix}$$

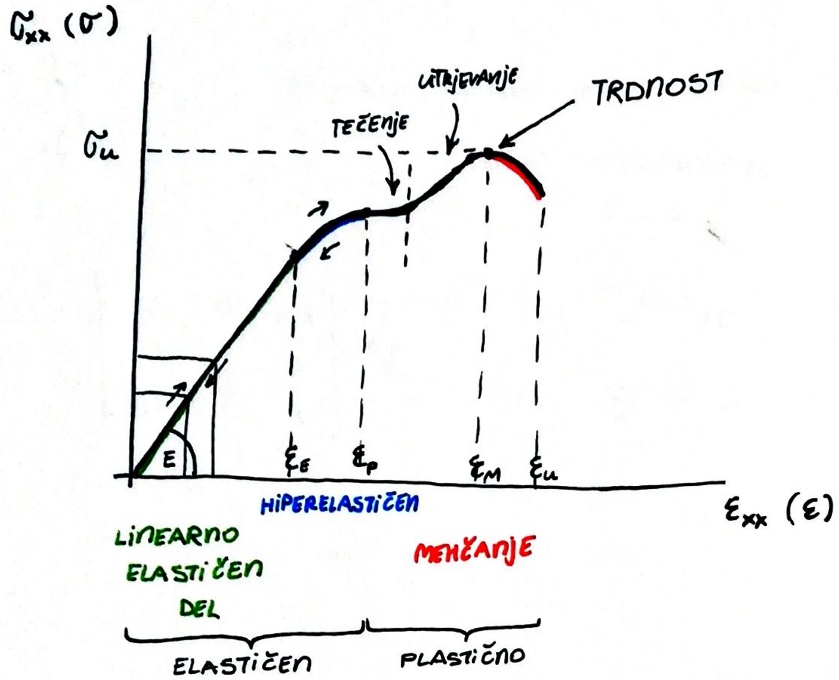
napetostno stanje

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Eva

σ - ε DIAGRAM

* IZPIT *

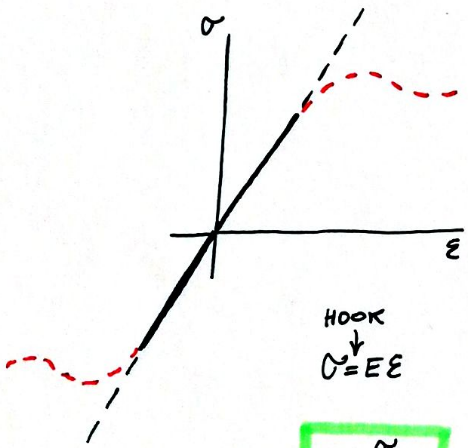


ε_p... meja elastičnosti

zveza med navadnimi napetostmi in navadnimi deformacijami

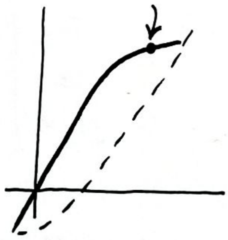
*elastičen = dokler se mi telo vrne nazaj v prvotno stanje *

računski modeli materiala



LINEARNO ELASTIČEN MATERIAL

mi se bomo omejili na tega



histerezne zanke

HOOK

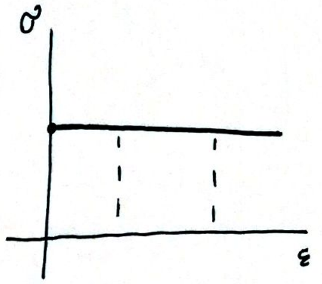
$\sigma = E\varepsilon$

⊖ omejeno območje

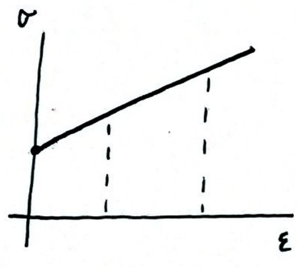
$$\boxed{E = \frac{\sigma_{xx}}{\varepsilon_{xx}}}$$

ELASTIČNI MODUL

PREISKUS → E, ν + PREDPOSTAVKA O IZOTROPIJI: vlakna se v vseh smereh obnašajo enako

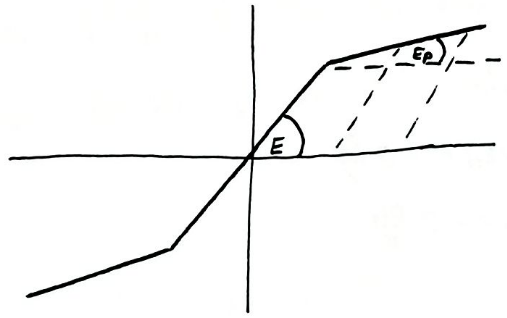


TOG IDEALNO PLASTIČEN MATERIAL



PLASTIČEN Z UTRDITVIJO

JEKLO



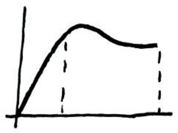
BILINEAREN ELASTIČEN Z UTRDITVIJO

posledica: manjši prerezi jeklenih konstrukcij

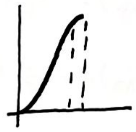
BETON
$E_b = 2000 - 4000 \frac{kN}{cm^2}$

JEKLO
$E_j = 19000 - 21000 \frac{kN}{cm^2}$

Ločimo še:



DUKTILNE materiale
"imamo še čas da se umaknemo pred porušitvijo" → VARNOST



KRHKE materiale

- $\frac{\varepsilon_{yy}}{\varepsilon_{xx}} \approx \text{konst.} \approx \frac{\varepsilon_{zz}}{\varepsilon_{xx}}$

$\varepsilon_{yy} = -\nu \varepsilon_{xx}$ ν Poissonov količnik
- lahko ⊖
- $-\infty < \nu \le 0,5$

$\varepsilon_{zz} = -\nu \varepsilon_{xx}$

BETON
$\nu_b = 0,1 \sim 0,3$

JEKLO
$\nu_j = 0,2 \sim 0,3$

MATERIALI
$-\infty < \nu \le 0,5$

triosno napetostno stanje v glavnih koordinatah

$$\sigma = \begin{bmatrix} \sigma_{11} & & \\ \bullet & \sigma_{22} & \\ & & \sigma_{33} \end{bmatrix}$$

$$\sigma_{11} = E\varepsilon_{11}$$

$$\varepsilon_{11} = \frac{\sigma_{11}}{E} = \varepsilon_{11}(\sigma_{11})$$

$$\varepsilon_{22}(\sigma_{11}) = -\nu\varepsilon_{11} = -\nu\frac{\sigma_{11}}{E}$$

$$\varepsilon_{33}(\sigma_{11}) = -\nu\varepsilon_{11} = -\nu\frac{\sigma_{11}}{E}$$

$$\sigma_{22} = E\sigma_{22}$$

$$\varepsilon_{22}(\sigma_{22}) = \frac{\sigma_{22}}{E}$$

$$\varepsilon_{11}(\sigma_{22}) = -\frac{\nu}{E}\sigma_{22}$$

$$\varepsilon_{33}(\sigma_{22}) = -\frac{\nu}{E}\sigma_{22}$$

$$\sigma_{33} = E\varepsilon_{33}$$

$$\varepsilon_{33}(\sigma_{33}) = \frac{\sigma_{33}}{E}$$

$$\varepsilon_{11}(\sigma_{33}) = -\frac{\nu}{E}\sigma_{33}$$

$$\varepsilon_{22}(\sigma_{33}) = -\frac{\nu}{E}\sigma_{33}$$

$$\boxed{\varepsilon = \alpha_T \Delta T}$$

α_T ... TEMPERATURNI RAZTEZNOSTNI KOEFICIENT

$$\varepsilon_{11}(\Delta T) = \alpha_T \Delta T$$

$$\varepsilon_{22}(\Delta T) = \alpha_T \Delta T$$

OSTALI VPLIVI

$$\varepsilon_{11} = \frac{\sigma_{11}}{E} - \frac{\nu}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} + \alpha_T\Delta T + \varepsilon_K$$

$$\boxed{\begin{aligned} \varepsilon_{11} &= \frac{1+\nu}{E}\sigma_{11} - \frac{\nu}{E}\overbrace{(\sigma_{11}+\sigma_{22}+\sigma_{33})}^{I_\sigma} + \alpha_T\Delta T + \varepsilon_K \\ \varepsilon_{22} &= \frac{1+\nu}{E}\sigma_{22} - \frac{\nu}{E}(\sigma_{11}+\sigma_{22}+\sigma_{33}) + \alpha_T\Delta T + \varepsilon_K \\ \varepsilon_{33} &= \frac{1+\nu}{E}\sigma_{33} - \frac{\nu}{E}(\sigma_{11}+\sigma_{22}+\sigma_{33}) + \alpha_T\Delta T + \varepsilon_K \end{aligned}}$$

POSPLOŠITEV HOOKOVEGA ZAKONA NA GLAVNE SMERI

$\varepsilon_{xx}, \varepsilon_{yy}$?

$$\varepsilon_{xx} = [\underline{e}_x]\left[\underline{\underline{\varepsilon}}_{\{1,2,3\}}\right]\left[\underline{e}_x\right] \qquad \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix}$$

$$\vec{e}_x = e_{x1}\vec{e}_1 + e_{x2}\vec{e}_2 + e_{x3}\vec{e}_3$$

$$e_{x1}^2 + e_{x2}^2 + e_{x3}^2 = 1$$ ENOTSKI VEKTOR

$$\varepsilon_{xx} = \varepsilon_{11}e_{x1}^2 + \varepsilon_{22}e_{x2}^2 + \varepsilon_{33}e_{x3}^2$$

$$= \frac{1+\nu}{E}(\sigma_{11}\cdot e_{x1}^2 + \sigma_{22}\cdot e_{x2}^2 + \sigma_{33}\cdot e_{x3}^2) - \frac{\nu}{E}I_\sigma \underbrace{(e_{x1}^2 + e_{x2}^2 + e_{x3}^2)}_{=1} + (\alpha_T\Delta T + \varepsilon_K)\underbrace{(e_{x1}^2 + e_{x2}^2 + e_{x3}^2)}_{=1}$$

$$\varepsilon_{xx} = \frac{1+\nu}{E}\sigma_{xx} - \frac{\nu}{E}(\sigma_{xx}+\sigma_{yy}+\sigma_{zz}) + \alpha_T\Delta T + \varepsilon_K$$

$$\boxed{\begin{aligned}\varepsilon_{xx} &= \frac{1+\nu}{E}\sigma_{xx} - \frac{\nu}{E}(\sigma_{xx}+\sigma_{yy}+\sigma_{zz}) + \alpha_T \Delta T + \varepsilon_K \\ \varepsilon_{yy} &= \frac{1+\nu}{E}\sigma_{yy} - \frac{\nu}{E}(\sigma_{xx}+\sigma_{yy}+\sigma_{zz}) + \alpha_T \Delta T + \varepsilon_K \\ \varepsilon_{zz} &= \frac{1+\nu}{E}\sigma_{zz} - \frac{\nu}{E}(\sigma_{xx}+\sigma_{yy}+\sigma_{zz}) + \alpha_T \Delta T + \varepsilon_K\end{aligned}}$$

$$\varepsilon_{xy} = [\, e_y \,]\left[\quad \underline{\underline{\varepsilon}}_{\{1,2,3\}} \quad\right]\left[\, e_x \,\right]$$

$$\vec{e}_y = e_{y1}\vec{e}_1 + e_{y2}\vec{e}_2 + e_{y3}\vec{e}_3$$
$$\vec{e}_x = e_{x1}\vec{e}_1 + e_{x2}\vec{e}_2 + e_{x3}\vec{e}_3$$
$$\perp$$

$$\boxed{e_{x1}e_{y1} + e_{x2}e_{y2} + e_{x3}e_{y3} = 0}$$

$$\varepsilon_{xy} = \varepsilon_{11}e_{x1}e_{y1} + \varepsilon_{22}e_{x2}e_{y2} + \varepsilon_{33}e_{x3}e_{y3} = \frac{1+\nu}{E}\underbrace{\left(\sigma_{11}e_{x1}e_{y1} + \sigma_{22}e_{x2}e_{y2} + \sigma_{33}e_{x3}e_{y3}\right)}_{=\sigma_{xy}} +$$
$$+\left(-\frac{\nu}{E}I_\sigma + \alpha_T\Delta T + \varepsilon_K\right)\underbrace{\left(e_{x1}e_{y1} \cancel{+ e_{x2}e_{y2}} + e_{x3}e_{y3}\right)}_{\emptyset}$$

$$\boxed{\begin{aligned}\varepsilon_{xy} &= \frac{1+\nu}{E}\sigma_{xy} \\ \varepsilon_{xz} &= \frac{1+\nu}{E}\sigma_{xz} \\ \varepsilon_{yz} &= \frac{1+\nu}{E}\sigma_{yz}\end{aligned}}$$

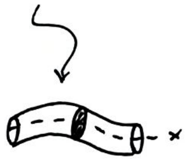
upogib nosilca

2.12.2021

način

$\omega_x \approx 0$

zanemarili bomo torzijske zasuke



$\boxed{\varepsilon_{xx} \neq 0}$

$\varepsilon_{yy} \approx 0$

$\varepsilon_{zz} \approx 0$

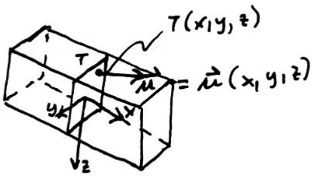
$\varepsilon_{yz} \approx 0$

$\varepsilon_{xy} \approx 0$

$\varepsilon_{xz} \approx 0$

MODEL

$$\varepsilon(x,y,z) = \begin{bmatrix} \varepsilon_{xx}(x,y,z) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

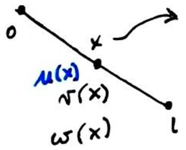


$\varepsilon_{yy} = 0 = \frac{\partial u_y}{\partial y} \rightarrow u_y$ Ni odvisen od y in z

$\varepsilon_{zz} = 0 = \frac{\partial u_z}{\partial z} \rightarrow u_z$ Ni odvisen od z in y

$\varepsilon_{yz} = 0 = \frac{1}{2}\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right)$

$\omega_x = 0 = \frac{1}{2}\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right) \rightarrow \frac{\partial u_z}{\partial y} = \frac{\partial u_y}{\partial z} = 0$



redukcija na referenčno os

$\Rightarrow u_y = u_y(x) \equiv v(x)$

$u_z = u_z(x) \equiv w(x)$

novi neznanki: prečna pomika referenčne osi nosilca

$u_x = ?$

$u_x = u_x(x,y,z)$

$$u_x = \int_{T_0}^{T} du_x$$

$$du_x = \frac{\partial u_x}{\partial x}dx + \frac{\partial u_x}{\partial y}dy + \frac{\partial u_x}{\partial z}dz$$

$\frac{\partial u_x}{\partial x} = \varepsilon_{xx}$

$\varepsilon_{xy} = 0 = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right)$

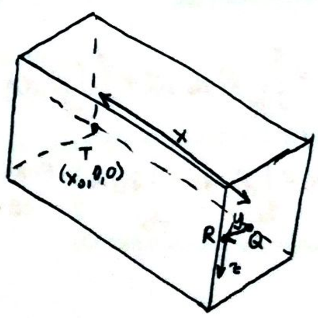
$\Rightarrow \frac{\partial u_x}{\partial y} = -\frac{\partial u_y}{\partial x} = -\frac{\partial v(x)}{\partial x} \equiv -\frac{dv}{dx}$

$\varepsilon_{xz} = 0 = \frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right)$

$\Rightarrow \frac{\partial u_x}{\partial z} = -\frac{\partial u_z}{\partial x} = -\frac{dw}{dx}$

$$\boxed{du_x = \frac{\partial u_x}{\partial x}dx - \frac{dv}{dx}dy - \frac{dw}{dx}dz}$$

integral $\int_{T_0}^{T}$ razstavimo na 3 koordinatne integrale



$T_0 \rightarrow Q \rightarrow R \rightarrow T$

$$\int_{T_0}^{T} = \int_{T_0}^{Q} + \int_{Q}^{R} + \int_{R}^{T}$$

$$\int_{T_0}^{T} du_x = \int_{T_0}^{Q} \frac{\partial u_x}{\partial x}\, dx - \int_{Q}^{R} \frac{\partial v}{\partial x}\, dy - \int_{R}^{T} \frac{\partial w}{\partial x}\, dz$$

$$= \underbrace{u_0(x_0,0,0) + \int_{x_0}^{x} \varepsilon_{xx}(\tilde{x},0,0)\, d\tilde{x}}_{\text{osni pomik} \equiv u(x)} - \frac{dv}{dx}\int_0^y dy - \frac{dw}{dx}\int_0^z dz$$

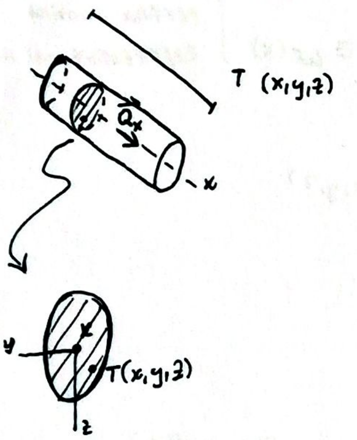
$$\boxed{u_x(x,y,z) = u(x) - \frac{dv}{dx}\, y - \frac{dw}{dx}\, z}$$

* $u(x)$, $w(x)$, $v(x)$ nadomeščajo $u(x,y,z)$ *

$$\omega_y = \omega_{zx} = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right) = -\frac{\partial u_z}{\partial x} = -\frac{dw}{dx}$$

$$\omega_z = \omega_{xy} = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) = -\frac{\partial u_y}{\partial x} = \frac{dv}{dx}$$

3.12.2021



① KINEMATIKA

$\varepsilon_{xx} \neq 0$, ostalo zanemarimo

$+ \omega_x = 0$

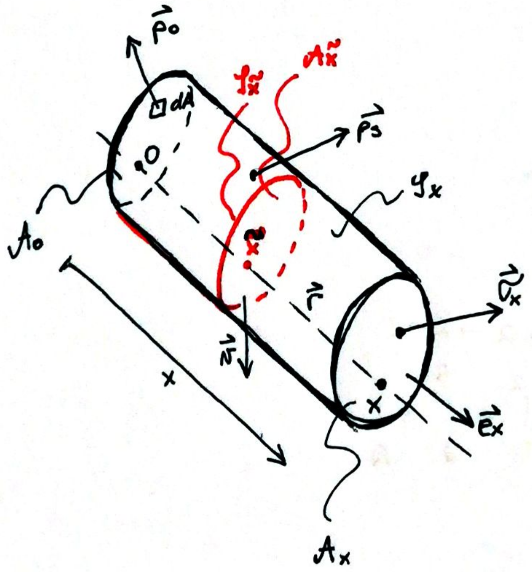
$$\boxed{u_x(x,y,z) = u(x) - \frac{dv}{dx}\, y - \frac{dw}{dx}\, z}$$

$u(x)$, $v(x)$, $w(x)$ postanejo neznanke

$$\varepsilon_{xx}(x,y,z) = \boxed{\varepsilon_{xx} = \frac{\partial u_x}{\partial x} = \frac{du}{dx} - \frac{d^2 v}{dx^2}\, y - \frac{d^2 w}{dx^2}\, z}$$ kinematična enačba nosilca

② RAVNOTEŽNE ENAČBE

nosilec prerežemo in zapišemo ravnotežje za levi del



$$\overbrace{\int_{A_0} \vec{p}_0\, dA}^{N_0} + \int_{S_x} \vec{p}_s\, ds + \int_{V_x} \vec{v}\, dV + \int_{A_x} \vec{\sigma}_x\, dA$$

$$\boxed{\vec{N} = \int_A \vec{\sigma}_x\, dA}$$

$$\vec{N}_0 + \int_0^x \int_{S_{\tilde{x}}} \vec{p}_s\, ds\, d\tilde{x} + \int_0^x \int_{A_{\tilde{x}}} \vec{v}\, dA\, dx + \vec{N}(x) = 0$$

$$\boxed{\vec{p}(x) = \int_{S_x} \vec{p}_s\, ds + \int_{A_x} \vec{v}\, dA}$$ PORAZDELJENA OBTEŽBA

$$\vec{N}_0 + \int_0^x \vec{p}(\tilde{x})\, d\tilde{x} + \vec{N}(x) = \vec{0} \quad / \frac{d}{dx}$$

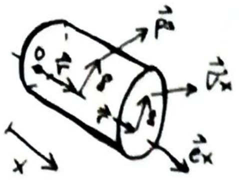
$$\boxed{\vec{p}(x) + \frac{d\vec{N}}{dx} = \vec{0}}$$

RAVNOTEŽJE SIL

$$\vec{N} = \int_A \vec{\sigma}_x \, dA$$

$$\vec{p}(x) = \int_{\ell_x} \vec{p}_s \, ds + \int_{A_x} \vec{n} \, dA$$

$$\vec{p}(x) + \frac{d\vec{N}}{dx} = \vec{0}$$



RAVNOTEŽJE MOMENTOV

$$\vec{M}_0 = \int_{A_0} \vec{\rho}_x \times \vec{p}_0 \, dA + \int_{\ell_x} \vec{r} \times \vec{p}_s \, ds + \int_{V_x} \vec{r} \times \vec{n} \, dV + \int_{A_x} \vec{r} \times \vec{\sigma}_x \, dA = \vec{0}$$

$$\vec{r} = \tilde{x}\vec{e}_x + \vec{\rho}$$

$$\vec{M}_0 + \int_{\ell_x} (\tilde{x}\vec{e}_x + \vec{\rho}) \times \vec{p}_s \, ds + \int_{V_x} (\tilde{x}\vec{e}_x + \vec{\rho}) \times \vec{n} \, dV + \int_{A_x} (x\vec{e}_x + \vec{\rho}) \times \vec{\sigma}_x \, dA = \vec{0}$$

$$\vec{M}_0 + \int_0^x \left[\int_{\ell_{\tilde{x}}} (\tilde{x}\vec{e}_x + \vec{\rho}) \times \vec{p}_s \, ds + \int_{A_{\tilde{x}}} (\tilde{x}\vec{e}_x + \vec{\rho}) \times \vec{n} \, dV \right] d\tilde{x} + x\vec{e}_x \times \int_{A_x} \vec{\sigma}_x \, dA + \int_{A_x} \vec{\rho} \times \vec{\sigma}_x \, dA$$

$$= \vec{N}$$

$$\boxed{\vec{M} = \int_{A_x} \vec{\rho} \times \vec{\sigma}_x \, dA}$$

REZULTANTNI MOMENT
NAVOR PREČNEGA PREREZA

$$\int_{\ell_x} (\tilde{x}\vec{e}_x + \vec{\rho}) \times \vec{p}_s \, ds + \int_{A_{\tilde{x}}} (\tilde{x}\vec{e}_x + \vec{\rho}) \times \vec{n} \, dA = \tilde{x}\vec{e}_x \times \vec{p}(x) + \underbrace{\int_{\ell_{\tilde{x}}} \vec{\rho} \times \vec{p}_s \, ds + \int_{A_{\tilde{x}}} \vec{\rho} \times \vec{n} \, dA}_{\text{porazdeljena momentna obtežba } \vec{\mathcal{M}}}$$

$$\boxed{\vec{\mathcal{M}}(x) = \int_{\ell_x} \vec{\rho} \times \vec{p}_s \, ds + \int_{A_x} \vec{\rho} \times \vec{n} \, dA}$$

$$\vec{M}_0 + \int_0^x \tilde{x}\vec{e}_x \times \vec{p}(\tilde{x}) \, d\tilde{x} + \int_0^x \vec{\mathcal{M}}(\tilde{x}) \, d\tilde{x} + x\vec{e}_x \times \vec{N} + \vec{M} = \vec{0} \quad / \frac{d}{dx}$$

$$x\vec{e}_x \times \vec{p}(x) + \vec{\mathcal{M}}(x) + \vec{e}_x \times \vec{N} + x\vec{e}_x \times \frac{d\vec{N}}{dx} + \frac{d\vec{M}}{dx} = \vec{0}$$

$$x\vec{e}_x \times \underbrace{\left(\vec{p}(x) + \frac{d\vec{N}}{dx} \right)}_{=0} + \vec{\mathcal{M}}(x) + \vec{e}_x \times \vec{N} + \frac{d\vec{M}}{dx} = \vec{0}$$

$$\boxed{\frac{d\vec{M}}{dx} + \vec{e}_x \times \vec{N} + \vec{\mathcal{M}} = \vec{0}}$$

POVZETEK

***** KAJ STA NOTRANJA SILA IN NOTRANJI MOMENT & KAKO JIH DOBIM?

izpit *****

$$\vec{N} = \int_{A_x} \vec{\sigma}_x \, dA \qquad \vec{N} = \vec{N}(x)$$

$$\vec{M} = \int_{A_x} \vec{\rho} \times \vec{\sigma}_x \, dA \qquad \vec{M} = \vec{M}(x)$$

$$\vec{\sigma}_x = \vec{\sigma}_x(x, y, z)$$

$$\frac{d\vec{N}}{dx} + \vec{p}(x) = \vec{0}$$

$$\frac{d\vec{M}}{dx} + \vec{e}_x \times \vec{N} + \vec{\mu} = \vec{0}$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 1 & 0 & 0 \\ N_x & N_y & N_z \end{vmatrix}$$

$$\vec{N} = N_x \vec{e}_x + N_y \vec{e}_y + N_z \vec{e}_z$$

$$\vec{\sigma}_x = \sigma_{xx} \vec{e}_x + \sigma_{xy} \vec{e}_y + \sigma_{xz} \vec{e}_z$$

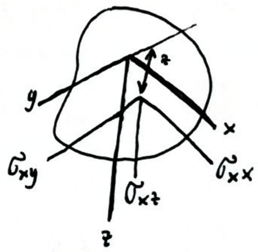
$$\boxed{N_x = \int_{A_x} \sigma_{xx} \, dA} \qquad N_y = \int_{A_x} \sigma_{xy} \, dA \qquad N_z = \int_{A_z} \sigma_{xz} \, dA$$

$$\vec{M} = M_x \vec{e}_x + M_y \vec{e}_y + M_z \vec{e}_z$$

$$\vec{\rho} = y \vec{e}_y + z \vec{e}_z$$

$$\vec{\rho} \times \vec{\sigma}_x = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & y & z \\ \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \end{vmatrix} = (y\sigma_{xz} - z\sigma_{xy})\vec{e}_x + z\sigma_{xx}\vec{e}_y - y\sigma_{xx}\vec{e}_z$$

$$M_x = \int_{A_x} (y\sigma_{xz} - z\sigma_{xy}) \, dA$$



$$\boxed{M_y = \int_{A_x} z\sigma_{xx} \, dA}$$

$$\boxed{M_z = -\int_{A_x} y\sigma_{xx} \, dA}$$

$$\boxed{\frac{dN_x}{dx} + p_x = 0}$$

$$\frac{dN_y}{dx} + p_y = 0$$

$$\frac{dN_z}{dx} + p_z = 0$$

$$\frac{dM_x}{dx} + \mu_x = 0$$

$$\boxed{\begin{aligned} &\frac{dM_y}{dx} - N_z + \mu_y = 0 \\ &\frac{dM_z}{dx} + N_y + \mu_z = 0 \end{aligned}} \xrightarrow[\text{OBLIKA}]{\text{ALTERNATIVNA}} \boxed{\begin{aligned} &\frac{d^2M_y}{dx^2} + P_z + \frac{d\mu_y}{dx} = 0 \\ &\frac{d^2M_z}{dx^2} - P_y + \frac{d\mu_z}{dx} = 0 \end{aligned}}$$

③ ENAČBE MATERIALA ob predpostavki $\sigma_{yy} \approx 0$, $\sigma_{zz} \approx 0$

$$\varepsilon_{xx} = \frac{1+\nu}{E}\sigma_{xx} - \frac{\nu}{E}(\sigma_{xx} + \cancel{\sigma_{yy}} + \cancel{\sigma_{zz}}) + \alpha_T \Delta T$$

$$0 = \varepsilon_{yy} = \frac{1+\nu}{E}\sigma_{yy} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) + \alpha_T \Delta T$$

$$0 = \varepsilon_{zz} = \frac{1+\nu}{E}\sigma_{zz} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) + \alpha_T \Delta T$$

$$\varepsilon_{xx}(x,y,z) = \frac{1}{E}\sigma_{xx} + \alpha_T \Delta T$$

$$\varepsilon_{xx} = \frac{du}{dx} - \frac{d^2v}{dx^2}y - \frac{d^2w}{dx^2}z$$

$$\sigma_{xx} = E(\varepsilon_{xx} - \alpha_T \Delta T)$$

$$\sigma_{xx}(x,y,z) = E\left(\frac{du}{dx} - \frac{d^2v}{dx^2}y - \frac{d^2w}{dx^2}z - \alpha_T \Delta T(x,y,z)\right)$$

$$\frac{dN_x}{dx} + P_x = 0$$

$$\frac{dM_y}{dx} - N_z + \mu_y = 0$$

$$\frac{dM_z}{dx} + N_y + \mu_z = 0$$

IDEJA: potek ΔT uskladim s kinematiko nosilca

$$\Delta T(x,y,z) = \Delta T_{x(x)} + \Delta T_{y(x)}\, y + \Delta T_{z(x)}\, z$$

$\uparrow$ predpostavka

$$\boxed{\sigma_{xx}(x,y,z) = E\left(\frac{du}{dx} - \alpha_T \Delta T_x\right) - E\left(\frac{d^2v}{dx^2} + \alpha_T \Delta T_y\right)y - E\left(\frac{d^2w}{dx^2} + \alpha_T \Delta T_z\right)z}$$

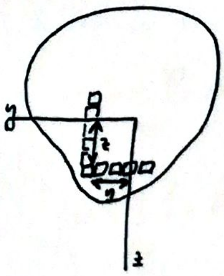
MODEL NORMALNIH NAPETOSTI NOSILCA

OPAZI: ravnotežne enačbe so že reducirane na os, materialne pa še ne!

$$N_x = \int_A \sigma_{xx}\, dA = E\int_A\left(\frac{du}{dx} - \alpha_T \Delta T\right)dA - E\int_A\left(\frac{d^2v}{dx^2} + \alpha_T \Delta T_y\right)y\, dA - E\int_A\left(\frac{d^2w}{dx^2} + \alpha_T \Delta T_z\right)z\, dA$$

$$= E\left(\frac{du}{dx} - \alpha_T \Delta T\right)\underbrace{\int_A dA}_{=A} - E\left(\frac{d^2v}{dx^2} + \alpha_T \Delta T_y\right)\underbrace{\cancel{\int_A y\, dA}}_{=S_z} - E\left(\frac{d^2w}{dx^2} + \alpha_T \Delta T_z\right)\underbrace{\cancel{\int_A z\, dA}}_{=S_y}$$

S_z, S_y = STATIČNA MOMENTA



v težišču prereza sta S_y in S_z enaka $\varnothing$

⊕ dodatno privzamemo, da je x TEŽIŠČNA OS!

$$N_x = EA\left(\frac{du}{dx} - \alpha_T \Delta T_x\right) \rightarrow \boxed{\frac{du}{dx} = \frac{N_x}{EA} + \alpha_T \Delta T_x}$$

$$M_y = \int_A z\,\sigma_{xx}\,dA \Rightarrow M_y = E\left(\frac{du}{dx} - \alpha_T\Delta T_x\right)\cdot \underbrace{\int_A z\,dA}_{=S_y=0} - E\left(\frac{d^2v}{dx^2} + \alpha_T\Delta T_y\right)\underbrace{\cancel{\int_A yz\,dA}}_{\text{deviacijski moment}} - E\left(\frac{d^2w}{dx^2} + \alpha_T\Delta T_z\right)\underbrace{\int_A z^2\,dA}_{I_y}$$

$$I_{yz} = -\int_A yz\,dA$$

najdemo lahko taki osi y in z, da bo $I_{yz} = 0$, rečemo jima glavni vztrajnostni osi

! IZPIT OD KJE PRIDE z^2

- redukcija nosilca (ročica in ...)

$$\boxed{\frac{d^2w}{dx^2} = -\frac{M_y}{EI_y} - \alpha_T\Delta T_z}$$

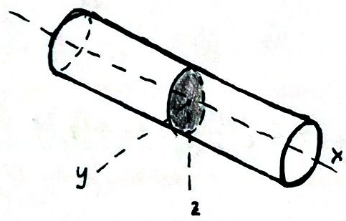
UPOGIBNA TOGOST

*notranji moment je povezan s prečnimi pomiki

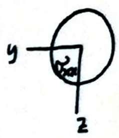
9. 12. 2021

$$\boxed{\varepsilon_{xx} = \frac{du}{dx} - \frac{d^2v}{dx^2}y - \frac{d^2w}{dx^2}z}$$

KINEMATIČNA ENAČBA



$$\sigma_{xx} = E(\varepsilon_{xx} - \alpha_T\Delta T_x)$$



$$\frac{dN_x}{dx} + p_x = 0$$

$$\frac{dM_y}{dx} - N_z + m_y = 0$$

$$\frac{dM_z}{dx} + N_y + m_z = 0$$

$$N_{xx} = \int_A \sigma_{xx}\,dA \overset{\text{TEŽIŠČNA OS}}{=} EA\left(\frac{du}{dx} - \alpha_T\Delta T\right)$$

$$\boxed{\frac{du}{dx} = \frac{N_x}{EA} + \alpha_T\Delta T_x}$$

$$M_y = \int z\,\sigma_{xx} \rightarrow \frac{d^2w}{dx^2} = -\frac{M_y}{EI_y} - \alpha_T\Delta T_z$$

$S_z = 0$ v težišču

$$M_z = -\int_A y\,\sigma_{xx} = -E\Big(\int_A \cancel{\left(\frac{du}{dx} - \alpha_T\Delta T_x\right)} y\,dA - \int_A\left(\frac{d^2v}{dx^2} + \alpha_T\Delta T_y\right)y^2\,dA - \int_A \cancel{\left(\frac{d^2w}{dx^2} + \alpha_T\Delta T_z\right)yz\,dA}$$

$I_{yz} = 0$

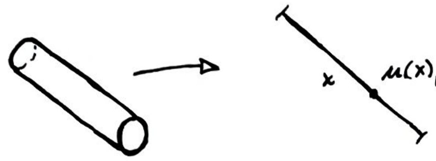
če sta y in z glavni vztrajnostni osi *

$$* \quad I_z = \int_A y^2 \, dA$$

$$M_z = E I_z \left(\frac{d^2 v}{dx^2} + \alpha_T \Delta T_y \right)$$

$$\boxed{\frac{d v^2}{dx^2} = \frac{M_z}{E I_z} - \alpha_T \Delta T_y}$$

POVZETEK



$u(x), v(x), w(x)$

EULER-BERNOULLIJEVE ENAČBE NOSILCA

DE 1. red: $\frac{du}{dx} = \frac{N_x}{EA} + \alpha_T \Delta T_x$ → spremember osnega pomika (skrček, raztezek) odvisna od N_x, E in geometrijske lastnosti osna togost A

NDE 2. red:

$$\frac{d^2 v}{dx^2} = \frac{M_z}{E I_z} - \alpha_T \Delta T_y$$

$$\frac{d^2 w}{dx^2} = -\frac{M_y}{E I_y} - \alpha_T \Delta T_z$$ → $I_y = \int_A z^2 \, dA$

1 z → iz modela deformacij

1 z → iz redukcije napetosti? na os nosilca

ENAČBI UPOGIBNIC

$$\frac{du}{dx} = f(x)$$

$$\int_0^x \frac{du}{dx} \, dx = \int_0^x f(\tilde{x}) \, d\tilde{x}$$

$$= u(x) - u(0) = F(x) - F(0)$$

} določen integral

2. možnost

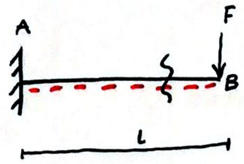
$$\frac{du}{dx} = f(x)$$

$$I = F + C + \text{ROBNI POGOJI}$$

* PRIMER 1

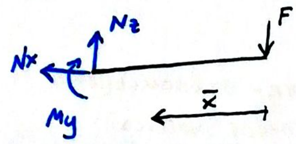
Določi pomike | previsnega nosilca s silo na prostem koncu.

(težišče osi ravninskega)



1. NAČIN

a) notranje sile



$\boxed{N_x = 0}$

$\boxed{M_y = -F\bar{x}}$

$M_y = Fx - Fl$

$\bar{x} = L - x$

b) enačbe nosilca

$$\frac{du}{dx} = \frac{N_x}{EA} \qquad \frac{d^2w}{dx^2} = -\frac{M_y}{EI_y}$$

$$\frac{du}{dx} = 0 \qquad \frac{d^2w}{dx^2} = \frac{1}{EI_y}(Fl - Fx)$$

$$u(x) = C$$

$$\frac{dw}{dx} = \frac{1}{EI_y}\left(Flx - F\frac{x^2}{2}\right) + D$$

$$w(x) = \frac{1}{EI_y}\left(Fl\frac{x^2}{2} - F\frac{x^3}{6}\right) + D_1x + D_2$$

c) robni pogoji

$u^A = 0 \rightarrow u(x=0) = 0$

$w^A = 0 \rightarrow w(x=0) = 0$

$\varphi^A = 0 \qquad \varphi_y = -\frac{dw}{dx} \rightarrow \left.\frac{dw}{dx}\right|_{x=0} = 0$

najprej odvedemo, šele potem vstavimo vrednost

$u(0) = \boxed{C = 0}$

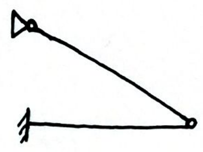
$\rightarrow \boxed{u(x) = 0}$

$w(0) = \boxed{D_2 = 0}$

$\left.\frac{dw}{dx}\right|_{x=0} = \boxed{D_1 = 0}$

$$w(x) = \frac{F}{6EI_y}(3lx^2 - x^3)$$

$$\boxed{w(l) = \frac{Fl^3}{3EI_y}}$$



ALTERNATIVNA OBLIKA ENAČB

$$\frac{dN_x}{dx} + p_x = 0$$

$$\frac{d^2u}{dx^2} = \frac{\frac{dN_x}{dx}}{EA} + \alpha_T \frac{dT_x}{dx}$$

$$\boxed{\frac{d^2u}{dx^2} = -\frac{p_x}{EA} + \alpha_T \frac{d\Delta T_x}{dx}}$$

$$\frac{dM_y}{dx} - N_z + M_y = 0$$

$$\frac{dN_z}{dx} + p_z = 0$$

$$\frac{d^2M_y}{dx^2} = -p_z - \frac{dM_y}{dx}$$

$$\boxed{\frac{d^4w}{dx^4} = \frac{p_z + \frac{dM_y}{dx}}{EI_y} - \alpha_T \frac{d^2\Delta T_z}{dx^2}}$$

$$\boxed{\frac{d^4v}{dx^4} = \frac{p_y - \frac{dM_z}{dx}}{EI_z} - \alpha_T \frac{d^2\Delta T_y}{dx^2}}$$

METODA POMIKOV

* PRIMER ①: 2. način

ENAČBE

a) $\frac{d^2u}{dx^2} = 0$ $\qquad$ $\frac{d^4w}{dx^4} = 0$

$\frac{du}{dx} = C_1$ $\qquad$ $\frac{d^3w}{dx^3} = D_1$

$u(x) = C_1 x + C_2$ $\qquad$ $\frac{d^2w}{dx^2} = D_2 + D_1 \cdot x$

$$\frac{dw}{dx} = D_1 \frac{x^2}{2} + D_2 x + D_3$$

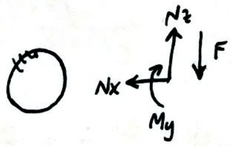
$$w(x) = D_1 \frac{x^3}{6} + D_2 \frac{x^2}{2} + D_3 x + D_4$$

b) ROBNI POGOJI

$C_2 = 0 \leftarrow u(0) = 0 \leftarrow u^A = 0$

$D_4 = 0 \leftarrow w(0) = 0 \leftarrow w^A = 0$

$D_3 = 0 \leftarrow \frac{dw}{dx}\Big|_{x=0} = 0 \leftarrow \varphi^A = 0$



$N_x(x=l) = 0 \rightarrow \frac{du}{dx}\Big|_{x=l} = 0 \rightarrow C_1 = 0$

$N_z(x=l) = F \rightarrow *$

$M_y(x=l) = 0 \rightarrow \frac{d^2w}{dx^2}\Big|_{x=l} = 0 \rightarrow D_1 l + D_2 = 0$

$* \ \frac{dM_y}{dx} = N_z \quad \frac{d^3w}{dx^3}\Big|_{x=l} = -\frac{l}{EI_y} \rightarrow D_1 = -\frac{F}{EI_y}$

$D_2 = \frac{Fl}{EI_y}$

$\boxed{u(x) = 0}$ ✓

$\boxed{w(x) = -\frac{Fx^3}{6EI_y} + \frac{Flx^2}{2EI_y}}$ ✓

$$\boxed{\frac{du}{dx} = \frac{N_x}{EA} + \alpha_T \Delta T_x}$$

1. način

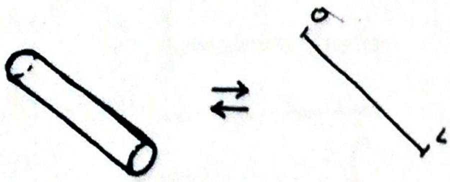
10. 12. 2021

$$\frac{d^2 v}{dx^2} = \frac{M_z}{EI_z} - \alpha_T \Delta T_y$$

$$\frac{d^2 w}{dx^2} = -\frac{M_y}{EI} - \alpha_T \Delta T_z$$

$I_{yz} = 0$

* IZPIT: na pamet!



2. način

$$\frac{d^2 u}{dx^2} = -\frac{p_x}{EA}$$

$$\frac{dN_x}{dx} + p_x = 0 \qquad \frac{dM_y}{dx} - N_z + m_y = 0$$

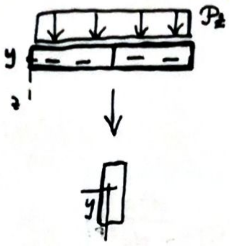
$$\frac{dN_y}{dx} + p_y = 0 \qquad \frac{dM_z}{dx} + N_y + m_z = 0$$

$$\frac{dN_z}{dx} + p_z = 0$$

$\xrightarrow[\Delta T = 0]{\text{če je}}$

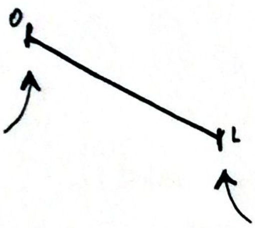
$$\frac{d^4 w}{dx^4} = -\frac{p_z + \frac{dM_y}{dx}}{EI_y}$$

$$\frac{d^4 v}{dx^4} = \frac{p_y - \frac{dM_z}{dx}}{EI_z}$$



ROBNI POGOJI

"na območju od 0 do l ne sme biti nobena je obtežba-ni obtežbe → lahko je obtežba, obtežba funkcije x, samo ne sme kar naenkrat izginit. Točkovni momenti, sile pa so lahko samo v točki 0 ali l, ne pa vmes"



$u(0)$
$w(0)$
$v(0)$
$\left.\frac{dw}{dx}\right|_{x=0}$
$\left.\frac{dv}{dx}\right|_{x=0}$

$u(l)$ $\quad \left.\frac{dw}{dx}\right|_{x=l}$
$w(l)$
$v(l)$ $\quad \left.\frac{dv}{dx}\right|_{x=l}$

← pogoji na osnovne neznanke (naravni robni pogoji)

$N_x(0)$ $\qquad N_x(l)$
$M_y(0), M_z(0)$ $\qquad M_y(l), M_z(l)$
$N_y(0), N_z(0)$ $\qquad N_y(l), N_z(l)$

$$N_x(0) = EA \left.\frac{du}{dx}\right|_{x=0}$$

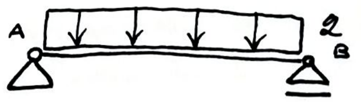
$$M_y(0) = -EI_y \left.\frac{d^2 w}{dx^2}\right|_{x=0}$$

$$N_z(0) = -EI_y \left.\frac{d^3 w}{dx^3}\right|_{x=0}$$

* če je $m_y = 0$

← pogoji na odvode neznank (statični robni pogoji)

* PRIMER ①



$w^A=0$ $\qquad$ $w^B=0$

$M_y^A=0$ $\qquad$ $M_y^B=0$

Robni pogoji

2. način

$$\frac{d^4w}{dx^4}=\frac{q}{EI_y}$$

$$\frac{d^3w}{dx^3}=\frac{qx}{EI_y}+D_1$$

$$\frac{d^2w}{dx^2}=\frac{qx^2}{2EI_y}+D_1x+D_2$$

$$\frac{dw}{dx}=\frac{qx^3}{6EI_y}+D_1\frac{x^2}{2}+D_2x+D_3$$

$$w(x)=\frac{qx^4}{24EI_y}+D_1\frac{x^3}{6}+D_2\frac{x^2}{2}+D_3x+D_4$$ nastavek

robni pogoji

1.) Levi rob

$w^A=0$

$w(x=0)=0 \rightarrow \boxed{D_4=0}$

$M_y^A=0$

$\left.\frac{d^2w}{dx^2}\right|_{x=0}=0 \rightarrow \boxed{D_2=0}$

$M_y^B=0$

$\left.\frac{d^2w}{dx^2}\right|_{x=l}=\frac{ql^2}{2EI_y}+D_1l=0$

$\rightarrow \boxed{D_1=-\frac{ql}{2EI_y}}$

$w|_{x=l}=0$

$\rightarrow \frac{ql^3}{24EI_y}+D_1\frac{l^2}{6}+D_3l=0$

$-\frac{ql^3}{24EI_y}+D_3=0$

$$\boxed{w(x)=\frac{q}{24EI_y}(x^4-2lx^3+l^3x)}$$ ✓

Pomik na sredi

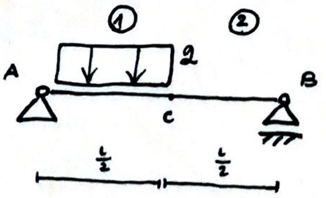
$$w\left(\frac{l}{2}\right)=\frac{5ql^4}{384EI_y}$$

$$M_y=-EI_y\frac{d^2w}{dx^2}=-\frac{q}{24}(12x^2-12lx)=-\frac{q}{2}x^2-lx$$

kvadratna parabola

$\left.\begin{array}{l}M_y(x=0)=0\\ M_y(x=l)=0\end{array}\right\}$ kontrola ✓

***PRIMER ②**



↑ 1. način
te enačbe veljajo za posamezna polja, ne za cel nosilec

$\Sigma Z: \; A_z + B_z + q\cdot\frac{L}{2} = 0$

$\Sigma M^A: \; -B_z\cdot L - q\cdot\frac{L}{2}\cdot\frac{L}{4} = 0$

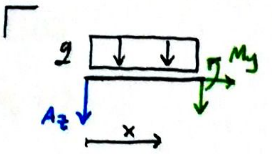
$B_z = -q\cdot\frac{L}{8}$

$A_z = -\frac{3qL}{8}$

$A_x = 0$

1. NAČIN ("ne moremo uporabiti moment v členku je $\emptyset$, v 2. načinu pa lahko)

polje ① $x \in [0, \frac{L}{2}]$



$M_y = -A_z x - q x \frac{x}{2}$

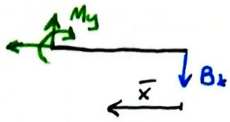
$M_y = -q\frac{x^2}{2} + \frac{3qL}{8}x$

$\frac{d^2w}{dx^2} = \frac{q}{EI_y}\left(\frac{x^2}{2} - \frac{3L}{8}x\right)$

$\frac{dw}{dx} = \frac{q}{EI_y}\left(\frac{x^3}{6} - \frac{3L}{16}x^2\right) + D_1$

$\boxed{w^{(1)}(x) = \frac{q}{EI_y}\left(\frac{x^4}{24} - \frac{3Lx^3}{48}\right) + D_1\cancel{x} \cancel{+ D_2}}$

polje ② z desne $\bar{x} \in [0, \frac{L}{2}]$



$M_y = -B_z \cdot \bar{x}$

$x = \frac{L}{2} - \bar{x}$

$x \in [0, \frac{L}{2}]$

$M_y = \frac{qL}{8}\left(\frac{L}{2} - x\right)$

$\frac{d^2w}{dx^2} = \frac{q}{EI_y}\left(\frac{Lx}{8} - \frac{L^2}{16}\right)$

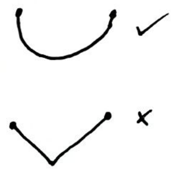
$\frac{dw}{dx} = \frac{q}{EI_y}\left(\frac{Lx^2}{16} - \frac{L^2 x}{16}\right) + E_1$

$\boxed{w^{(2)}(x) = \frac{q}{EI_y}\left(\frac{Lx^3}{48} - \frac{L^2x^2}{32}\right) + E_1 x + E_2}$

D_1, D_2, E_1, E_2 ... 4 konstante → rabimo 4 robne pogoje

robni pogoji

$w^A = 0$ $\quad w_c^{(1)} = w_c^{(2)}$ $\quad \varphi_c^{(1)} = \varphi_c^{(2)}$ $\quad w^B = 0$



$\varphi_c^{(1)} = \varphi_c^{(2)}$

$w^{(1)}(x=0) = 0 \rightarrow \boxed{D_2 = 0}$

$w^{(2)}(x=\frac{L}{2}) = 0$

$\frac{q}{EI_y}\left(\frac{L^4}{48\cdot 8} - \frac{L^4}{32\cdot 4}\right) + E_1\frac{L}{2} + E_2 = 0$ (1)

$\left.\frac{dw^{(1)}}{dx}\right|_{x=\frac{L}{2}} = \left.\frac{dw^{(2)}}{dx}\right|_{x=0}$

$\frac{q}{EI_y}\left(\frac{L^3}{48} - \frac{3L^3}{54}\right) + D_1 = E_1$ (2)

$w^{(1)}(x=\frac{L}{2}) = w^{(2)}(x=0)$

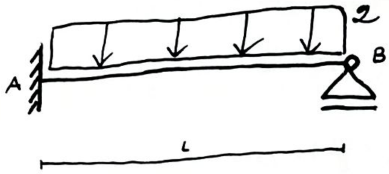
$\frac{q}{EI_y}\left(\frac{L^4}{24\cdot 16} - \frac{3L^4}{48\cdot 8}\right) \cdot D_1\frac{L}{2} = E_2$ (3)

$D_1 = \frac{3qL^3}{128EI_y}$

$E_1 = -\frac{qL^3}{384EI_y}$

$E_2 = \frac{5qL^3}{768EI_y}$

* PRIMER 3



II. način

$$\frac{d^4w}{dx^4} = \frac{q}{EI_y}$$

$$\frac{d^3w}{dx^3} = \frac{qx}{EI_y} + D_1$$

$$\frac{d^2w}{dx^2} = \frac{qx^2}{2EI_y} + D_1x + D_2$$

$$\frac{dw}{dx} = \frac{qx^3}{6EI_y} + D_1\frac{x^2}{2} + D_2x + D_3$$

$$\boxed{w(x) = \frac{qx^4}{24EI_y} + D_1\frac{x^3}{6} + D_2\frac{x^2}{2} + \cancel{D_3}x + \cancel{D_4}} \quad (1)$$

robni pogoji (potrebujemo 4 → 4 konstante)

$w^A = 0$

$\varphi^A = 0$

$w(0) = 0 \rightarrow \boxed{D_4 = 0}$

$\frac{dw}{dx}\Big|_{x=0} = 0 \rightarrow \boxed{D_3 = 0}$

$w^B = 0$

$M_y^B = 0$

$\frac{d^2w}{dx^2}\Big|_{x=L} = 0 \rightarrow$

$\rightarrow \boxed{\frac{qL^2}{2EI_y} + D_1L + D_2 = 0} \quad (2)$

$\frac{qL^4}{24EI_y} + D_1\frac{L^3}{6} + D_2\frac{L^2}{2} = 0 \quad /\cdot\frac{6}{L^2}$

$\boxed{D_1L + 3D_2 = -\frac{qL^2}{4EI_y}} \quad (3)$

$\ominus \quad D_1L + D_2 = -\frac{ql^2}{2EI_y} \quad (2)$

$2D_2 = \frac{ql^2}{4EI_y}$

$\rightarrow \boxed{D_2 = \frac{ql^2}{8EI_y}}$

$\rightarrow \boxed{D_1 = -\frac{5ql}{8EI_y}}$

D_1, D_2, D_3 in D_4 vstavimo v (1)

$$w(x) = \frac{q}{24EI_y}\left(x^4 - \frac{5L}{2}x^3 + \frac{3L^2}{2}x^2\right)$$

$$\boxed{w(x) = \frac{q}{48EI_y}\left(2x^4 - 5Lx^3 + 3L^2x^2\right)}$$

vstavim $x=0$, $x=l$ } kontrola

KAj PA momenti M_y?

$$M_y = -\frac{d^2w}{dx^2}\cdot EI_y$$

$$= -\left(\frac{qx^2}{2} - \frac{5qL}{8}x + \frac{qL^2}{8}\right) = \frac{q}{8}\left(-4x^2 + 5Lx - l^2\right)$$

vstavim $x = L$ → moment v členku mora biti $\emptyset$ → $M_y(l) = 0$ ✓

$$M_y = -\frac{ql^2}{8}$$

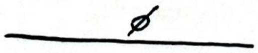
teme parabole dobimo tako, da parabolo odvajamo, zraven dobimo še prečno silo.

$$N_z = \frac{dM_y}{dx} = \frac{q}{8}(-8x + 5l) \qquad \text{ničla: } -8x + 5l = 0$$

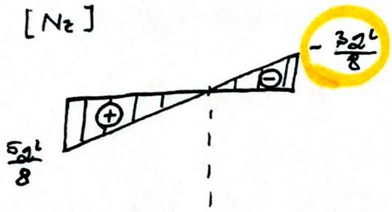
$$N_z(0) = \frac{5ql}{8} \qquad x_N = \frac{5l}{8}$$

$$N_z(l) = -\frac{3ql}{8} \qquad M_E = M_y\left(\frac{5l}{8}\right) = DN$$

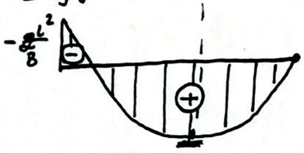
$[N_x]$



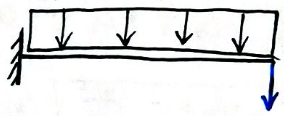
$[N_z]$



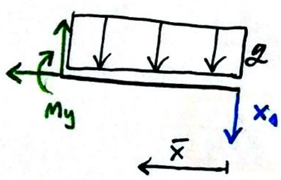
$[M_y]$



KAJ PA I. NAČIN?



$X_1 = B_z \leftrightarrow w_B = 0$



$$M_y = -X_1 \bar{x} - q\bar{x}\frac{\bar{x}}{2} \quad \leftarrow \quad \bar{x} = l - x$$

$$M_y = -\frac{q}{2}(l-x)^2 - X_1(l-x)$$

$$\frac{d^2w}{dx^2} = \frac{1}{EI_y}\left(\frac{qx^2}{2} - qlx + \frac{ql^2}{2} - X_1 x + X_1 l\right)$$

$$\frac{dw}{dx} = \frac{1}{EI_y}\left(\frac{qx^3}{6} - ql\frac{x^2}{2} + ql^2\frac{x}{2} - X_1\frac{x^2}{2} + X_1 l x\right) + D_1$$

$$w(x) = \frac{1}{EI_y}\left(\frac{qx^4}{24} - \frac{qlx^3}{6} + \frac{qx^2l^2}{4} - X_1\frac{x^3}{6} + X_1 l\frac{x^2}{2}\right) + D_1 x + D_2$$

robni pogoji → 3 robne enačbe

$w^A = 0 \qquad \varphi^A = 0 \qquad w^B = 0$

$w(x=0) = 0 \;\rightarrow\; \boxed{D_2 = 0}$

$\left.\frac{dw}{dx}\right|_{x=0} = 0 \;\rightarrow\; \boxed{D_1 = 0}$

$w(x=l) = 0$

$$\frac{1}{EI_y}\left(\frac{ql^4}{24} - \frac{ql^4}{6} + \frac{ql^4}{4} - X_1\frac{l^3}{6} + X_1\frac{l^3}{2}\right) = 0$$

$$\frac{3ql^4}{24} + X_1\frac{l^3}{3} = 0$$

$$X_1 = -\frac{3ql}{8} \quad ✓$$

$$M_y = -\frac{q}{2}\left(x^2 - 2lx + l^2 + \frac{3ql}{8}(l-x)\right)$$

$$M_y = \frac{q}{8}\left(-4x^2 + 8lx - 4l^2 + 3l^2 - 3lx\right)$$

$$M_y = \frac{q}{8}\left(-4x^2 + 5lx - l^2\right.$$

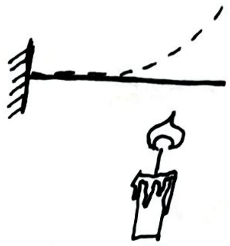
16\. 12. 2021

$$\frac{du}{dx} = \frac{N_x}{EA_x} + \alpha_T \Delta T_x$$

TEMPERATURNI VPLIV

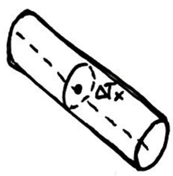
$$\frac{d^2w}{dx^2} = -\frac{M_y}{EI_y} - \alpha_T \Delta T_z$$

upogibna togost



spomnimo se: $\Delta T(x,y,z) = \Delta T_x + \Delta T_y \cdot y + \Delta T_z \cdot z$

model!

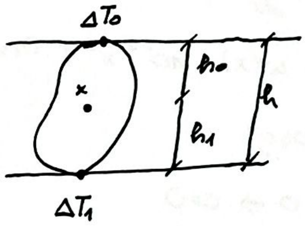


$\Delta T_x\ (x)$

$\Delta T_y\ (x)$

$\Delta T_z\ (x)$

naj bo $\Delta T_y = 0$



$$\Delta T(-h_0) = \Delta T_x - \Delta T_z h_0 = \Delta T_0$$

$$\Delta T(h_1) = \Delta T_x + \Delta T_z h_1 = \Delta T_1 \quad \ominus$$

$$\Delta T_z (h_0+h_1) = \frac{\Delta T_1 - \Delta T_0}{1}$$

$$\boxed{\Delta T_z = \frac{\Delta T_1 - \Delta T_0}{h}} \quad \left[\frac{K}{m}\right]$$

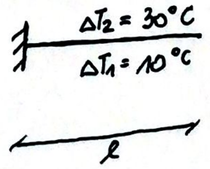
$$\Delta T_x h_0 + \Delta T_x h_1 = \Delta T_0 h_1 + \Delta T_1 h_0$$

$$\Delta T_x = \frac{\Delta T_0 h_1 + \Delta T_1 h_0}{h}$$

če je $h_0 = h_1 = \frac{h}{2}$

$$\boxed{\Delta T_x = \frac{\Delta T_0 + \Delta T_1}{2}}$$

* NALOGA



$E = 10\ 000\ \frac{kN}{cm^2}$

$A = 10\ cm^2$

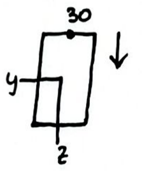
$I_y = 100\ cm^4$

$\alpha_T = 10^{-5}\ \frac{1}{K}$

$h = 10cm$ prerez je simetričen

$$\Delta T_x = \frac{\Delta T_1 + \Delta T_2}{2} = 20K$$

$$\Delta T_z = \frac{\Delta T_1 - \Delta T_2}{10cm} = \frac{-20K}{10cm} = -2\ \frac{K}{cm}$$



če mehanskih vplivov ni, je:

$$\frac{du}{dx} = \alpha_T \Delta T_x \qquad \frac{du}{dx} = 10^{-5} \cdot 20 = 2 \cdot 10^{-4}$$

$$u(x) = 2 \cdot 10^{-4} x + C$$

$$\frac{d^2w}{dx^2} = -\alpha_T \Delta T_z$$

$$\frac{d^2w}{dx^2} = -\alpha_T \Delta T_z = 2 \cdot 10^{-5}$$

$$\frac{dw}{dx} = 2 \cdot 10^{-5} x + D_1$$

$$w(x) = 10^{-5} x^2 + D_1 x + D_2$$

Robni pogoji

$u(0) = 0 \rightarrow C = 0$

$w(0) = 0 \rightarrow D_2 = 0$

$\left.\frac{dw}{dx}\right|_{x=0} = 0 \rightarrow D_1 = 0$

$u(x) = 2 \cdot 10^{-4} x \qquad u(100\ cm) = 0.02\ cm$

$w(x) = 10^{-5} x^2\ [cm]$! $\qquad w(100\ cm) = 0.1\ cm$

IZPIT

* STATIČNO NEDOLOČENA KONSTRUKCIJA

- metoda sil → teorija II

Geometrijske karakteristike prečnega prereza

$A = \int_A dA$ [m] PLOŠČINA

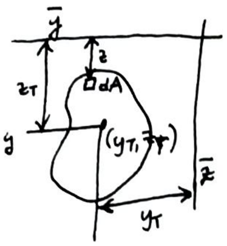
* statični moment ≠ moment

morski pes ≠ pes

$S_y = \int_A z\, dA$ [m^3]

$S_z = \int_A y\, dA$ [m^3]

STATIČNI MOMENT



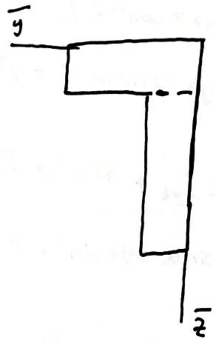
premik ⊥ KS v težišče

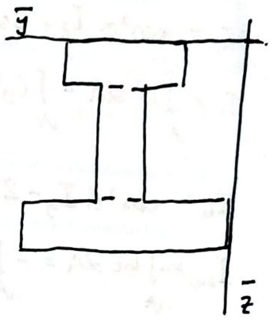
→ statična momenta = 0

$I_y = \int_A z^2 dA$

$I_z = \int_A y^2 dA$

VZTRAJNOSTNI MOMENT (lastnosti oblike telesa) [m^4]





$I_{yz} = -\int_A yz\, dA$ DEVIACIJSKI MOMENT

TEŽIŠČE

$$\boxed{z_T A = S_y \qquad y_T A = S_z \qquad z_T = \frac{S_y}{A} \qquad y_T = \frac{S_z}{A}}$$

VPLIV POMIKA iz TEŽIŠČA in v TEŽIŠČE

$\bar{y} = y_T + y$

$\bar{z} = z_T + z$

$$I_{\bar{y}} = \int_A (z + z_T)^2 dA = \int_A (z^2 + 2z_T z + z_T^2)\, dA$$

$$I_{\bar{y}} = I_y + 2z_T \underbrace{\cancel{S_y}}_{=0} + z_T^2 A$$

= 0 → zaradi premika v težišče

3 enačbe premika KS

$$\boxed{I_{\bar{y}} = I_y^T + z_T^2 A}$$ STEINERJEVI IZREKI

$$I_{\bar{z}} = \int_A (y + y_T)^2 dA = \int_A (y^2 + 2y_T y + y_T^2)\, dA$$

$$\boxed{I_{\bar{z}} = I_z^T + y_T^2 A}$$

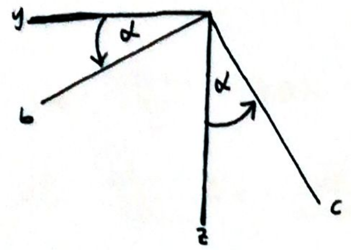
"bolj kot se oddaljujem od osi prereza večji navidezni vztrajnostni moment imamo"

$$I_{\bar{y}\bar{z}} = -\int_A \bar{y}\bar{z}\, dA = -\int_A (yz + y_T z + z_T y + y_T z_T)\, dA$$

$$\boxed{I_{\bar{y}\bar{z}}^T = I_{yz} - y_T z_T A}$$

$$I_y^T = I_y - z_T^2 A$$
$$I_z^T = I_z - y_T^2 A$$
$$I_{yz}^T = I_{yz} - y_T z_T A$$

STEINERJEVE FORMULE



$$b = y\cos\alpha + z\sin\alpha$$
$$c = -y\sin\alpha + z\cos\alpha$$

$$I_b = \int_A c^2 dA = \int_A (y^2\sin^2\alpha - 2yz\sin\alpha\cos\alpha + z^2\cos^2\alpha)\, dA$$

$$\boxed{I_b = \sin^2\alpha\, I_z + 2\sin\alpha\cos\alpha\, I_{yz} + \cos^2\alpha\, I_y}$$

$$I_c = \int_A b^2 dA = \int_A (y^2\cos^2\alpha + 2yz\sin\alpha\cos\alpha + z^2\sin^2\alpha)\, dA$$

$$\boxed{I_c = \cos^2\alpha\, I_y - 2\sin\alpha\cos\alpha\, I_{yz} + \sin^2\alpha\, I_z}$$

$$I_{bc} = -\int_A bc\, dA = -\int_A (-y^2\cos\alpha\sin\alpha - yz\sin^2\alpha + yz\cos^2\alpha + z^2\sin\alpha\cos\alpha)\, dA$$

$$= I_z\sin\alpha\cos\alpha - I_{yz}\sin^2\alpha + I_{yz}\cos^2\alpha - I_y\sin\alpha\cos\alpha$$

radi bi našli tak α, da bo $I_{bc} = 0$

$$I_{bc} = (I_z - I_y)\sin\alpha\cos\alpha + I_{yz}(\cos^2\alpha - \sin^2\alpha)$$

$$0 = \frac{1}{2}(I_z - I_y)\sin 2\alpha + I_{yz}\cos^2\alpha$$

$$(I_y - I_z)\sin 2\alpha = 2 I_{yz}\cos 2\alpha$$

$$\frac{\sin 2\alpha}{\cos 2\alpha} = \frac{2I_{yz}}{I_y - I_z}$$

$$\operatorname{tg} 2\alpha = \frac{2I_{yz}}{I_y - I_z} \rightarrow \boxed{\alpha_0 = \frac{1}{2}\operatorname{arctg}\frac{2I_{yz}}{I_y - I_z}}$$

$$I_{1,2} = \frac{I_y + I_z}{2} \pm \sqrt{\frac{1}{4}(I_y - I_z)^2 + I_{yz}^2}$$

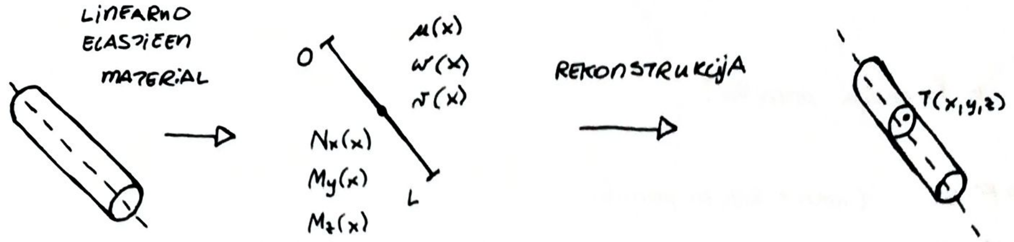
$$\boxed{\begin{bmatrix} I_b & I_{bc} \\ I_{bc} & I_c \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} I_y & I_{yz} \\ I_{yz} & I_z \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}}$$

$$= \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} I_y\cos\alpha + I_{yz}\sin\alpha & * \\ I_{yz}\cos\alpha + I_{z}\sin\alpha & * \end{bmatrix}$$

$$= \begin{bmatrix} I_y\cos^2\alpha + I_{yz}\sin\alpha\cos\alpha + I_{yz}\sin\alpha\cos\alpha + I_z\sin^2\alpha & * \\ -I_y\sin\alpha\cos\alpha - I_{yz}\sin^2\alpha + I_{yz}\cos^2\alpha + I_z\sin\alpha\cos\alpha & * \end{bmatrix}$$

$$I_{yz} = -\int yz\, dA$$

Rekonstrukcija normalnih napetosti iz rezultantnih notranjih sil



$$\frac{d^2v}{dx^2} = \frac{M_z}{EI_z}$$

$$\boxed{\varepsilon_{xx}(x,y,z) = \frac{du}{dx} - \underbrace{\frac{d^2v}{dx^2}\,y - \frac{d^2w}{dx^2}\,z}_{\text{ukrivljenosti}}} \longrightarrow \boxed{\sigma_{xx}(x,y,z) = \frac{N_x}{A} + \frac{M_y}{I_y}z - \frac{M_z}{I_z}y}$$

največje na robovih

če sta druga dva člena ∅ → je ta člen tak

$$\sigma_{xx} = E\varepsilon_{xx} \qquad \frac{du}{dx} = \frac{N_x}{EA} \qquad \frac{d^2w}{dx^2} = -\frac{M_y}{EI_y}$$

IZREK O DOPOLNILNEM VIRTUALNEM DELU in NJEGOVA UPORABA

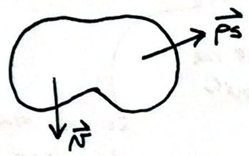
$f(x)$

$\mathcal{F}(\vec{u}(x,y,z))$ → funkcija pomika?

- delo sile: $W = F \cdot s$ (delo = sila in pomik)

  $W = \vec{F} \circ \vec{u}$ (skalarni produkt: iz 2 vektorjev → skalar)

  $W = \sum_i \vec{F}_i \cdot \vec{u}_i + \sum_j \vec{M}_j \cdot \vec{\varphi}_j$ delo = skupek vplivov sil in ~~pomikov~~ odzivov (pomikov)



$$\boxed{W = \int_S \vec{p}_s \cdot \vec{u}\, ds + \int_V \vec{v} \cdot \vec{u}\, dV}$$

DELO ZUNANJIH SIL

$W \in \mathbb{R}$ → funkcionalne preslikave

$W : (\vec{u}, \vec{p}_s, \vec{v}) \to \mathbb{R}$

$\mathcal{F}(\vec{u})$ ← variacijski račun

$$\vec{u}_{PER} = \vec{u} + \varepsilon\, \delta\vec{u}$$

ε – SKALARNI FAKTOR KI GA POŠLJEMO PROTI Ø

$\delta\vec{u}$ – POLJUBNA SPREMEMBA = VARIACIJA $\vec{u}$

> funkcija 1 spremenljivke $f(x)$
>
> $\underbrace{\text{min, max}}_{\text{ekstrem}} f(x)$
>
> $$\lim_{\Delta x \to 0} \frac{f(\overset{x_{PER}}{x + \Delta x}) - f(x)}{\Delta x} = \frac{df}{dx}$$

$$\delta\mathcal{F} = \lim_{\varepsilon \to 0} \frac{\mathcal{F}(\vec{u} + \varepsilon\,\delta\vec{u}) - \mathcal{F}(\vec{u})}{\varepsilon}$$

$$\boxed{\delta W = \int_S \vec{p}_s\, \delta\vec{u}\, ds + \int_V \vec{v}\, \delta\vec{u}\, dV}$$

VIRTUALNO DELO

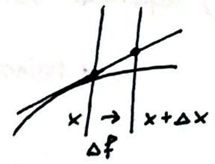
$$\boxed{\delta W_z^* = \int_S \delta\vec{p}_s \cdot \vec{u}\, ds + \int_V \delta\vec{v} \cdot \vec{u}\, dV}$$

DOPOLNILNO VIRTUALNO DELO ZUNANJIH SIL, kjer sta $\delta\vec{p}_s$ in $\delta\vec{v}$ poljubni spremembi obtežbe, $\vec{u}$ pa dejanski pomiki telesa!

δW_z^*: VIRTUALNO → δ, DELO → W, DOPOLNILNO → *, ZUNANJIH SIL → $_z$

$$\delta W_z = \int_S \delta \vec{p}_S \, \vec{u} \, ds + \int_V \delta \vec{v} \, \vec{u} \, dV$$

funkcija 1 spremenljivke



TUDI VIRTUALNA OBTEŽBA MORA ZADOSTATI RAVNOTEŽNIM POGOJEM

$$\frac{\partial \delta \vec{\sigma}_x}{\partial x} + \frac{\partial \delta \vec{\sigma}_y}{\partial y} + \frac{\partial \delta \vec{\sigma}_z}{\partial z} + \delta \vec{v} = \vec{0}$$

$$\delta \vec{\sigma}_x e_{sx} + \delta \vec{\sigma}_y e_{sy} + \delta \vec{\sigma}_z e_{sz} = \delta \vec{p}_S$$

$$\int_S (\delta \vec{\sigma}_x e_{sx} + \delta \vec{\sigma}_y e_{sy} + \delta \vec{\sigma}_z e_{sz}) \cdot \vec{u} \, ds = \int_V \left(\frac{\partial(\delta \vec{\sigma}_x \cdot \vec{u})}{\partial x} + \frac{\partial(\delta \vec{\sigma}_y \cdot \vec{u})}{\partial y} + \frac{\partial(\delta \vec{\sigma}_z \cdot \vec{u})}{\partial z} \right) dV$$

$$\delta W_z^* = \int_V \left(\frac{\partial \delta \vec{\sigma}_x}{\partial x} \cdot \vec{u} + \frac{\partial \delta \vec{\sigma}_y}{\partial y} \vec{u} + \frac{\partial \delta \vec{\sigma}_z}{\partial z} \vec{u} \right) dV + \int_V \left(\delta \vec{\sigma}_x \cdot \frac{\partial \vec{u}}{\partial x} + \delta \vec{\sigma}_y \cdot \frac{\partial \vec{u}}{\partial y} + \delta \vec{\sigma}_z \frac{\partial \vec{u}}{\partial z} \right) dV$$

* spomnimo se

$$\varepsilon_{x*} = \varepsilon_{xx} \vec{e}_x + \varepsilon_{xy} \vec{e}_y + \varepsilon_{xz} \vec{e}_z$$

$$\varepsilon_x = \frac{\partial u_x}{\partial x} \vec{e}_x + \frac{1}{2} \left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \right) \vec{e}_y + \frac{1}{2} \left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right) \vec{e}_z$$

$$\varepsilon_x = \frac{\partial \vec{u}}{\partial x} - \vec{\omega}_x \vec{e}_x$$

$$\frac{\partial \vec{u}}{\partial x} = \vec{\varepsilon}_x + \vec{\omega} \times \vec{e}_x$$

$$\frac{\partial \vec{u}}{\partial y} = \vec{\varepsilon}_y + \vec{\omega} \times \vec{e}_y$$

$$\frac{\partial \vec{u}}{\partial z} = \vec{\varepsilon}_z + \vec{\omega} \times \vec{e}_z$$

* nismo se spomnili kdaj hudiča smo to delali ?

$$\delta W_z^* = \int_V \overbrace{\left(\frac{\partial \delta \vec{\sigma}_x}{\partial x} + \frac{\partial \delta \vec{\sigma}_y}{\partial y} + \frac{\partial \delta \vec{\sigma}_z}{\partial z} + \vec{v} \right)}^{=\emptyset} \cdot \vec{u} \, dV + \int_V (\delta \vec{\sigma}_x \cdot \vec{\varepsilon}_x + \delta \vec{\sigma}_y \cdot \vec{\varepsilon}_y + \delta \vec{\sigma}_z \cdot \vec{\varepsilon}_z) \, dV +$$

$$+ \int_V \underbrace{\left(\delta \vec{\sigma}_x (\vec{\omega} \times \vec{e}_x) + \delta \vec{\sigma}_y (\vec{\omega} \times \vec{e}_y) + \delta \vec{\sigma}_z (\vec{\omega} \times \vec{e}_z) \right)}_{=\emptyset} dV$$

ZARADI RAVNOTEŽJA MOMENTOV

izrek za splošno telo

$$\boxed{\delta W_z^* = \int_S \delta \vec{p}_S \cdot \vec{u} \, ds + \int_V \delta \vec{v} \cdot \vec{u} \, dV = \int_V (\delta \vec{\sigma}_x \cdot \vec{\varepsilon}_x + \delta \vec{\sigma}_y \cdot \vec{\varepsilon}_y + \delta \vec{\sigma}_z \cdot \vec{\varepsilon}_z) \, dV = W_n^*}$$

IZREK O DOPOLNILNEM VIRTUALNEM DELU:

dopolnilno virtualno delo zunanjih sil je enako dopolnilnemu virtualnemu delu notranjih sil

OBLIKA IZREKA ZA UPOGIB NOSILCA

① dejanska obtežba: N_x, M_y, M_z

- DEJANSKE NAPETOSTI $\sigma_{xx} = \frac{N_x}{A} + \frac{M_y}{I_y} z - \frac{M_z}{I_z} y$

- DEJANSKE DEFORMACIJE $\boxed{\varepsilon_{xx} = \frac{N_x}{EA} + \frac{M_y}{EI_y} z - \frac{M_z}{EI_z} y}$

② poljubna (virtualna) obtežba: $\delta N_x, \delta M_y, \delta M_z$

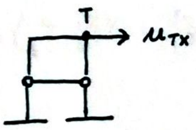
- $\boxed{\delta\sigma_{xx} = \frac{\delta N_x}{A} + \frac{\delta M_y}{I_y} z - \frac{M_y}{I_z} y}$

$$\delta W^*_{n,\text{nosilca}} = \int_V \delta\sigma_{xx} \cdot \varepsilon_{xx} \, dV = \int_0^L \left(\int_A \delta\sigma_{xx} \cdot \varepsilon_{xx} \, dA \right) dx =$$

$$= \int_0^L \frac{N_x \delta N_x}{EA^2} \underbrace{\int_A dA}_{A} + \frac{M_y \delta N_x}{EI_y A} \underbrace{\int_A z \, dA}_{S_y = 0} + \dots \emptyset + \frac{M_y \delta M_y}{EI_y^2} \underbrace{\int_A z^2 \, dA}_{I_y} + \frac{M_z \delta M_z}{EI_z^2} \underbrace{\int_A y^2 dA}_{I_z})$$

* OPOMBA: dobimo 6 statičnih in deviacijskih momentov, ki pa so zaradi predpostavk enaki $\emptyset$? *

$$\boxed{\delta W^*_{n,\text{nosilca}} = \int_0^L \left(\frac{N_x \delta N_x}{EA} + \frac{M_y \delta M_y}{EI_y} + \frac{M_z \delta M_z}{EI_z} \right) dx}$$



UPORABA: če me zanima dejanski pomik / zasuk v neki smeri temu priredim VIRTUALNO OBTEŽBO in postavim virtualno silo velikosti 1 v smeri iskanega pomika/zasuka!

$$\delta W^*_z = \int_S \underbrace{\cancel{\delta \vec{p}_s \vec{u}}}_{\emptyset} ds + \int_V \underbrace{\cancel{\delta \vec{v} \vec{u}}}_{\emptyset} dV + \sum \delta \vec{F}_i \cdot \vec{u}_i + \sum \delta \vec{M}_j \vec{\varphi}_j$$

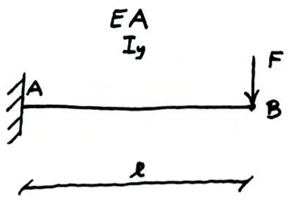
$$\delta W^*_z = \delta \vec{F}_T \vec{u}_T = \delta F_T u_T = 1 \cdot u_T$$

$$\delta \vec{M}_a \vec{\varphi}_a = \delta M_a \varphi_a = 1 \cdot \varphi_a$$

$$\boxed{u_T, \varphi_T = \underbrace{\sum}_{\text{vsota po poljih}} \int_0^L \left(\frac{N_x \delta N_x}{EA} + \frac{M_y \delta M_y}{EI_y} + \frac{M_z \delta M_a}{EI_z} \right) dx}$$

PRIMERI:

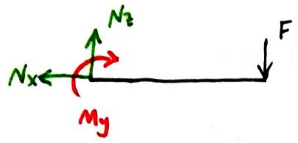
(1)



določi / izrazi navpični pomik na prostem koncu.

$w_B = ?$

a) notranje sile zaradi F



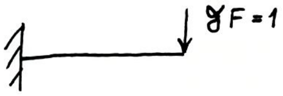
$\boxed{N_x = 0}$

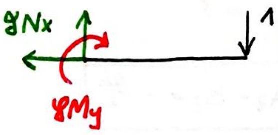
$\boxed{M_y = -F\bar{x}}$

$\boxed{M_y = Fx - Fl}$

$\bar{x} = l - x$

b) virtualna obtežba velikosti 1 v smeri iskanega pomika





$\delta N_x = 0$

$\delta M_y = -\bar{x}$

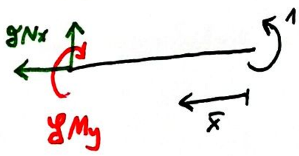
$\boxed{\delta M_y = x - l}$

$$w_B = \int_0^l \frac{1}{EI_y}(Fx - Fl)(x-l)\,dx = \frac{F}{EI_y}\int_0^l (x-l)^2\,dx = \frac{F}{EI_y}\int_0^l (x^2 - 2lx + l^2)\,dx =$$

$$= \frac{F}{EI_y}\left(\frac{x^3}{3} - lx^2 + l^2x\right)\Big|_0^l = \frac{Fl^3}{3EI_y}$$

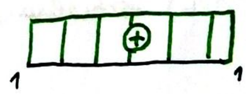
c) določi še zasuk na prostem koncu φ_B?

virtualna obtežba velikosti 1 v smeri iskanega zasuka



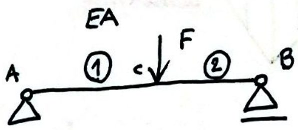
$\delta M_y = 1$

$[\delta M_y]$



$$\varphi_B = \frac{1}{EI_y}\int_0^l (Fx - Fl)\cdot 1\,dx = \frac{F}{EI_y}\left(\frac{x^2}{2} - lx\right)\Big|_0^l = -\frac{Fl^2}{2EI_y} \quad \text{(v rad?)}$$

(2)



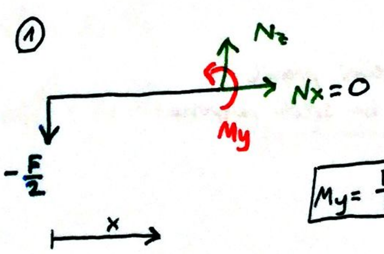
$w_c = ?$

$A_z = B_z = -\frac{F}{2}$

$A_x = 0$

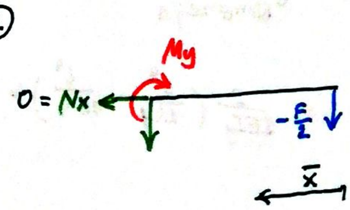
a) dejanska obtežba

(1)



$\boxed{M_y = \frac{Fx}{2}}$

(2)

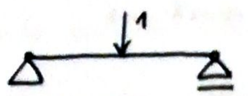


$\bar{x} = \frac{l}{2} - x$

$M_y = \frac{F\bar{x}}{2}$

$\boxed{M_y = \frac{Fl}{4} - \frac{Fx}{2}}$

b) virtualna obtežba



Polje ①

$\delta M_y = \frac{x}{2}$

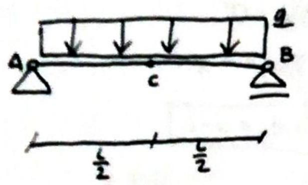
Polje ②

$\delta M_y = \frac{l}{4} - \frac{x}{2}$

$$w_x = \int_0^{\frac{l}{2}} \frac{1}{EI_y} \frac{Fx}{2} \frac{x}{2} \, dx + \int_0^{\frac{l}{2}} \frac{1}{EI_y} \left(\frac{Fl}{4} - \frac{Fx}{2}\right)\left(\frac{l}{4} - \frac{x}{2}\right) dx =$$

$$= \left(\frac{F}{EI_y} \frac{x^3}{12}\right)\Big|_0^{\frac{l}{2}} + \frac{F}{EI_y}\left(\frac{x^3}{12} - l\frac{x^2}{8} + \frac{l^2}{16}x\right)\Big|_0^{\frac{l}{2}} = \frac{Fl^3}{48EI_y}$$

③

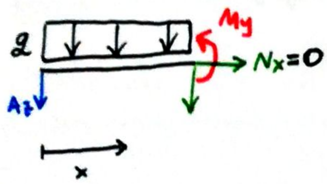


$w_c = ?$

$\boxed{A_z = B_z = -\frac{ql}{2}}$

$\boxed{A_x = 0}$

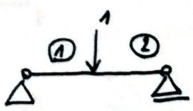
a) dejanska obtežba



$M_y = -A_z x - q\frac{x^2}{2}$

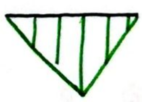
$\boxed{M_y = ql\frac{x}{2} - q\frac{x^2}{2}}$

b) virtualna obtežba



① $\delta M_y = \frac{x}{2}$

② $\delta M_y = \frac{l}{4} - \frac{x}{2}$



simetrija

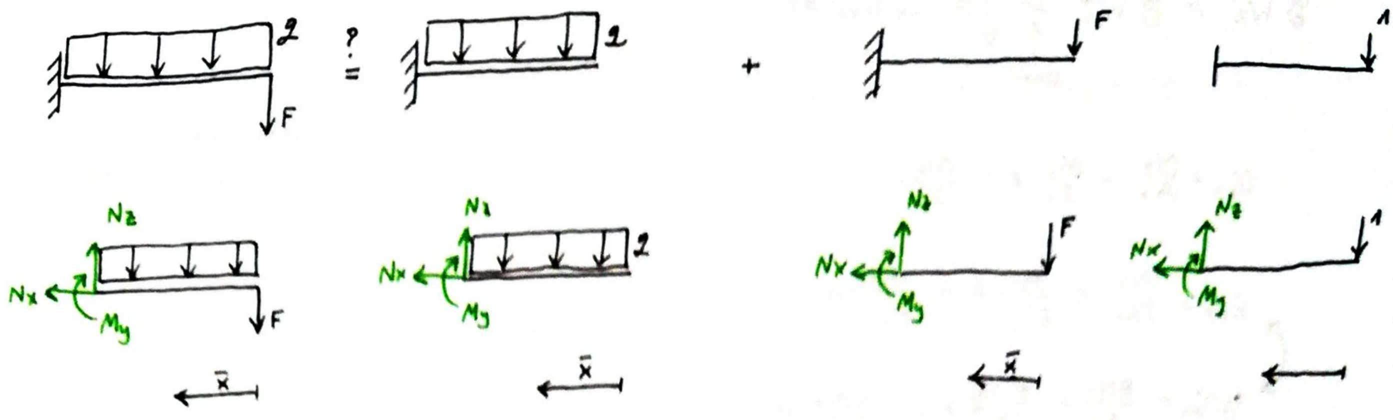
$$w_c = 2\int_0^{\frac{l}{2}} \frac{1}{EI_y}\left(ql\frac{x}{2} - q\frac{x^2}{2}\right)\frac{x}{2}\,dx = \frac{q}{2EI_y}\int_0^{\frac{l}{2}}(x^2 l - x^3)\,dx = \frac{q}{2EI_y}\left(\frac{x^3 l}{3} - \frac{x^4}{4}\right)\Big|_0^{\frac{l}{2}} =$$

simetrija

$$= \frac{q}{2EI_y}\left(\frac{l^4}{24} - \frac{l^4}{16\cdot 4}\right) = \frac{5ql^4}{128EI_y 3}$$

prečni premik na sredini razpona

Motivacija



$$N_x = 0 \qquad\qquad N_x = 0 \qquad\qquad N_x = 0 \qquad\qquad \left\{ N_x = 0 \right.$$

$$N_z = F + q\bar{x} \quad = \quad N_z = q\bar{x} \quad + \quad N_z = F \quad \leftarrow F\cdot \left\{ N_z = 1 \right.$$

$$M_y = -F\bar{x} - q\frac{\bar{x}^2}{2} \qquad M_y = -q\frac{\bar{x}^2}{2} \qquad M_y = -F\bar{x} \qquad \left\{ M_y = -\bar{x} \right.$$

$\updownarrow$

DIAGRAME LAHKO VČABIH KAR SEŠTEJEMO = **SUPER POZICIJA**

ko so deformacije majhne

$$N_x^{q+F} = N_x^{q} + F\cdot N_x^{F=1}$$

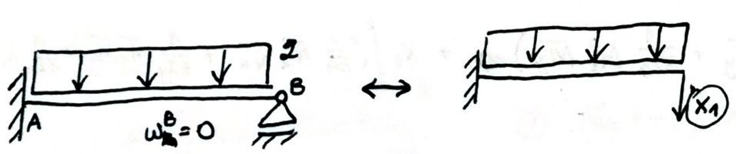
$\bar{N}_x$ ... notranja sila zaradi nominalne obtežbe velikosti 1.

$$N_x^{q+F} = N_x^{q} + F\cdot \bar{N}_x$$

$$M_y^{q+F} = M_y^{q} + F\,\bar{M}_y$$

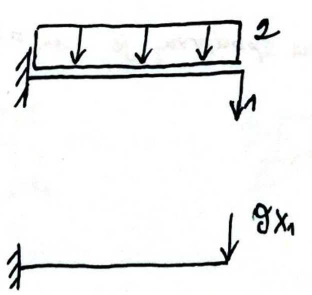
$$N_z^{q+F} = N_z^{q} + F\bar{N}_z$$

notranje sile lahko sestavimo iz ločenih in nominalnih obteženih primerov



ANALIZIRAMO

$$\begin{cases} N_x^{nk} = N_x^{q} + X_1\bar{N}_x \\ M_y^{nk} = M_y^{q} + X_1\bar{M}_y \\ N_z^{nk} = N_z^{q} + X_1\bar{N}_z \\ M_z^{nk} = M_z^{q} + X_1\bar{M}_z \end{cases}$$



Izrek o DVD (dopolnilnem virtualnem delu)

$$\delta W_z^* = \delta W_n^* \underset{\substack{\uparrow \\ \text{nosilec}}}{=} \int_V \delta \sigma_{xx}\, \varepsilon_{xx}\, dV$$

nk... nedoločena konstrukcija

$$\sigma_{xx} = \frac{N_x}{A_x} + \frac{M_y}{I_y} z - \frac{M_z}{I_z} y$$

$$\varepsilon_{xx} = \frac{N_x}{EA_x} + \frac{M_y}{EI_y} z - \frac{M_z}{EI_z} y$$

$$\delta\sigma_{xx} = \frac{\delta N_x}{A_x} + \frac{\delta M_y}{I_y} z - \frac{\delta M_z}{I_z} y$$

$$\delta W_z^* = \int_0^L \left(\frac{N_x \delta N_x}{EA} + \frac{M_y \delta M_y}{EI_y} + \frac{M_z \delta M_z}{EI_z} \right) dx \qquad \text{DVD}$$

iščemo primerno virtualno obtežbo

→ ker je na mestu sprostitve dejanski pomik / zasuk enak ∅ (če je bila tam podpora)

izberemo δX_1 za VIRTUALNO OBTEŽBO

δX_1 ... sila poljubne velikosti v smeri neznane sile X_1

$q + X_1$... predstavlja dejansko obtežbo → N_x^{nk}, M_y^{nk}, M_z^{nk}

δX_1 ... predstavlja virtualno obtežbo → δN_x, δM_y, δM_z

$$\left.\begin{aligned} \delta N_x &= \delta X_1 \overline{N}_x \\ \delta M_y &= \delta X_1 \overline{M}_y \\ \delta M_z &= \delta X_1 \overline{M}_z \end{aligned}\right\} \text{ vstavimo v DVD}$$

$$\delta X_i u_i = \int_0^L \left(\frac{1}{EA} (N_x^0 + X_1 \overline{N}_x) \delta X_1 \overline{N}_x + \frac{1}{EI_y} (M_y^0 + X_1 \overline{M}_y) \delta X_1 \overline{M}_y + \frac{1}{EI_z} (M_z^0 + X_1 \overline{M}_z) \delta X_1 \overline{M}_z \right) dx$$

$$u_1 \delta X_1 = \delta X_1 \cdot \left\{ \underbrace{\int_0^L \left(\frac{1}{EA} N_x^0 \cdot \overline{N}_x + \frac{1}{EI_y} M_y^0 \overline{M}_y + \frac{1}{EI_z} M_z^0 \overline{M}_z \right) dx}_{b_1} + X_1 \underbrace{\int_0^L \left(\frac{1}{EA} \overline{N}_x \overline{N}_x + \frac{1}{EI_y} \overline{M}_y \overline{M}_y + \frac{1}{EI_z} \overline{M}_z \overline{M}_z \right) dx}_{a_{11}} \right\}$$

$$u_1 \cancel{\delta X_1} = \cancel{\delta X_1} (b_1 + X_1 a_{11}); \quad \delta X_1 \text{ je poljuben}$$

$$b_1 + X_1 a_{11} = u_1 \qquad \text{če je točka nosilca podprta, je } u_i = 0$$

$$\downarrow \text{ ROBNI POGOJI}$$

$$b_1 + X_1 a_{11} = 0$$

ALGORITEM za enkrat nedoločene konstrukcije

① $\tilde{m}_{ps} = -1$

$m = 1$ enkrat nedoločena

② SPROSTITEV: ~~poiščemo~~ izberemo statično določeno konstrukcijo, kjer eno podporo ali vez sprostimo tako, da konstrukcija postane statično določena.

Labilne sprostitve so prepovedane!

Tej konstrukciji rečemo NADOMESTNA, STATIČNO DOLOČENA KONSTRUKCIJA

③ za nadomestno konstrukcijo določimo:

a) notranje sile zaradi dejanske obtežbe Q

N_x^Q, M_y^Q, M_z^Q

b) notranje sile zaradi nominalne ~~obtež~~ vrednosti $(X_1) = 1$

$\overline{N_x}, \overline{M_y}, \overline{M_z}$

④ DOLOČITEV PRAVE VREDNOSTI X_1

$$X_1 = \frac{-b_1}{a_{11}}$$

$$b_1 = \int_0^L \left(\frac{N_x \overline{N_x}}{EA} + \frac{M_y \overline{M_y}}{EI_y} + \frac{M_z \overline{M_z}}{EI_z} \right) dx$$

$$a_{11} = \int_0^L \left(\frac{\overline{N_x} \overline{N_x}}{EA} + \frac{\overline{M_y} \overline{M_y}}{EI_y} + \frac{M_z \overline{M_z}}{EI_z} \right) dx$$

⑤ REKONSTRUKCIJA → SUPERPOZICIJA

$N_x^{nk} = N_x^Q + X_1 \overline{N_x}$

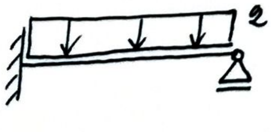
$M_y^{nk} = M_y^Q + X_1 \overline{M_y}$

...

↓

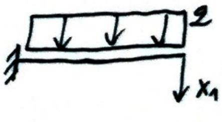
račun notranjih sil za znan X_1

* PRIMER

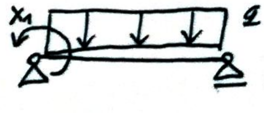


① $\tilde{m}_{ps} = -1$, $m = 1$

② I. možnost



II. možnost

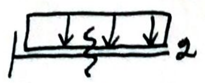


I. možnost

$X_1 = ?$

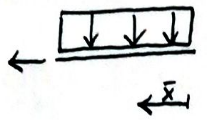
③ NSK za 2. obtežbena problema

1) zunanja obtežba



$\bar{x} = l - x$

$[N_x]$



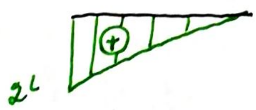
$N_x = 0$

$M_y = -q\frac{\bar{x}^2}{2}$

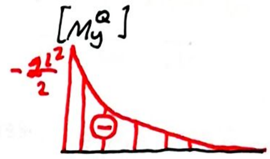
$M_y = -\frac{q}{2}(l-x)^2$

$$\boxed{M_y = -\frac{q}{2}(x^2 - 2lx + l^2)}$$

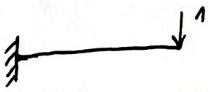
$[N_z]$



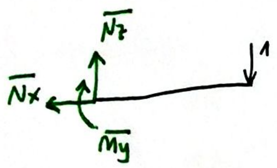
$[M_y^q]$



2) $X_1 = 1$



$[\bar{N}_x]$



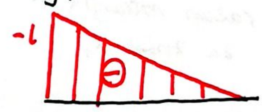
$N_x = 0$

$\bar{M}_y = -\bar{x} = -(l-x)$

$\bar{M}_y = x - l$

$[\bar{N}_z]$

$[\bar{M}_y]$



3) določitev X_1

$$a_{11} = \int_0^l \frac{1}{EI_y}(x-l)^2\,dx$$

$$a_{11} = \frac{1}{EI_y}\int_0^l (x^2 - 2lx + l^2)\,dx$$

$$a_{11} = \frac{1}{EI_y}\left(\frac{x^3}{3} - lx^2 + lx\right)\Big|_0^l$$

$$a_{11} = \frac{l^3}{3EI_y}$$

$$b_1 = \frac{1}{EI_y}\int_0^l -\frac{q}{2}\left(x^2-2lx+l^2\right)\cancel{dx}\,(x-l)\,dx$$

$$b_1 = \frac{q}{2EI_y}\int_0^l \left(x^3-3lx^2+3l^2x-l^3\right)dx$$

$$b_1 = -\frac{q}{2EI_y}\left(\frac{x^4}{4} - lx^3 + 3l^2\frac{x^2}{2} - l^3x\right)\Big|_0^l$$

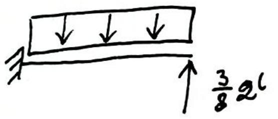
$$b_1 = -\frac{q}{2EI_y}\left(\frac{1}{4}\left(l^4-4l^4+6l^4-4l^4\right)\right)$$

$$b_1 = \frac{ql^4}{8EI_y}$$

$$X_1 = -\frac{b_1}{a_{11}} = -\frac{ql^4\, 3\cancel{EI_y}}{8\cancel{EI_y}\, l^3}$$

$$X_1 = -\frac{3ql}{8\cancel{EI_y}}$$

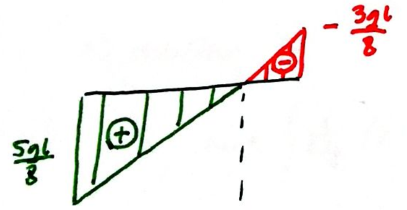
4)



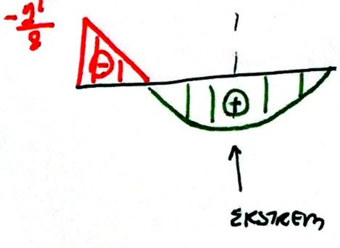
diagramu iz točke 1) prištejemo $X_1 \cdot 2)$ $\left(-\frac{3ql}{8}\right)$

$[N_x^{nk}]$

$[N_z^{nk}]$

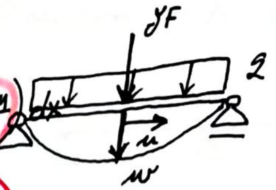


$[M_y^{nk}]$



izrek o virtualnih silah

$$u \cdot \cancel{\delta F}^{\,1} = \sum_{el} \int_0^{L_{el}} \left(\frac{N_x \delta N_x}{EA_x} + \frac{N_z \delta N_z}{GA_z} + \frac{M_y \delta M_y}{EI_y} \right) dx$$



* virtualna sila ne sme biti 0! *

* ponavadi izberemo virtualno silo 1! *

$$\frac{N_x}{EA_x} \delta N_x \qquad \frac{M_y}{EI_y} M_y$$

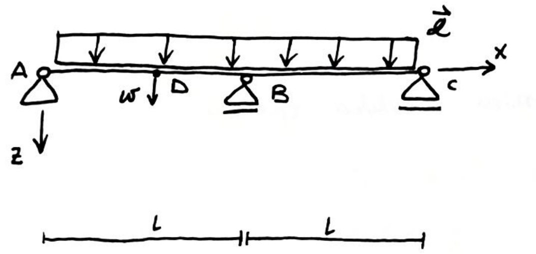
notranje delo virtualnih sil

$$\frac{du}{dx} \qquad \frac{d^2 w}{dx}$$

IZRAČUN STATIČNO NEDOLOČENIH KONSTRUKCIJ

Metoda sil (uporaba izreka o virtualnih silah)

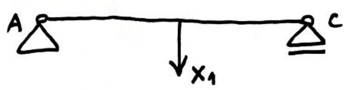
5.11.



1. $n = -\tilde{m}_{ps} = -(3 \cdot 1 - (2+1+1)) = 1$ → 1x STATIČNO NEDOLOČENA KONSTRUKCIJA

2. vpeljemo sprostitve (ker imamo 1 stopnjo preveč) - SPROSTITVE (da tvorimo statično določeno konstrukcijo)

- odstranimo podporo B → vpeljemo X_1



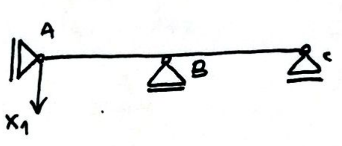
✓

X_1 nadomestna sila, ko jo bomo izračunali bo to reakcija na tej točki

- odstranimo podporo C

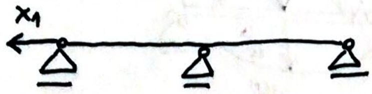
✓

- odstranimo podporo A



✓

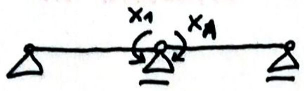
• odstranimo podporo A → LABILNA? (ne znamo določiti notranjih sil)



⊗ LABILNA

$X_1 \neq 0$

• dodamo členek → iz togega elementa sprostimo zasuk



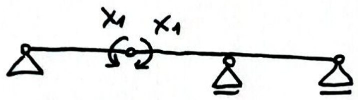
$m = -m_{ps} = -(3\cdot 2 - (2+1+1) - 2) = 0$

dodamo še

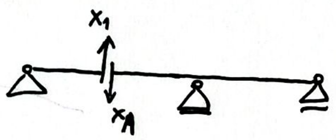
* to lahko naredimo kjerkoli na konstrukciji

vrinemo členek

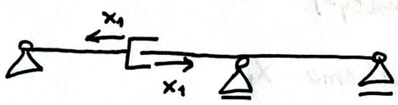
• vrinemo členek – lahko kjerkoli



• sprostitev vertikalnega pomika – lahko kjerkoli

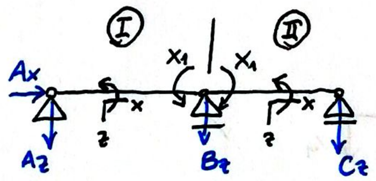


• sprostimo horizontalen pomik v poljubni točki



⊗ LABILNA

② izbrali bi lahko katerokoli



*če je 1X statično ~~predo~~ nedoločena → lahko naredimo samo 1 sprostitev, sicer je statično predoločena *

X_1 ... enak nadomestnemu momentu → nadomestna sila

③ NOTRANJE SILE

reakcije

• zaradi sile sproščitve: $X_1 = 1$

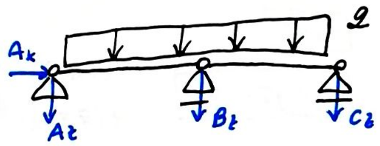
ΣX: $\boxed{A_x = 0}$

~~Zh~~ ΣM^A: $-B_z \cdot l - C_z \cdot 2l = 0 \rightarrow B_z = -2C_z \rightarrow \boxed{B_z = \frac{2}{l}}$

ΣM^B_{Cz}: $-C_z l - X_1 = 0 \rightarrow \boxed{C_z = -\frac{X_1}{l} = -\frac{1}{l}}$

ΣZ: $C_z + A_z + B_z = 0 \rightarrow \boxed{A_z = -\frac{1}{l}}$

• reakcije zaradi zunanje obtežbe (računamo brez X_1)



ΣX: $\boxed{A_x = 0}$

ΣM^B_{BC}: $-C_z l - q l \frac{l}{2} = 0 \rightarrow \boxed{C_z = -\frac{ql}{2}}$

ΣM^A: $-B_z \cdot l - C_z \cdot 2l - q \cdot 2l \cdot l = 0$

$B_z = -C_z \cdot 2l - 2ql$

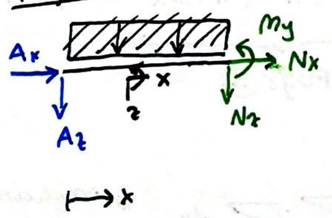
$B_z = ql - q \cdot 2l$

$\boxed{B_z = -ql}$

ΣZ: $A_z + B_z + C_z + q2l = 0 \rightarrow \boxed{A_z = -\frac{ql}{2}}$

• notranje sile zaradi $X_1 = 1$ (brez q)

polje ① $x \in [0, l]$



$\boxed{N_x = 0}$

$N_z = -A_z \rightarrow \boxed{N_z = \frac{1}{l}}$

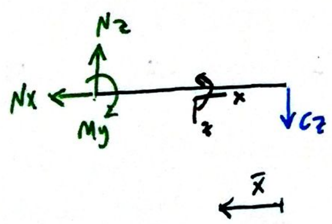
$M_y = -A_z \cdot x = \frac{1}{l} x$

$\boxed{M_y = \frac{1}{l} x}$ LINEARNA f.

$M_y(0) = 0$

$M_y(l) = 1$

polje ② $x \in [0, l]$



$\boxed{N_x = 0}$

$\boxed{N_z = -\frac{1}{l}}$

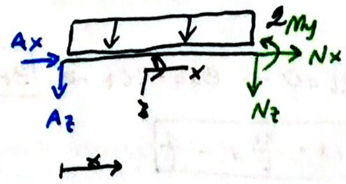
$M_y = -C_z \cdot \bar{x} \rightarrow \boxed{M_y = \frac{1}{l} \bar{x}}$

$M_y(\bar{x} = l) = 1$

$M_y(\bar{x} = 0) = 0$

• notranje sile zaradi zunanje obtežbe q

Polje (I) $x \in [0,l]$



$\boxed{N_x=0}$

$N_z=-qx+\frac{ql}{2}$ → $\boxed{N_z=-qx+\frac{ql}{2}}$

$N_z(0)=\frac{ql}{2}$

$N_z(1)=-\frac{ql}{2}$

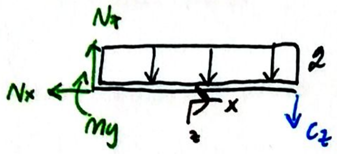
$\boxed{M_y=-q\frac{x^2}{2}+\frac{ql}{2}x}$

$M_y(0)=0$

$M_y(1)=-\frac{ql}{2}$

$M_y(\frac{l}{2})=\frac{ql^2}{2}$ ekstrem

Polje (II) $\bar{x} \in [0,l]$



$\boxed{N_x=0}$

$\boxed{N_z=q\bar{x}-\frac{ql}{2}}$ $N_z(\bar{x}=l)=\frac{ql}{2}$ $N_z(\bar{x}=0)=\frac{ql}{2}$ ničla: $\bar{x}=\frac{l}{2}$

$\boxed{M_y=-q\frac{\bar{x}^2}{2}+\frac{ql}{2}\bar{x}}$

$M_y(\bar{x}=l)=0$

$M_y(\bar{x}=0)=0$ ekstrem $M_y(\bar{x}=\frac{l}{2})=\frac{ql^2}{8}$

④ določitev koeficientov a_{ij} in b_j

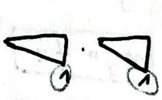
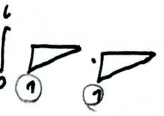
$$\boxed{a_{11}=\sum_{polja}\int_0^{l_{polja}}\left(\frac{\bar{N}_{x1}\bar{N}_{x1}}{EA_x}+\cancel{\frac{\bar{N}_{z1}\bar{N}_{z1}}{GA_z}}^{\,0}+\frac{\bar{M}_{y1}\bar{M}_{y1}}{EI_y}\right)dx} =$$

zanemarimo vpliv prečnih sil

vredi tamle

$\bar{N}_{x1}$ ... osna sila zaradi nadomestne sile X_1

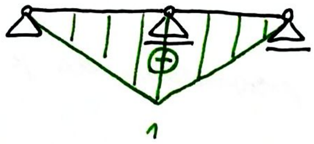
$$= \underbrace{\int_0^l \frac{1}{EI_y}\left(\frac{1}{l}x\cdot\frac{1}{l}x\right)dx}_{\text{polje (I)}} + \underbrace{\int_0^l \frac{1}{EI_y}\left(\frac{1}{l}\bar{x}\cdot\frac{1}{l}\bar{x}\right)d\bar{x}}_{\text{polje (II)}} = 2\frac{l^3}{3l^2}\frac{1}{EI_y} = \frac{2l}{3}\frac{1}{EI_y}$$

$= \frac{1}{EI_y}\int_0^l$  $dx + \frac{1}{EI_y}\int_0^l$  dx *preglednice*

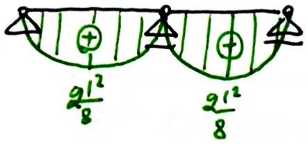
$= \frac{1}{EI_y}\frac{1}{3}\cdot l\cdot 1\cdot 1 + \frac{1}{EI_y}\frac{1}{3}\cdot l\cdot 1\cdot 1 = \frac{1}{EI_y}\frac{2l}{3}$ ✓

$$\boxed{a_{11}=\frac{1}{EI_y}\frac{2l}{3}}$$

$L1 \cdot M_{yg} =$



[$\overline{M}_{yg}$] polje (I) polje (II)



0 – osne sile odpadejo

$$\boxed{b_1 = \sum_{poljih} \int_0^l \left(\frac{\overline{N}_{xg}\,\overline{N}_{x1}}{EA_x} + \frac{M_{yg}\,\overline{M}_{y1}}{EI_y} \right) dx}$$

$$= \int_0^l \frac{1}{EI_y} \left(-q\frac{x^2}{2} + \frac{ql}{2}x\right)\left(\frac{1}{l}x\right)dx + \int_0^l \frac{1}{EI_y}\left(-\frac{q\bar{x}^2}{2} + \frac{ql}{2}\bar{x}\right)\left(\frac{1}{l}\bar{x}\right)d\bar{x}$$

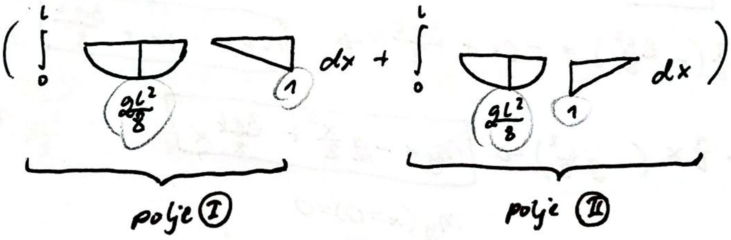
$$= \frac{1}{EI_y}\int_0^l \left(-\frac{q}{2l}x^3 + \frac{q}{2}x^2\right)dx + \frac{1}{EI_y}\int_0^l \left(-\frac{q}{2l}\bar{x}^3 + \frac{q}{2}\bar{x}^2\right)d\bar{x}$$

$$= \frac{1}{EI_y}\left[-\frac{qx^4}{8l} + \frac{qx^3}{6}\right]_0^l \cdot 2$$

$$= \frac{1}{EI_y}\frac{ql^3}{24}\cdot 2 = \frac{1}{EI_y}\frac{ql^3}{12}$$

iz preglednice

$b_1 = \frac{1}{EI_y}\Big($



$\Big)$

$$= \frac{1}{EI_y}\left(\frac{1}{3}\cdot l \cdot \frac{ql^2}{8}\cdot 1 + \frac{1}{3}\,l\,\frac{ql^2}{8}\cdot 1\right) = \frac{1}{EI_y}\frac{ql^3}{12}$$

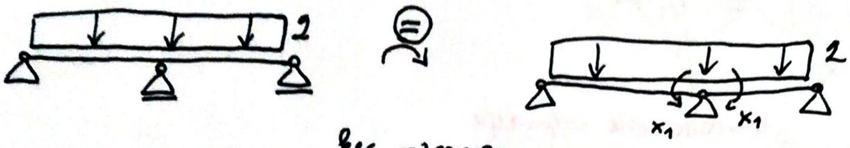
$$\boxed{b_1 = \frac{1}{EI_y}\frac{ql^3}{12}}$$

$a_{11} = X_1 + b_1 = 0$

$$X_1 = -\frac{b_1}{a_{11}} = \frac{\cancel{\frac{1}{EI_y}}\,\frac{ql^3}{12}}{\cancel{\frac{1}{EI_y}}\,\frac{2l}{3}} = -\frac{ql^2}{8}$$

$\boxed{X_1 = -\frac{ql^2}{8}}$ velikost sile sprostitve (mi smo predpostavili, da je 1)

⑤ Določitev notranjih sil v statično nedoločeni konstrukciji



$X_1 = \frac{ql^2}{8}$

ker poznamo silo sprostitve

superpozicija

$N_x = N_{xq} + \overline{N_{x1}} \cdot X_1$

$N_z = N_{zq} + \overline{N_{z1}} \cdot X_1$

$M_y = M_{yq} + \overline{M_{y1}} \cdot X_1$

$X_1 = -\frac{ql^2}{8}$

* velja samo za ~~plastično~~ linearno plastičen material*

POLJE I

$N_x = 0 + 0 \cdot X_1 = 0 \rightarrow \boxed{N_x = 0}$

$N_z = -qx + \frac{ql}{2} + \frac{1}{l}\left(-\frac{ql^2}{8}\right) = -qx + \frac{3ql}{8} \rightarrow \boxed{N_z = -qx + \frac{3ql}{8}}$

$N_z(0) = \frac{3ql}{8}$

$N_z(x=l) = -\frac{5ql}{8}$

$M_y = -q\frac{x^2}{2} + \frac{ql}{2}x + \frac{1}{l}x\left(-q\frac{l^2}{8}\right) \rightarrow \boxed{M_y = -q\frac{x^2}{2} + \frac{3ql}{8}x}$

ničla: $x = \frac{3l}{8}$

$M_y(x=0) = 0$

$M_y(x=l) = -\frac{ql^2}{8}$

ekstrem: $M_y\left(x = \frac{3l}{8}\right) = -q\frac{9l^2}{128} + \frac{3ql}{8}\frac{3l}{8} =$

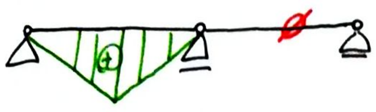
$= -\frac{9ql^2}{128}$

ta druga ničla simetrična

$[\gamma N_x = 0]$

od obtežbe

$[\gamma M_y]$



$\frac{L}{4}$

$[M_y]$

$-\frac{qL^2}{8}$

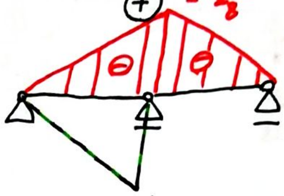
osnovna Σ. - ni členka!

⊜

$\frac{qL^2}{8}$ $\frac{qL^2}{8}$

zunanja obtežba

⊕ $-\frac{qL^2}{8}$



← realno stanje, ko upoštevamo vrednost

to je bilo ko je bilo predpostavljena sila 1

$$W_D = \frac{1}{EI_y}\int_0^L M_y \gamma M_y \, dx = \frac{1}{EI_y}\int_0^L \quad \cdot \quad dx =$$

$$= \frac{1}{EI_y}\int_0^L \left(\underset{\frac{qL^2}{8}}{} + \quad ^{-\frac{qL^2}{8}} \right)\cdot \underset{\frac{L}{4}}{} \, dx$$

$$= \frac{1}{EI_y}\int_0^L \left(\underset{\frac{qL^2}{8}}{\overset{\frac{L}{2}\ \frac{L}{2}}{}} \cdot \underset{\frac{L}{4}}{\overset{\frac{L}{2}\ \frac{L}{2}}{}} + \quad ^{-\frac{qL^2}{8}} \quad \underset{\frac{L}{4}}{} \right) dx$$

$-\frac{qL^2}{16}$ $-\frac{qL^2}{8}$

ker drugače ne bomo mogli gledat razpredelnic

$$= \frac{1}{EI_y}\int_0^{\frac{L}{2}} \left(\left(\underset{\frac{qL^2}{8}}{} \ \underset{\frac{L}{4}}{} + \underset{\frac{qL^2}{8}}{} \ \underset{\frac{L}{4}}{} \right) + \left(\quad^{-\frac{qL^2}{16}} \cdot \underset{\frac{L}{4}}{} + \quad^{-\frac{qL^2}{16}\ \ -\frac{qL^2}{8}} \cdot \underset{\frac{L}{4}}{} \right)\right) dx$$

$$= \frac{1}{EI_y}\left(\left(\frac{5}{12}\cdot\frac{L}{2}\cdot\frac{qL^2}{8}\cdot\frac{L}{4}\cdot 2 \right) + \left(\frac{1}{3}\cdot\frac{L}{2}\left(-\frac{qL^2}{16}\right)\cdot\frac{L}{4} + \frac{1}{6}\cdot\frac{L}{2}\left(2\cdot\left(-\frac{qL^2}{16}\right)+\left(-\frac{qL^2}{8}\right)\right)\cdot\frac{L}{4} \right)\right)$$

$$= \frac{1}{EI_y}\left(\frac{5qL^4}{384} - \frac{qL^4}{384} - \frac{qL^4\cdot 2}{192\cdot 2} \right) = \frac{3qL^4}{384\, EI_y} = W_D$$

* Enote!

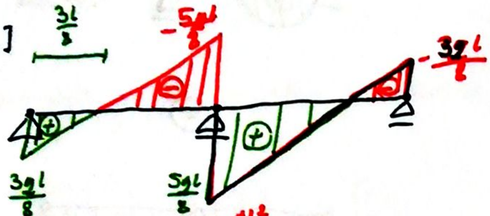
$EI_y \to \frac{kN}{cm^2}\cdot cm^4 = \frac{kN}{cm^2}$ ~~[illegible]~~

$kN(10^{-2})^2 m^2 = kNm^2 \cdot 10^{-4}$

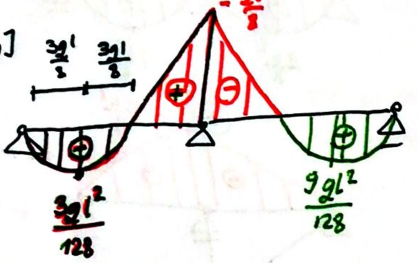
• diagrami

[Nx]

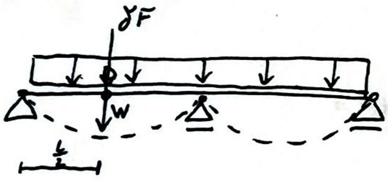
[Nz]



[My]

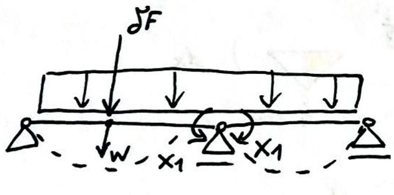


⑥ vertikalen pomik v točki D



na mesto in na smer kjer nas zanima pomik, damo virtualno silo $\delta F = 1$ namesto pomika w

$$W \cdot \cancel{\delta F}^{1} = \sum_{polja} \int_0^{l_{polja}} \left(\frac{N_x \, \delta N_x}{EA_x} + \frac{M_y \, \delta M_y}{EI_y} \right) dx$$

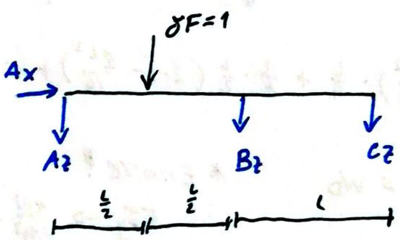


pomik w bo na tej konstrukciji enak kot na zgornji konstrukciji

RAČUN vertikalnega pomika v točki D

• notranje sile zaradi virtualne sile ~~δF=1~~

$\delta F = 1$ (na razširjeni statično določeni konstrukciji)



$A_x = 0$

$C_z = 0$

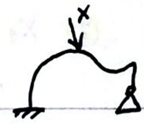
$B_z = -\frac{1}{2}$

$A_z = -\frac{1}{2}$

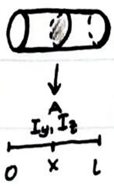
* prejšnjo polje ① moramo zdaj razdeliti na 2 polji, ker imamo vmes še δF ! *

STATIČNO NEDOLOČENE LINIJSKE KONSTRUKCIJE

$\bar{X}=1$



ENAČBE UPOGIBA



METODA POMIKOV
D.E. 4. reda

METODA SIL
algebrajske enačbe

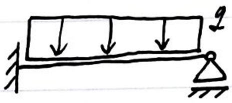
$$X_1 = -\frac{b_1}{a_{11}}$$

$$b_1 = \int_0^l \left(\frac{N_x \bar{N}_x}{EA} + \frac{M_y \bar{M}_y}{EI_y} + \frac{M_z \bar{M}_z}{EI_z} \right) dx$$

$$a_{11} = \int_0^l \left(\frac{\bar{N}_x \bar{N}_x}{EA} + \frac{\bar{M}_y \bar{M}_y}{EI_y} + \frac{\bar{M}_z \bar{M}_z}{EI_z} \right) dx$$

sile s $\bar{\ }$ ... nominalne sile, ne virtualne!

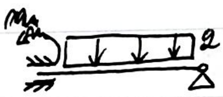
PRIMER : 2. način



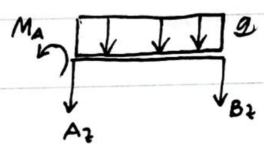
* Če sprostimo LABILNO dobimo sistem 0=0 ali protislovje – ne nadaljujemo ampak najdemo STABILNO *

LABILNA SPROSTITEV

*OPOMBA: LABILNE SPROSTITVE NISO DOVOLJENE *



$m_{ps} = 3-2-1=0$



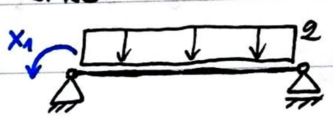
$\Sigma M^A: \; A_z \cdot l + M_A + q \cdot l \frac{l}{2} = 0$

$\Sigma M^B: \; -B_z \cdot l + M_A - ql\frac{l}{2} = 0$

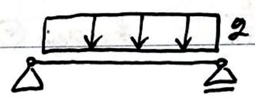
$\Sigma x: \; \boxed{0=0}$ STOP! ∞ rešitev → neprimerna sprostitev

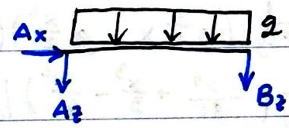
$\Sigma z: \; A_z + B_z + ql = 0$

SPROSTITEV



① ZUNANJA OBTEŽBA Q





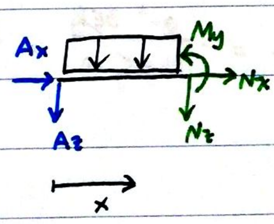
$\boxed{A_x = 0}$

$\boxed{B_z = A_z = -q\frac{l}{2}}$

$\boxed{N_z = -A_z - qx}$

$N_z(0) = q\frac{l}{2}$

$N_z(l) = -q\frac{l}{2}$

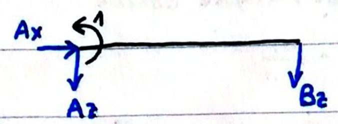


$N_x = 0$

$M_y = -xA_z - qx\frac{x}{2}$

$\boxed{M_y = q\frac{l}{2}x - q\frac{x^2}{2}}$

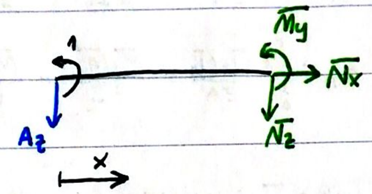
② $X_1 = 1$



$\boxed{A_x = 0}$

$\Sigma M^B = A_z l + 1 = 0 \rightarrow \boxed{A_z = -\frac{1}{l}}$

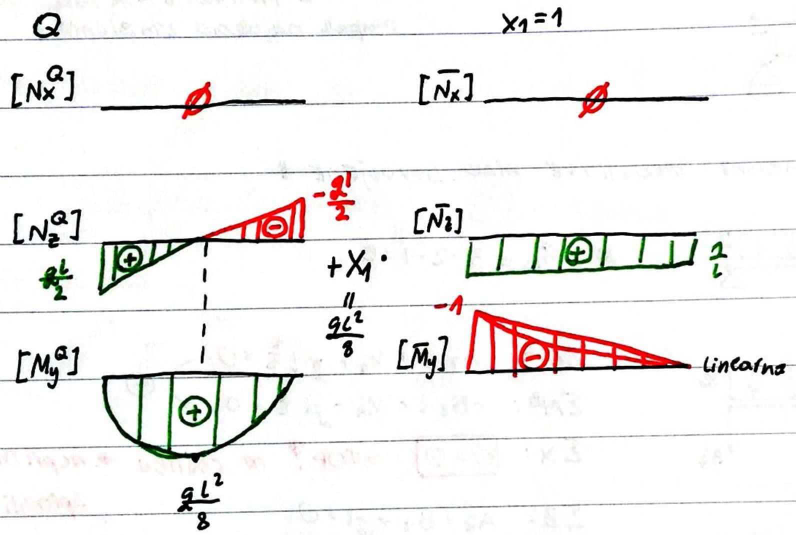
$\Sigma Z: \; A_z + B_z = 0 \rightarrow \boxed{B_z = \frac{1}{l}}$



$\boxed{\bar{N}_x = 0}$

$\bar{M}_y = -1 - A_z x \rightarrow \boxed{\bar{M}_y = -1 + \frac{x}{l}}$

$\boxed{\bar{N}_z = \frac{1}{l}}$



③ DOLOČITEV X_1

$$a_{11} = \int_0^l \frac{1}{EI_y}\left(-1 + \frac{x}{l}\right)^2 dx = \frac{1}{EI_y}\int_0^l \left(\frac{x^2}{l^2} - 2\frac{x}{l} + 1\right) dx = \frac{1}{EI_y}\left(\frac{x^3}{3l^2} - \frac{x^2}{l} + x\right)\Big|_0^l$$

$$a_{11} = \frac{l}{3EI_y}$$ *vedno ⊕*

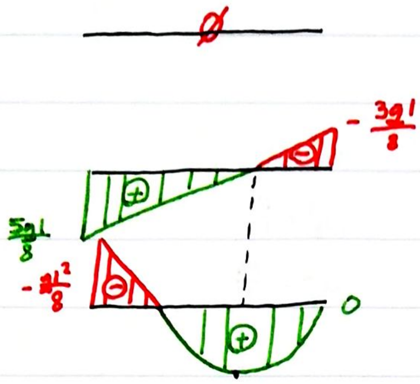
$$b_1 = \int_0^l \frac{1}{EI_y}\left(q\frac{l}{2}x - q\frac{x^2}{2}\right)\left(-1 + \frac{x}{l}\right) dx = \frac{q}{EI_y}\int_0^l \left(-\frac{x^3}{2l} + x^2 - \frac{lx}{2}\right) dx =$$

$$= \frac{q}{EI_y}\left(-\frac{x^4}{8l} + \frac{x^3}{3} - \frac{lx^2}{4}\right)\Big|_0^l = \frac{q}{EI_y}\left(-\frac{l^3}{8} + \frac{l^3}{3} - \frac{l^3}{4}\right) = -\frac{ql^3}{24EI_y}$$

$$X_1 = -\frac{b_1}{a_{11}} = -\,-\frac{ql^3\, 3\cancel{EI_y}}{24\cancel{EI_y} \cdot l} = \frac{ql^2}{8}$$

④ SUPERPOZICIJA

$[N_x^{mk}]$

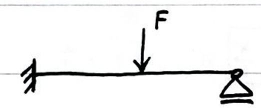


HITREJŠI način:
(s tabelo)

$$EI_y b_1 = \int_0^l \frac{ql^2}{8} \;\; -1 \; dx = \frac{1}{3} l \cdot \frac{ql^2}{8} \cdot (-1) = -\frac{ql^3}{24}$$

$$EI_y a_{11} = \int_0^l -1 \;\; -1 \; dx = \frac{1}{3} l (-1)(-1) = \frac{l}{3}$$

*PRIMER:



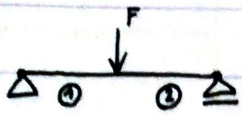
POSPLOŠITEV:

$$a_{11} = \sum_{\text{po poljih}} \int_0^{l\,\text{polja}} \left(\frac{\overline{N_x}\,\overline{N_x}}{EA} + \frac{\overline{M_y}\,\overline{M_y}}{EI_y} + \frac{\overline{M_z}\,\overline{M_z}}{EI_z} \right) dx$$

$$b_1 = \sum_{\text{po poljih}} \int_0^{l\,\text{polja}} \left(\frac{N_x^Q\,\overline{N_x}}{EA} + \frac{M_y^Q\,\overline{M_y}}{EI_y} + \frac{M_z^Q\,\overline{M_z}}{EI_z} \right) dx$$

SPROSTITEV

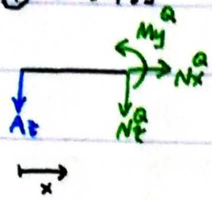
1 ZUNANJA OBTEŽBA



$A_x = 0$

$A_z = B_z = -\frac{F}{2}$

POLJE 1 $x \in [0, \frac{l}{2}]$

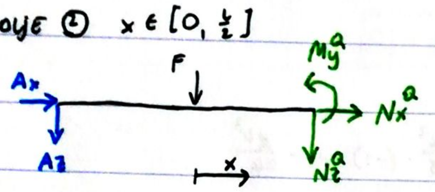


$N_x^Q = 0$

$N_z^Q = \frac{F}{2}$

$M_y^Q = \frac{Fx}{2}$

POLJE 2 $x \in [0, \frac{l}{2}]$



$N_x^Q = 0$

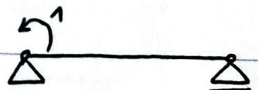
$N_z^Q = -A_z - F = -\frac{F}{2}$

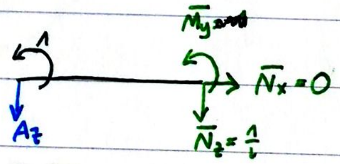
$M_y^Q = -A_z\left(\frac{l}{2} + x\right) - Fx$

$M_y^Q = \frac{F}{2}\left(\frac{l}{2} + x\right) - Fx$

$\boxed{M_y^Q = \frac{Fl}{4} - \frac{Fx}{2}}$

2 $x_1 = 1$ $x \in [0, l]$





$B_z = \frac{1}{l}$

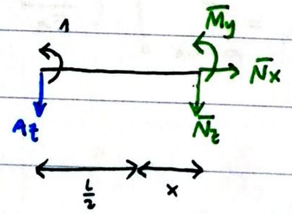
$A_z = -\frac{1}{l}$

$\boxed{\bar{M}_y = -1 + \frac{x}{l}}$

polje 1

$\bar{M}_y = -1 + \frac{x}{l}$

polje 2

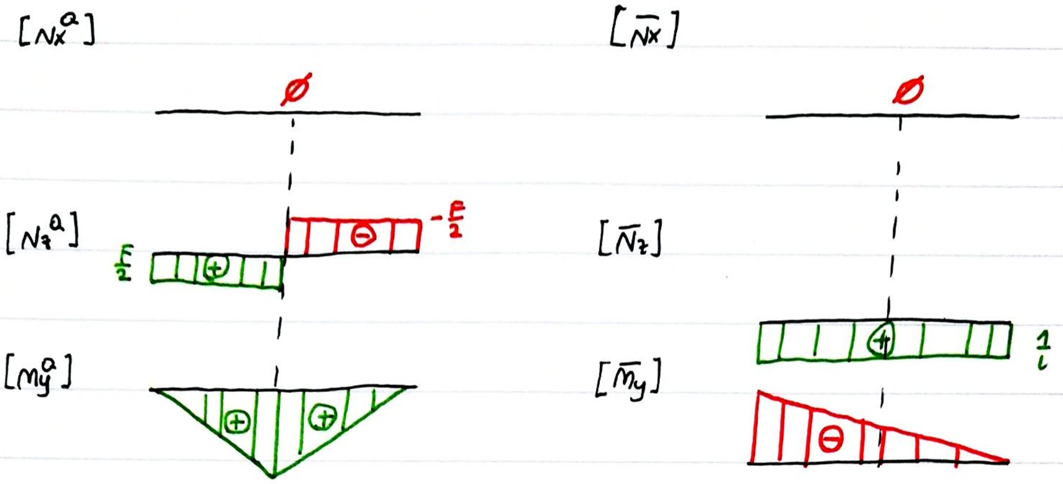


$\bar{M}_y = -A_z\left(\frac{l}{2} + x\right) - 1$

$\bar{M}_y = \frac{1}{2} - \frac{x}{l} - 1$

$\boxed{\bar{M}_y = -\frac{1}{2} - \frac{x}{l}}$

Q $\qquad\qquad x_1 = 1$



1. način

$$EI_y a_{11} = \int_0^l \; {}^{-1} \; {}^{1} \; dx = \int_0^l \left(-1+\frac{x}{l}\right)^2 dx$$

$$\parallel \quad \frac{1}{3} l$$

za a_1 diagrame množim same s sabo

$$EI_y b_1 = \int_0^{\frac{l}{2}} \frac{Fx}{2}\left(-1+\frac{x}{l}\right) dx + \int_0^{\frac{l}{2}} \left(\frac{Fl}{4} - \frac{Fx}{2}\right)\left(-\frac{1}{2} - \frac{x}{l}\right) dx$$

$$= F\left(\int_0^{\frac{l}{2}} \left(\frac{x^2}{2l} - \frac{x}{2}\right) dx + \int_0^{\frac{l}{2}} \left(\frac{x^2}{2l} - \frac{l}{8}\right) dx\right) = F\left(\left(\frac{x^3}{6l} - \frac{x^2}{4}\right)\Big|_0^{\frac{l}{2}} + \left(\frac{x^3}{6l} - \frac{l}{8}x\right)\Big|_0^{\frac{l}{2}}\right)$$

$$= F\left(\frac{l^2}{24} - \frac{l^2}{16} - \frac{l^2}{16}\right) = -\frac{2Fl^2}{24} = -\frac{Fl^2}{12}$$

2. način (tabelce)

$$EI_y b_1 = \int_0^{\frac{l}{2}} \; {}^{\frac{Fl}{4}} \; {}^{-1} \; {}^{-\frac{1}{2}} \; dx + \int_0^{\frac{l}{2}} \; {}^{\frac{Fl}{4}} \; {}^{-\frac{1}{2}} \; dx =$$

$$= \frac{1}{6}\frac{l}{2}\left(-1+2\left(-\frac{1}{2}\right)\right)\frac{Fl}{4} + \frac{1}{3}\frac{l}{2}\frac{Fl}{4}\left(-\frac{1}{2}\right) = -\frac{Fl^2}{24} - \frac{Fl^2}{48} = -\frac{3Fl^2}{48}$$