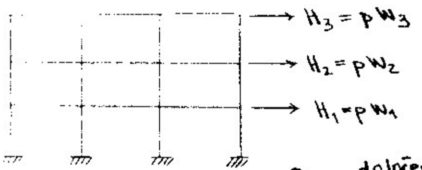
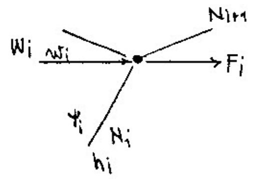
IZBIRA VEKTORJA ZUNANJE OBTEŽBE $\{F\}$

METODA HORNE



p.... določen procent vertikalne obtežbe



$$w_i = w_{i-1} + h_i \psi_i$$

$$F_i = p\, W_i$$

$$W_i = N_i - N_{i+1}$$

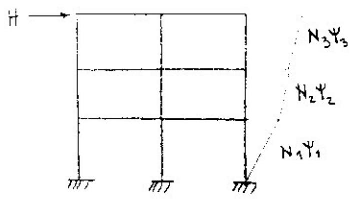
$$\{\tilde{v}\}\{F\} = \sum_i^n w_i F_i = p \sum_{i=1}^n h_i \psi_i N_i \mapsto \gamma_{CR} = \frac{p \sum_{i=1}^n h_i \psi_i N_i}{\alpha \sum \psi_i^2 h_i N_i} > \frac{p \sum \psi_i h_i N_i}{\alpha \sum \psi_i \psi_{max} h_i N_i} = \frac{p}{\alpha \psi_{max}}$$

$$\boxed{\gamma_{CR} \approx \frac{p}{\alpha \psi_{max}}}$$

$\alpha = 1{,}2$ enoetažni okvirji

$\alpha = 1{,}11$ ostali

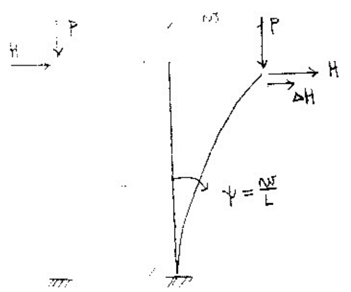
PBAB



$$\gamma_{CR} = \frac{\pi^2}{12} \frac{H}{(N\psi)_{max}}$$

P – Δ POSTOPEK

P–Δ postopek je metoda s katero ocenimo vpliv geometrijske nelinearnosti na pomike in notranje sile na podlagi rezultatov analize po TPR.



$$H = k \cdot w = \frac{3EJ}{L^3} \cdot w$$

$$\Delta H = P \cdot \psi = P \cdot \frac{w}{L}$$

$$H + \Delta H = H + P\frac{w}{L} = kw$$

$$w\left(k - \frac{P}{L}\right) = H$$

$$w = \frac{H}{\left(k - \frac{P}{L}\right)} \mapsto P_{CR} = k \cdot L = \frac{3EJ}{L^2}$$

Iteracija:

1. $\underbrace{w_0 = 0 \quad \Delta H = 0 \quad H = k w_1}_{w_1 = \frac{H}{k}}$

2. $\underbrace{\Delta H_1 = P\frac{w_1}{L} \quad H + \Delta H_1 = k w_2}_{w_2 = \frac{H}{k}\left(1 + \frac{P w_1}{LH}\right)}$, $\frac{P w_1}{LH} = \beta$

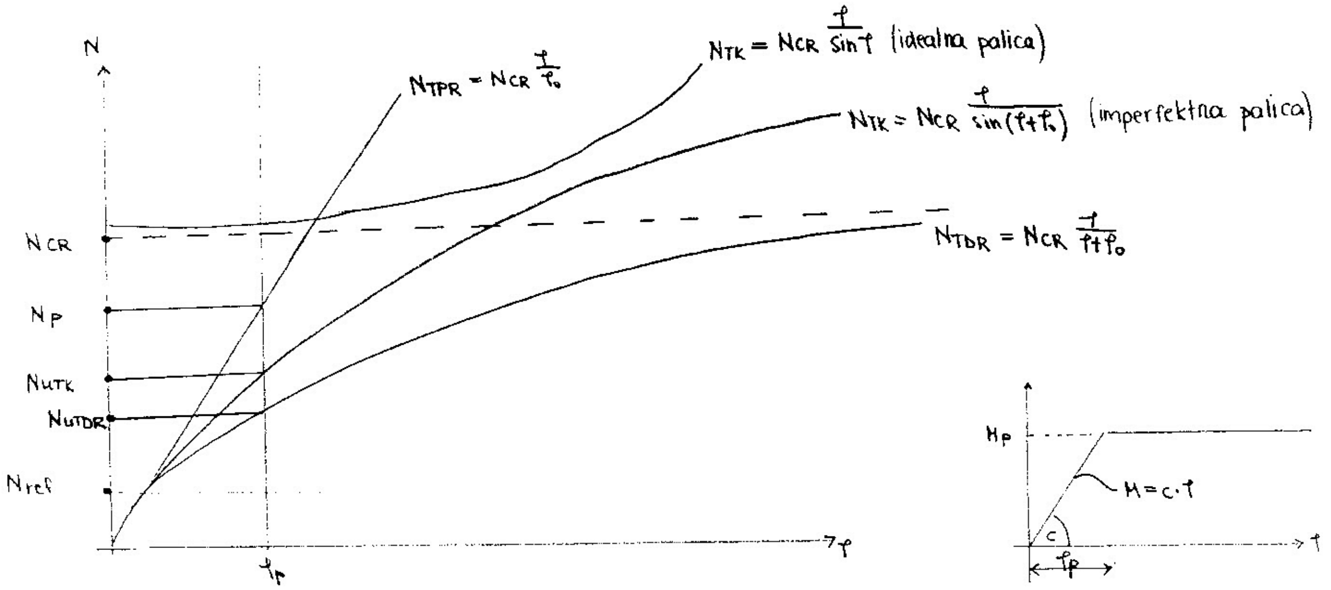
3. $\underbrace{\Delta H_2 = P\frac{w_2}{L} \quad H + \Delta H_2 = k w_3}_{w_3 = \frac{H}{k}\left(1 + \beta + \beta^2\right)}$

4. $w_4 = \frac{H}{k}\left(1 + \beta + \beta^2 + \beta^3\right)$

$$u_{TPR} = \frac{P}{k} = \frac{H}{kw}$$

$$\left.\begin{array}{l} u_1 = 0 \\ \psi = \psi_0 + \frac{u_1}{h} \end{array}\right\} \begin{array}{l} \Delta H = \alpha(P \cdot \psi) \\ u = \frac{H + \Delta H}{k} \end{array}$$

3.2. MEJNA STANJA KONSTR Z UPOŠTEVANJEM MATERIALNE IN GEOMETRIJSKE NELINEARNOSTI



Nelinearna vzmet, idealno elasto plastična

MEJNA STANJA:

1. ELASTIČNA TEORIJA I. REDA: mejno stanje N_P .... mejno stanje po elastični teoriji 1. reda
   geometrijska linearnost
   začetek plastifikacije prereza

2. PLASTIČNA TEORIJA I. REDA: mejno stanje N_P .... mejno stanje po plastični teoriji 1. reda
   $N = \gamma \cdot N_{ref}$
   $N_P = \gamma_P \cdot N_{ref}$
   ↳ mejni obtežni koeficient po plastični teoriji 1. reda
   geometrijska linearnost
   materialna nelinearnost

3. STABILNOSTNA ANALIZA IDEALNE KONSTR. (geometrijsko nelinearna, elastična analiza, idealne konstrukcije)
   $N_{CR} = \gamma_{CR} \cdot N_{ref}$
   ↳ uklonski obtežni faktor idealne konstr
   geometrijska nelinearnost, elastični material

4. PLASTIČNA TEORIJA II. REDA: (geometrijska nelinearnost, elasto-plastična analiza, imperfektna konstr)
   $N_u = \gamma_u \cdot N_{ref}$
   ↳ Mejni obtežni faktor z upoštevanjem obeh nelinearnosti in imperf.

5. ELASTIČNA TEORIJA II REDA: geometrijska nelinearnost
   materialna nelinearnost
   realna konstr.

PROBLEM ZAČETNE NESTABILNOSTI (PZN)

Vzemimo, da so pomiki 0, ter da smo notranje sile izračunali po TPR.

Ker pri TPR velja načelo superpozicije, lahko pišemo $N = \vartheta^n N_{ref}$, $\vec{b}_i = \vec{b}_{oi}$, $\vec{u}_A = \vec{u}_B = 0$, $l = l_0$

$$\Rightarrow \boxed{K_{ij} = \underbrace{\frac{EA_0}{l_0} b_{oi} b_{oj}}_{K_0} + \vartheta^n \underbrace{\frac{N_{ref}}{l_0}\left(\delta_{ij} - b_{oi} b_{oj}\right)}_{K_G}}$$

- K_0
  - → togostna matrika po TPR - majhne deform.
  - → elasticitetna matrika
  - → linearna tangentna m. (enaka kot pri TPR)
- K_G
  - → geometrijska matrika ali matrika začetnih napetosti

V matrični obliki za matriko celotne konstrukcije dobimo

$$\boxed{[K] = [K_0] + \vartheta^n [K_G]}$$

n - število prostostnih stopenj; dimenzija matrike

K_0 in K_G sta konstanti, γ edina neznanka

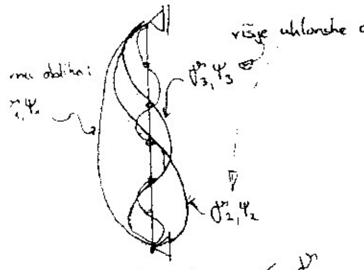
- Pogoj stabilnosti: $\det[K] = 0$ nam da rešitev za ϑ^n_{cr} (limitna ali bifurkacijska točka) (polinom reda n)
- Ker je $\det[K]$ polinom n-te stopnje, dobimo n-rešitev za ϑ^n_{cr}; ničle polinoma so krit. obt. faktorji
- merodajna rešitev je najmanjša (min γ_{cr})
- problem lahko prevedemo na posplošen problem lastnih vrednosti

$$\left([K_0] + \vartheta^n [K_G]\right)\{\Psi\} = \{0\}$$

$\{\Psi\}$ - lastni vektorji problema

→ je lažje rešiti kot iskanje posameznih ničel

- lastni vektor problema lastnih vrednosti predstavlja „uklonsko obliko" (Ψ_i .. uklonska oblika)



$$\vartheta^n_{cr1} < \vartheta^n_{cr2} < \ldots < \vartheta^n_{crn}$$

→ eksperimentalno lahko določimo le prvo kritično obtežbo, ostale so le „matematičen konstrukt"

→ imperfektnost konstrukcije je velikokrat proporcionalna prvi uklonski obliki

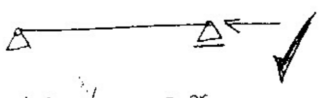
$$\boxed{\{X\}_{imper.} = \{X_0\} + \alpha \{\overline{\Psi}\}}$$

$\{X\}_{imper.}$ - imperfektna geometrija; $\{X_0\}$ - koordinate idealne konstrukcije; α - faktor; $\{\overline{\Psi}\}$ - koordinate vodič imperfektne konst.

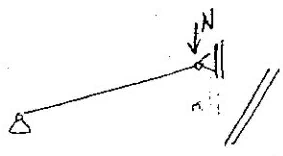
$$\{\overline{\Psi}\} = \frac{1}{\max(\{\Psi\})} \cdot \{\Psi\}$$

⇒ veljavnost rezultatov analize PZN je omejena na primere, kjer pomiki malo vplivajo na smer in razporeditev notranjih sil!

⇒ PZN je osnova formul v pravilnikih



→ če povečuješ silo se smer ne spremeni in notranje sile tudi ne drastično



→ spremeni se smer in notranje sile - veliko

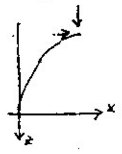
Opiši DINAMIČEN in ENERGIJSKI kriterij stabilnosti. Napiši osnovne enačbe in povej v katerih primerih veljata!

Vzamemo mehanski sistem katerega stanje lahko opišemo z m posplošenimi pomiki – (m prostostnimi stopnjami)

$q = \{q_1, q_2, q_3, \dots q_m\}$

in naj bo sistem v statičnem ravnotežju za vrednosti pomikov $q = q^e$ v času $t = t_0$

$q = \{u, v, w\}$



Ⓐ DINAMIČEN KRITERIJ STABILNOSTI

Ravnotežna lega $q = q^e$ je stabilna, če majhna perturbacija in hitrost pomikov rezultira v omejenem gibanju sistema okoli ravnotežne lege.

δ_q ... perturbacija pomikov

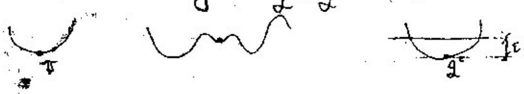
$\delta_{\dot{q}}$ ... hitrost pomikov

za $\forall t > 0$ $\quad \|q(t) - q^e\| < \varepsilon \quad$ in $\quad \|\delta \dot{q}(t)\| < \varepsilon$ $\qquad$ ε ... dovolj majhno pozitivno število

Dinamičen kriterij stabilnosti je splošen, vendar ga je zahtevno preveriti v primeru realnih konstr.

Ⓑ ENERGIJSKI KRITERIJ STABILNOSTI

Če ima totalna potencialna energija sistema v ravnotežni legi $q = q^e$ relativni (lokalni) minimum, potem je ravnotežna lega stabilna



$0 < \Pi(q) < \varepsilon$
v okolici ravnotežne točke

DOKAZ:

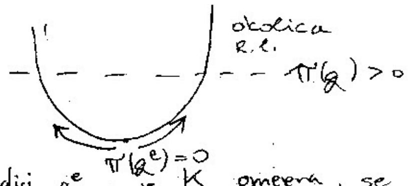
Iz izreka o ohranitvi energije sledi, da je vsota potencialne (Π) in kinetične energije (K) konstantna (c)

$$\Pi(q) + K(\dot{q}) = c$$

$c > 0$

$K > 0$

$0 < \Pi < \varepsilon$



⇒ Ker je K pozitivna, C konstantna in Π pozitivna in omejena v okolici q^e, je K omejena, se pravi, da je gibanje omejeno in ravnotežna lega stabilna.

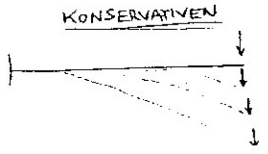
Energijski kriterij je ZADOSTEN in POTREBEN pogoj za stabilno ravnotežje sistema s potencialom.

Dinamičen kriterij → poljubni problemi

Energijski kriterij → konservativni problemi (elastičen in hiperelastičen mat., konservativna obtežba)

PRIMER:

KONSERVATIVEN



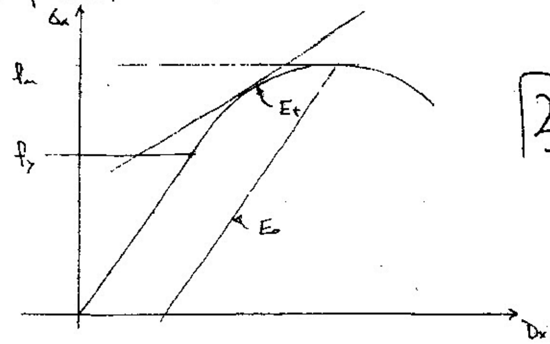
NEKONSERVATIVEN

⇒ po energijskem kriteriju stabilno

⇒ po dinamičnem kriteriju nestabilno

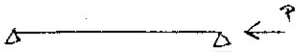
ELASTO - PLASTIČEN UKLON

⇒ elasto - plastičen material



2

⇒ vzemimo nosilec konstantnega prereza obremenjen z osno silo



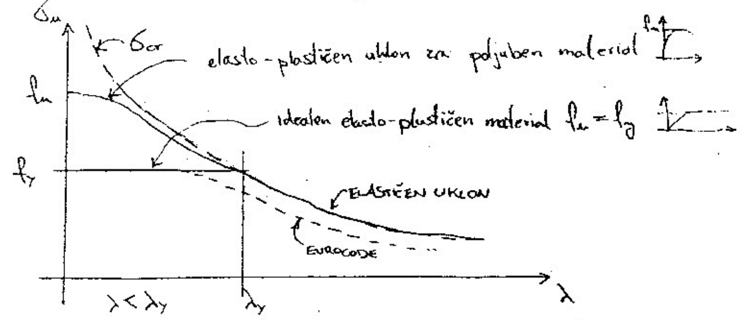
$\boxed{N_{cr} = \frac{\pi^2 EI}{l_u^2}}$ ⇒ elastična kritična obtežba

Območje veljavnosti Eulerjeve rešitve:

$$\sigma_{cr} = \frac{N_{cr}}{A} = \frac{\pi^2 EI}{A \cdot l_u^2} = \frac{\pi^2 E i^2}{l_u^2} = \frac{\pi^2 E}{\lambda^2} < f_y \qquad i = \sqrt{\frac{I}{A}}$$

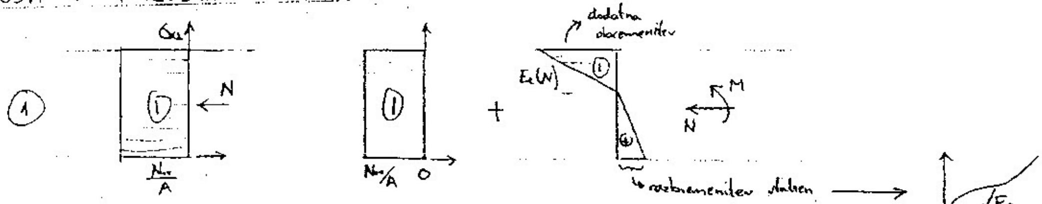
↳ vitkost

$$\Rightarrow \lambda > \lambda_y = \pi \sqrt{\frac{E}{f_y}}$$ (jeklo: $E = 21000$; $f_y = 23{,}5\ kN/cm^2$ ⇒ $\lambda_y = 92$)



3

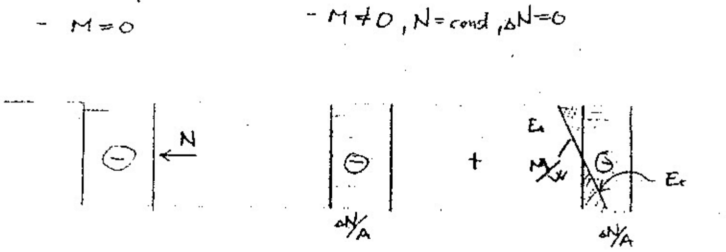
⇒ NAPETOSTI V POLJUBNEM PREREZU:



4

- PRED UKLONOM
- ravna palica
- $M = 0$

- PO UKLONU
- ukrivljena palica
- $M \neq 0$, $N = const$, $\Delta N = 0$



$\Delta N \neq 0$
$M \neq 0$

v procesu uklona se poveča osna sila in moment
$N + \Delta N = N_{cr}$

- ker je material elasto - plastičen, je pojav NEKONZERVATIVEN
- rešitev (N_{cr}) je odvisna od poti obremenjevanja

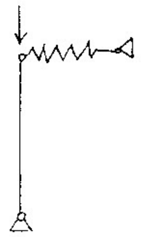
MOŽNE POTI:
ⓐ - ob uklonu ostane sila konstantna, moment pa se povečuje
- zaradi M se vlakna nad nično osjo za M razbremenijo.
- TEORIJA REDUCIRANEGA MODELA

ⓑ - ob uklonu se povečata tako sila kot moment
- vsa vlakna se dodatno obremenijo; razbremenitve ni, ker dodatna osna sila nevtralizira

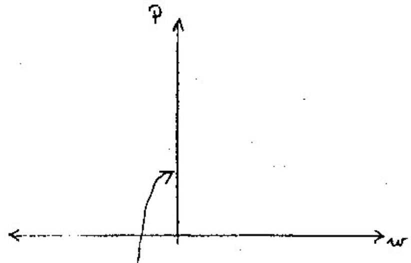
ANALIZIRAJ OBTEŽNE POTI PODANE KONSTRUKCIJE ZA PRIMER:

ⓐ TPR
ⓑ TDR
ⓒ TK

ⓐ $\boxed{\text{TPR}}$



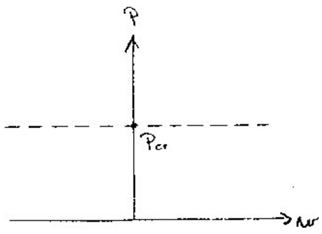
$\rightarrow$



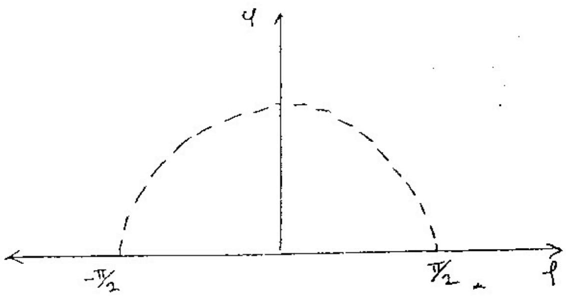
za b je tak graf

– ker se TPR dogaja pri NEDEFORMIRANI konstr. $\Rightarrow$
ni pomikov, P pa se lahko veča do neskončnosti

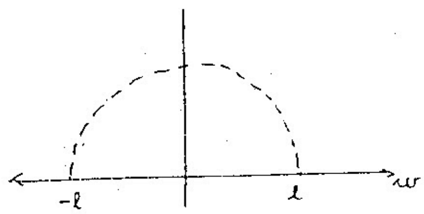
ⓑ $\boxed{\text{TDR}}$

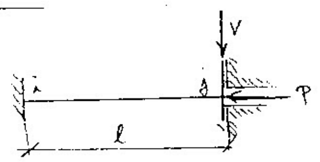


ⓒ $\boxed{\text{TK}}$ $(\varphi_0 = 0)$



$\Rightarrow$



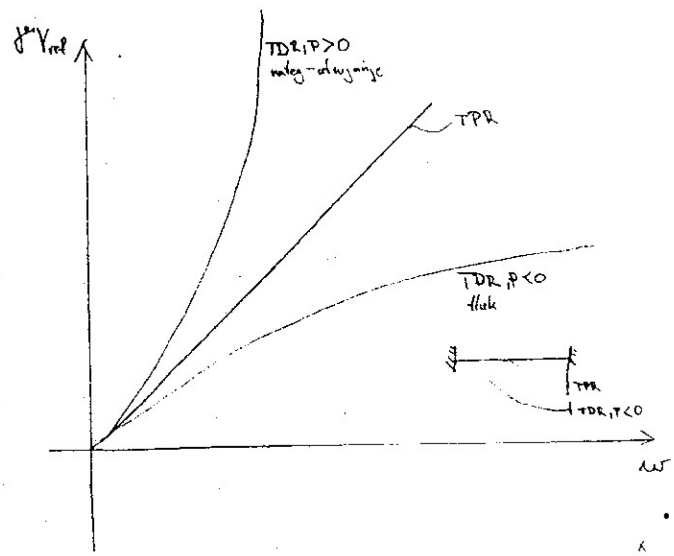


$$V = k \cdot w_j \Rightarrow w_j = \frac{V}{k}$$

ⓐ TPR $\quad k = k(EI)$

ⓑ TDR $\quad P > 0 \; ; \; k = k(EI, P) \Rightarrow$ večji k (zmanjšuje pomike)

$P < 0 \; ; \; k = k(EI, -P) \Rightarrow$ manjši k (povečuje pomike)



Izpelji Rayleighov količnik. Kaj z njim določamo? Kakšne predpostavke so uporabljene pri izpeljavi

→ Problem začetne nestabilnosti (PZN) ⇒ pomiki so 0; notranje sile po TPR

→ Predpostavimo, da je razporeditev notranjih sil neodvisna od nivoja obtežbe.

$$K = K_0 + K_\sigma = K_0 + \vartheta^{\mu} \underbrace{K_\sigma(N = N_{ref})}_{\overline{K}_\sigma} ; \qquad N = \vartheta^{\mu} N_{ref}$$

K ↳ matrika konstrukcije

$\overline{K}_\sigma$ ↳ geometrijska matrika je linearna funkcija osne sile N_{ref}

- predpostavka velja tudi na primer za P3 element in P-Δ geometrijsko matriko.

Pogoj kritične obtežbe $\det K = 0$ prevedemo na reševanje posplošenega problema lastnih vrednosti

$$\boxed{\left([K_0] + \vartheta_i^{\mu} [\overline{K}_\sigma]\right)\{\Psi_i\} = \{0\}} \qquad i = 1, 2, \ldots, n_F^{\text{št. p.s.}}$$

ϑ_i^{μ} – LASTNA VREDNOST; $\{\Psi_i\}$ – LASTNI VEKTOR

Reševanje posplošenega problema lastnih vrednosti je numerično zahtevno. Problem se poenostavi, če poznamo lastni vektor $\{\Psi_i\}$.

$$\left([K_0] + \vartheta^{\mu}[\overline{K}_\sigma]\right)\{\Psi_i\} = \{0\} \qquad / \cdot \{\Psi_i\}^T$$

$$\{\Psi_i\}^T [K_0]\{\Psi_i\} + \vartheta^{\mu}\{\Psi_i\}^T[\overline{K}_\sigma]\{\Psi_i\} = 0$$

$$\boxed{\vartheta^{\mu}_{cr} = -\frac{\{\Psi_i\}^T[K_0]\{\Psi_i\}}{\{\Psi_i\}^T[\overline{K}_\sigma]\{\Psi_i\}}} \rightarrow \text{RAYLEIGH-ev KOLIČNIK}$$

→ točen $\{\Psi_i\}$ ⇒ točen ϑ^{μ}_{cr}

→ točen $\{\Psi_i\}$ ne poznamo, lahko pa ga za regularne konstrukcije predpostavimo (pomembna le oblika, ne velikost!)

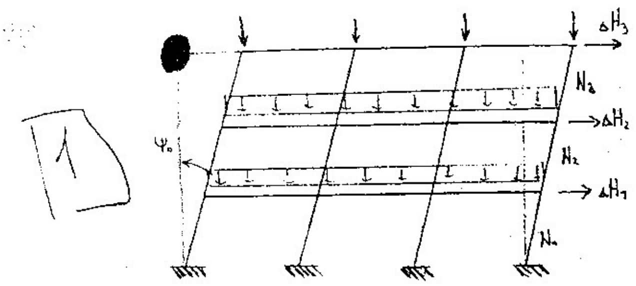
Naštej metode za približno oceno elastične kritične obtežbe regularnih okvirnih konstrukcij. Podrobneje opiši P–Δ postopek ocene (približnega določanja) pomikov konstrukcije po TDR in kritične obtežbe okvirne konstrukcije.

METODE:
- → poenostavljena metoda (P–Δ)
- → k_δ metoda
- → Rayleigh-ev količnik
- → za okvirje:
  - Horne
  - PBAB
- → P–Δ metoda

$\boxed{\text{P–Δ postopek}}$

P–Δ postopek je metoda s katero ocenimo vpliv geometrijske nelinearnosti na pomike in notranje sile na podlagi rezultatov analize po TPR.

→ ORTOGONALNI OKVIR (začetna nepopolnost ψ_0)



$\Leftarrow \Delta H_3 = N_3 \psi_0$ $\qquad$ ψ_0 – začetna nepopolnost

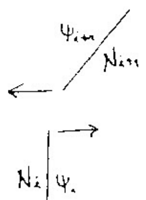
$\Leftarrow \Delta H_2 = (N_2 - N_3)\psi_0$

$\Leftarrow \Delta H_1 = (N_1 - N_2)\psi_0$

S programom po TPR rešujemo problem

$$\boxed{[K]\{v\} = \underbrace{\{F\}}_{\text{dana zunanja obtežba}} + \underbrace{\{\Delta H(v)\}}_{\text{P–Δ efekt}}}$$

$$\boxed{\Delta H_i = (N_i - N_{i+1})\psi_0 + \underbrace{1.2}_{\text{korekcijski faktor}}(N_i \psi_i - N_{i+1}\psi_{i+1})}$$



1. iteracija: $[K]\{v_1\} = \{F\} \Rightarrow w_1, \psi_1, N_1, M_1$ po TPR

2. iteracija: $[K]\{v_2\} = \{F\} + \Delta H_i(v_1) \Rightarrow w_2, \psi_2, N_2$

→ postopek ponavljamo dokler razlika dveh korakov ni dovolj majhna

$$\boxed{\|\{v_i\} - \{v_{i-1}\}\| < \varepsilon}$$

→ dobimo w_{TDR}, M_{TPR}

→ z uporabo amplifikacijskega faktorja k_δ nato ocenimo kritično obtežbo

$$k_\delta = \frac{w_{TDR}}{w_{TPR}} = \frac{1}{1 - \frac{\gamma^n}{\gamma_{cr}}}$$

$\gamma^n = 1 \Rightarrow$ projektna obtežba

$\frac{\gamma^n_{cr}}{\gamma^n} > 10 \Rightarrow$ ni nevarnosti uklona, konstr. lahko računamo po TPR

Klancek.si

⑤ ELASTIČNA TEORIJA II. REDA:

→ geometrijska nelinearnost
→ materialna linearnost
→ realna konstrukcija

MERCHANT - RANKINOVA FORMULA!

Radi bi izpeljali izraz za določitev ϑ_u^r, če poznamo ϑ_p^r in ϑ_{cr}.

φ_p – mejni zasuk, pri katerem določimo ϑ_p^r in ϑ_u^r

plastična TPR

① $$N_p = N_{cr} \cdot \frac{\varphi_p}{\varphi_o} \Rightarrow \frac{N_{cr}}{N_p} = \frac{\varphi_o}{\varphi_p}$$

- predpostavimo majhne pomike!

elastična TDR

② $$N_u \approx N_{u_{TDR}} = N_{cr} \cdot \frac{\varphi_p}{\varphi_o + \varphi_p} = N_{cr} \frac{1}{\frac{\varphi_o}{\varphi_p} + 1}$$

$$① \to ② \Rightarrow N_u = N_{cr} \frac{1}{\frac{N_{cr}}{N_p} + 1} \Rightarrow \frac{1}{N_u} = \frac{\frac{N_{cr}}{N_p} + 1}{N_{cr}} = \frac{\cancel{N_{cr}}}{N_p \cdot \cancel{N_{cr}}} + \frac{1}{N_{cr}} = \frac{1}{N_p} + \frac{1}{N_{cr}}$$

$$\frac{1}{N_u} = \frac{1}{N_p} + \frac{1}{N_{cr}} \qquad / \; N_u = \vartheta_u^r \cdot N_{ref}, \; N_p = \vartheta_p^r N_{ref}, \; N_{cr} = \vartheta_{cr} \cdot N_{ref}$$

$$\to \boxed{\frac{1}{\vartheta_u^r} = \frac{1}{\vartheta_p^r} + \frac{1}{\vartheta_{cr}^r}} \quad \text{MERCHANT - RANKINE}$$

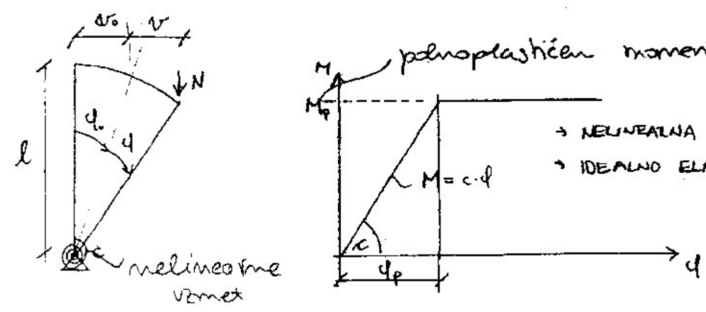
mejni obtežni faktor po plastični TDR

Formula velja ob predpostavkah:

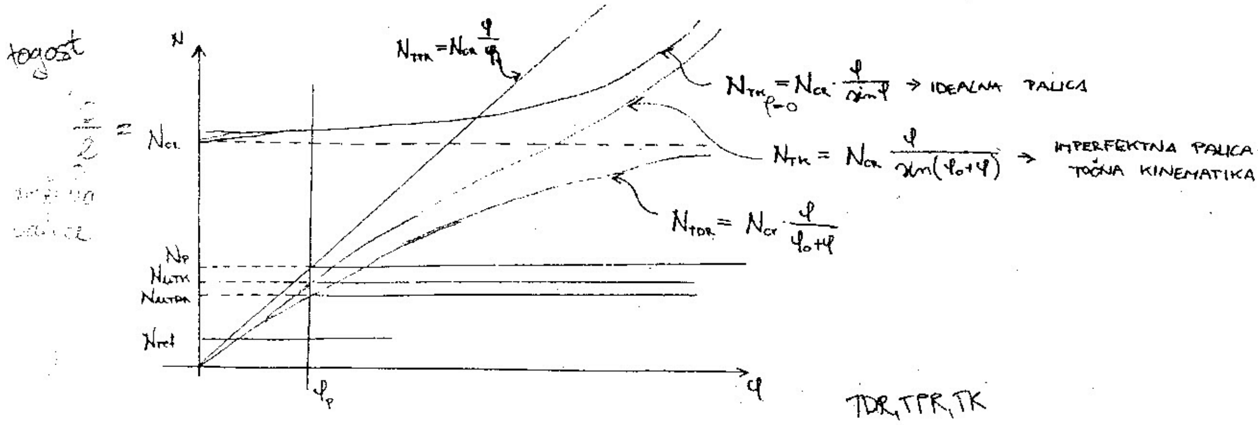
→ majhni (realni?) pomiki in ~~zasuki~~
→ idealen-elasto-plastičen material
→ problem z eno prostostno stopnjo
→ majhna imperfektnost

Formulo uporabimo za približno oceno vpliva geometrijske nelinearnosti na mejno stanje konstrukcije.

Opiši možna stanja konstrukcije glede na upoštevanje materialne in geometrijske neenakosti. Kaj izraža Marchant - Rankinova formula?



→ NELINEARNA VZMET
→ IDEALNO ELASTO-PLASTIČNA VZMET



TDR, TPR, TK

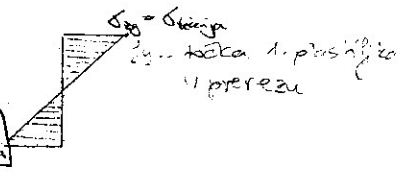
Mejno stanje je v vsakem primeru, ne glede na način računa, doseženo, ko je vzmet plastificirana, to je, ko je $\boxed{\varphi = \varphi_p}$

MEJNA STANJA

① ELASTIČNA TEORIJA I. REDA

- geometrijska linearnost, materialna
- elastičen material
- plastifikacija prve točke konstrukcije → mejno stanje po elast. TPR $\boxed{N_p = \gamma_r N_{ref}}$

obtežni faktor
referenčna obtežba



② PLASTIČNA TEORIJA I. REDA

→ geometrijska linearnost
→ materialna nelinearnost

→ N_p - mejno stanje po plastični teoriji prvega reda
→ vpeljemo obtežne faktorje (γ^r) in referenčno obtežbo (N_{ref})

$N = \gamma^r \cdot N_{ref}$ $\boxed{N_p = \gamma^r_p N_{ref}}$

za naš primer isto (na splošno pa ne)

③ STABILNOSTNA ANALIZA IDEALNE KONSTRUKCIJE

→ geometrijska nelinearnost
→ elastični material
→ idealna konstrukcija
→ materialna linearnost

$\boxed{N_{cr} = \gamma^r_{cr} N_{ref}}$

γ^r_{cr} = kritični uklonski obtežni faktor idealne konstr.

④ PLASTIČNA TEORIJA II. REDA

→ geometrijska nelinearnost
→ elasto-plastična analiza
→ imperfektna konstr.
→ materialna ...

$\boxed{N_{u_{TDR}} = \gamma^r_u N_{ref}}$

γ^r_u = mejni obtežni faktor z upoštevanjem geometrijske in materialne ...

① trivialna rešitev : $w_{mn} = 0$

② netrivialna rešitev za m,n-ti člen

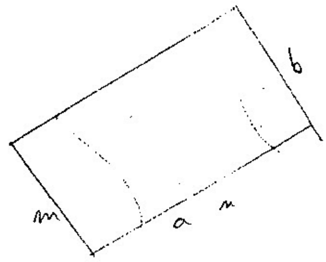
$$D\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^2 - N_{cr}\left(\frac{m\pi}{a}\right)^2$$

$$\boxed{N_{cr} = D\left(\frac{m\pi}{a} + \left(\frac{n\pi}{b}\right)^2\frac{a}{m\pi}\right)^2 = D\pi^2\left(\frac{m}{a} + \frac{n^2}{b^2}\frac{a}{m}\right)^2}$$

m - št. polvalov v x smeri

n - št. polvalov v y smeri

$$\alpha = \frac{a}{b}$$



- merodajen je minimalen $N_{cr}(m,n)$

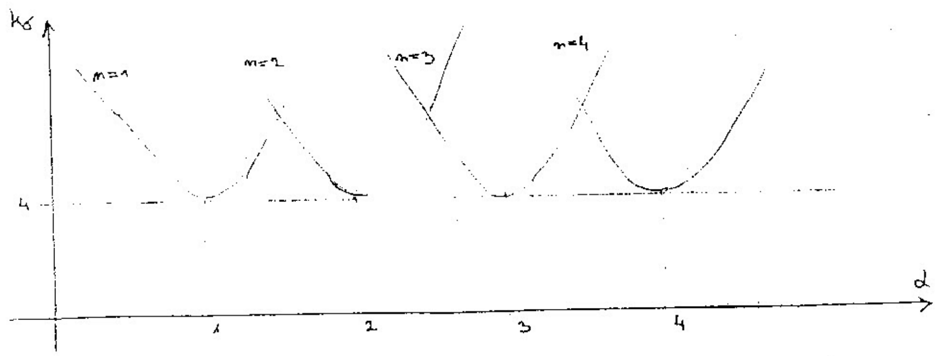
$$N_{cr} = D\pi^2\frac{1}{b^2}\left(\frac{mb}{a} + \frac{n^2 a}{mb}\right)^2 = \frac{D\pi^2}{b^2}\underbrace{\left(\frac{m}{\alpha} + \frac{n^2\alpha}{m}\right)^2}_{k_\sigma} = \frac{D\pi^2}{b^2}k_\sigma \quad \Rightarrow \quad \boxed{N_{cr} = \frac{D\pi^2}{b^2}k_\sigma}$$

merodajen: ① $n = 1$ $\quad k_\sigma = \left(\frac{m}{\alpha} + \frac{\alpha}{m}\right)^2$

② $\frac{\partial k_\sigma}{\partial m} = 0 = 2\left(\frac{m}{\alpha} + \frac{\alpha}{m}\right)\underbrace{\left(\frac{1}{\alpha} - \frac{\alpha}{m^2}\right)}_{=0}$ $\quad \rightarrow \quad \frac{1}{\alpha} - \frac{\alpha}{m^2} = 0$

$\frac{1}{\alpha} = \frac{\alpha}{m^2} \Rightarrow \boxed{\alpha = m}$

$$k_\sigma = \left(\frac{\alpha}{\alpha} + \frac{\alpha}{\alpha}\right)^2 = 4$$



$\alpha > 1 \qquad N_{cr} \approx 4D\frac{\pi^2}{b^2}$

$\alpha < 1 \qquad N_{cr} = D\frac{\pi^2}{b^2}\left(\frac{1}{\alpha} + \alpha\right)^2$

$$D = \frac{Et^3}{12(1-\nu^2)}$$ ... konstanta upogibne togosti

→ vzdolžne ojačitve (m) povečajo nosilnost kvadratov, prečne ojačitve (n) začnejo povečevati nosilnost šele ko je $a/b < 1$!

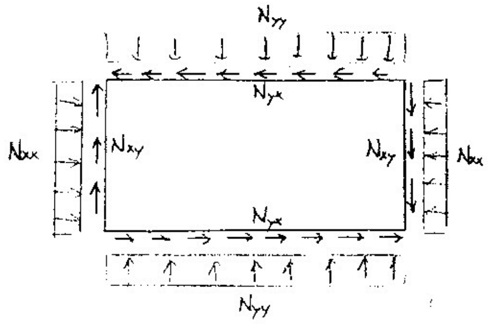
• Opiši fenomen izbočanja ploskovnih konstrukcij. Kako vplivajo na velikost kritične napetosti izbočenja vrtljivo podprte pravokotne plošče (a·b) na skici?

ⓐ razmerje a/b

ⓑ vzdolžne ojačitve

ⓒ prečne ojačitve

D.E IZBOČENJA PLOŠČE



$p_x = p_y = 0$; $p_z = 0$

Če je obtežba taka, da je $N_{xx,x} = N_{yy,y} = N_{xy,x} = N_{yx,y} = 0$ potem iz ravnotežja v smeri Z v izbočeni legi dobimo:

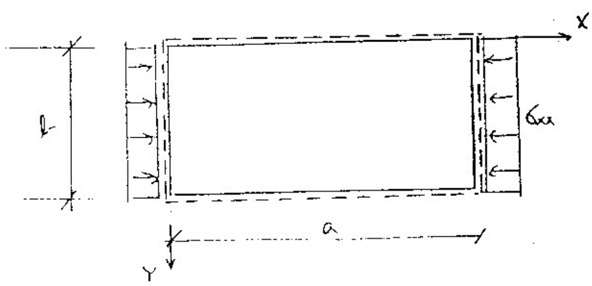
$$\underbrace{Q_{x,x} + Q_{y,y}}_{-D\cdot\Delta\Delta w} + N_{xx} w_{,xx} + 2\,N_{xy} w_{,xy} + N_{yy} w_{,yy} = 0$$

D.E izbočenja: $\boxed{D\,\Delta\Delta w = N_{xx} w_{,xx} + 2\,N_{xy} w_{,xy} + N_{yy} w_{,yy}}$

POSTOPEK: ① iz ravnotežnih pogojev v X in Y določimo N_{xx}, N_{yy}, N_{xy}

② določimo netrivialno rešitev D.E izbočenja $\Rightarrow -N_{cr}$
- uklonska oblika w

Stabilnost vrtljivo podprte, enoosno obremenjene plošče.



R.E v X in Y smeri

$\Rightarrow N_{xx} = -\sigma_{xx}\cdot t = -N_{cr}$

$N_{yy} = N_{xy} = 0$

D.E izbočenja: $D\cdot\Delta\Delta w + N_{cr} w_{,xx} = 0$

ROBNI POGOJI:

$$\left.\begin{array}{l} x=0 \\ x=a \\ y=0 \\ y=b \end{array}\right\} w = 0 \qquad \left.\begin{array}{l} x=0 \\ x=a \end{array}\right\} M_{xx}=0 \Rightarrow w_{,xx}=0 \qquad \left.\begin{array}{l} y=0 \\ y=b \end{array}\right\} M_{yy}=0 \Rightarrow w_{,yy}=0$$

$$\Delta\Delta w = w_{,xxxx} + 2\,w_{,xxyy} + w_{,yyyy}$$

$$D\left(w_{,xxxx} + 2\,w_{,xxyy} + w_{,yyyy}\right) + N_{cr}\cdot w_{,xx} = 0$$

Izberemo nastavek za w v obliki trigonometrične vrste:

$$\boxed{w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn} \sin\frac{n\pi y}{b} \sin\frac{m\pi x}{a}}$$

- robni pogoji so enolično izpolnjeni za vsak člen vrste.

D.E.

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn} \sin\frac{n\pi y}{b} \sin\frac{m\pi x}{a}\left(D\left[\left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4\right] - N_{cr}\left(\frac{m\pi}{a}\right)^2\right) = 0$$

Bilanca stanja (BS) je temeljni računovodski izkaz, ki pokaže premoženje organizacije na aktivni strani in lastništvo na pasivni strani, najpogosteje na zadnji dan v letu, tj. 31. 12. Premoženje organizacije oziroma vsa sredstva, ki jih je nekdo prispeval, so izražena v aktivi bilance stanja, pasiva pa prikazuje vse vire sredstev oziroma lastništvo, s katerimi organizacija razpolaga (Čokelc 1994, 18). Sredstva navadno prispevajo delničarji ali družabniki v denarni ali nedenarni obliki. V vsakem primeru se sredstva izrazijo in knjižijo v denarni obliki, ki je skupni imenovalec za vse vrste sredstev. Kdo je sredstva prispeval, v kakšni višini in kakšne so njegove pravice, pa se odraža v BS na pasivni strani. Obveznosti do virov sredstev kažejo, koliko obveznosti ima organizacija do lastnikov in koliko obveznosti do drugih (Turk idr. 2004, 233).

Obveznost do virov sredstev se poenostavljeno in najpogosteje deli na kapital in dolgove. S kapitalom se izraža denarno vrednost vseh sredstev, ki so jih vložili lastniki v organizacijo. Organizacija pa za svoje delovanje potrebuje še dodatna sredstva, saj je vložek lastnikov za nemoteno delovanje premajhen. Dodatno pridobljena sredstva se imenujejo dolgovi. Delitev obveznosti na kapital predstavlja tiste vire financiranja, ki v času poslovanja ne zapadejo, dolgovi pa v času poslovanja zapadejo, in sicer nekateri prej, nekateri kasneje (Turk idr. 2004, 237). Bilanca stanja je poenostavljeno prikazana v preglednici 1.

Preglednica 1: Poenostavljena shema bilance stanja

Bilanca stanja na dan	
Sredstva - aktiva	*Obveznosti do virov sredstev - pasiva*
A Stalna sredstva	A Kapital
1. Stvari	1. Osnovni kapital
2. Pravice	2. Druge sestavine kapitala
B Gibljiva sredstva	B Dolgovi
1. Stvari	1. Posojila
2. Pravice	2. Druge sestavine dolgov
3. Denar	

Vir: Melavc in Milost 2003, 140.

Dolgovi se torej delijo na dolgoročne in kratkoročne oziroma povedano drugače, na dolgoročne in kratkoročne obveznosti. Dolgoročni dolgovi so tisti, ki jih je organizacija dolžna vrniti v obdobju, daljšem od enega leta, kratkoročni pa vsi tisti dolgovi, ki imajo dospelost krajšo od enega leta oziroma dolgovi, ki jih mora organizacija vrniti v roku do enega leta.

2.1 Pojem kratkoročne obveznosti do dobaviteljev

Najpogostejša delitev kratkoročnih obveznosti je delitev na kratkoročne dolgove in kratkoročne pasivne časovne razmejitve. Kratkoročni dolgovi so glede na vir financiranja

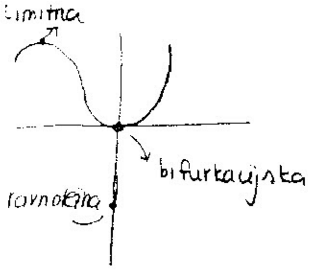
Klancek.si

OPIŠI KARAKTERISTIČNE TOČKE RAVNOTEŽNE POTI & NAČINI IZGUBE STABILNOSTI glede na obliko ravnotežne poti

1. Limitna točka oz. mičelna točka je točka skozi katero je tangenta na ravnot pot vzporedna z horizontalno

Bifurkacijska točka je točka v kateri se sekata dve ravnotežni poti

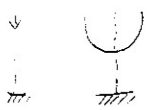
Ravnotežna točka ena izmed točk na ravnotežni poti



NAČINI IZGUBE STABILNOSTI :

* klasičen uklon (stabilna simetrična bifurkacija)
* nestabilna nesimetrična bifurkacija
* nestabilna simetrična bifurkacija
* preskok sistema

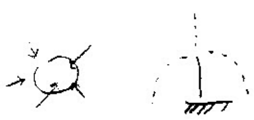
KLASIČEN UKLON



Silo povečujemo do P_{CR}, ko dosežemo P_{CR} se palica ukloni → zavzame neko drugo lego, vendar še vedno prevzema obtežbo

POST KRITIČNA NOSILNOST

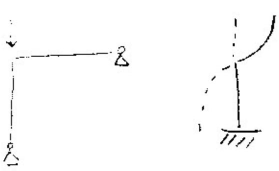
NESTABILNA SIMETRIČNA :



Ko silo povečamo nad P_{CR} pride do porušitve

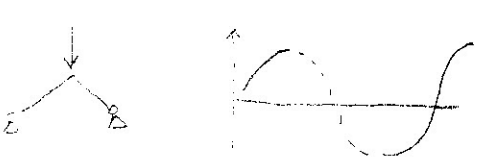
PRIMER : hidrostat nap stanje

NESTABILNA NESIMETRIČNA



Silo povečamo nad P_{CR}, konstr pa lahko zavzame stabilno ali nestabilno lego

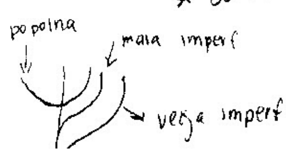
PRESKOK SISTEMA



STABILNOST REALNE KONSTR

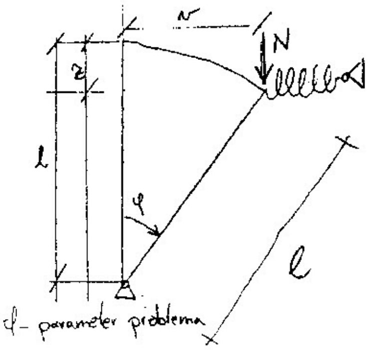
↳ imperfektnosti : * geometrijska
* materialna
* obtežbe

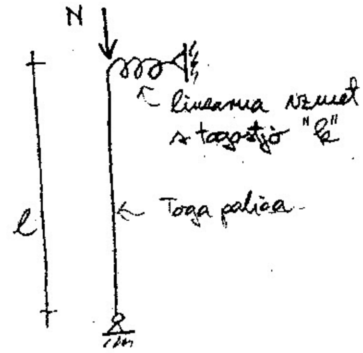
↳ odziv



[redacted]

Izpelji kritično uklonsko silo N_{cr} ob različnih predpostavkah za spodnjo konstrukcijo. Nariši tudi bifurkacijski diagram.





- PREDPOSTAVIM MAJHNE POMIKE IN ZASUKE

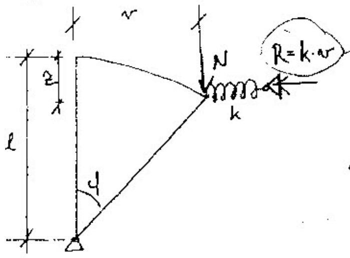
$\sin\varphi \approx \varphi$

$\cos\varphi \approx 1 - \frac{\varphi^2}{2}$

$tg\,\varphi \approx \varphi \approx \frac{v}{l}$

$l - z \approx l$

1 RAVNOTEŽNA (Eulerjeva) metoda



→ nastavimo ravnotežje v deformirani legi ob predpostavki majhnih pomikov

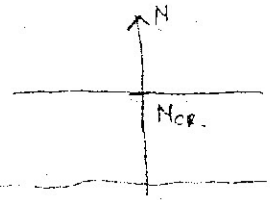
→ imamo majhne pomike $l \cong l - z$

$\Sigma M^B: \; R \cdot l - N \cdot v = 0$

$k \cdot v \cdot l - N \cdot v = 0$

$\boxed{v(kl - N) = 0}$ ravnotežna enačba

OBTEŽNI DIAGRAM



metoda nam da bif. točke, → 2 REŠITVI:

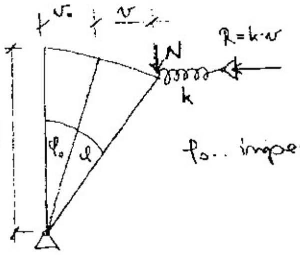
① $v = 0$ (trivialno)

② $kl - N = 0 \Rightarrow N_{cr} = N = kl$

"v"... je poljuben

in limitne točke, ne pove pa nam narave navadnih ravn. točk.

2 METODA IMPERFEKTNOSTI - NEPOPOLNOSTI



→ pri kateri obtežbi, bo imela konstrukcija ~~nekritične~~ neskončne pomike pri majhni imper

φ_0... imperfektnost

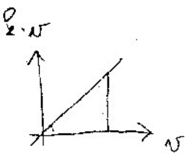
$\Sigma M^B: \; k \cdot v \cdot l - N(v + v_0) = 0$

$k \cdot v \cdot l - Nv - Nv_0 = 0$

$v(kl - N) - Nv_0 = 0$

$\boxed{v = \frac{Nv_0}{kl - N}}$ če gre imenovalec proti 0 gre pomik → ∞

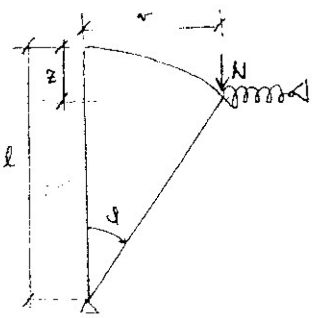
$v \to \infty \;\Rightarrow\; N = N_{cr} = kl$



3 ENERGIJSKA METODA (iščemo, kdaj bo sistem v nevtralnem ravnotežju)

Imamo MAJHNE POMIKE: $\cos\varphi \cong 1 - \frac{\varphi^2}{2}$

$tg\,\varphi \cong \varphi$



$\to tg\,\varphi = \frac{v}{l} = \varphi \;\Rightarrow\; v = \varphi \cdot l$

Potencialna en. notr. in zun. sil:

$\Pi^n = \frac{1}{2} k v^2 = \frac{1}{2} \cdot k \cdot \varphi^2 l^2$

$\Pi^z = -N \cdot z = -\frac{1}{2} N \varphi^2 l$

$z = l - l\cos\varphi =$

$= l\left(1 - 1 + \frac{\varphi^2}{2}\right) = \frac{1}{2}\varphi^2 l$

Klancek.si

1. odvod: $\frac{\partial \Pi}{\partial \varphi} = k \cdot l^2 \cdot \varphi - N \cdot l \cdot \varphi = 0$

$\Rightarrow l\varphi \left(kl - N \right) = 0$

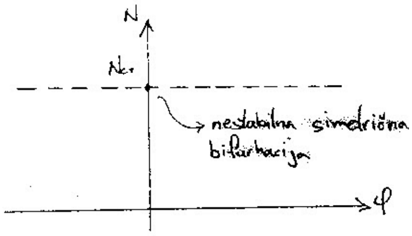
Rešitve: ① $\varphi = 0$ (trivialno)

② $kl - N = 0 \Rightarrow \underline{N_{kr} = k \cdot l}$

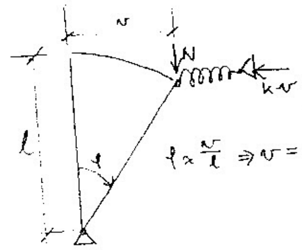
2. odvod: $\frac{\partial^2 \Pi}{\partial \varphi^2} = kl^2 - Nl = l(kl - N) \longrightarrow$ >0; $\underline{N < kl;}$ stabilna lega

$=0$; $\underline{N = kl,}$ nevtralna lega / kritična obt

<0; $\underline{N > kl;}$ nestabilna lega



4 DINAMIČEN PRISTOP



$\varphi \approx \frac{v}{l} \Rightarrow v = \varphi l$

→ I - masni vztrajnostni moment palice okoli B

→ Dinamično ravnotežje sistema

$\cancel{I\ddot{\varphi} = M}$ ... dinamična enačba

$M = N \cdot v - kv \cdot l =$

$= N\varphi l - k\varphi l^2$

$= \varphi(Nl - kl^2)$

$\Rightarrow I\ddot{\varphi} - \varphi(Nl - kl^2) = 0 \;/ : I \quad = I\ddot{\varphi} + \varphi(k l^2 - Nl)$

$\ddot{\varphi} - \frac{l(N - kl)}{I} \cdot \varphi = 0$

poljubna konst

REŠITEV: $\boxed{\varphi = C_1 \sin \sqrt{\frac{l(N-kl)}{I}}\, t}$

Harmonično nihanje s frekvenco: $\omega = \sqrt{\frac{l(N-kl)}{I}}$ $\qquad \omega = \sqrt{\frac{l(k \cdot l - N)}{I}}$

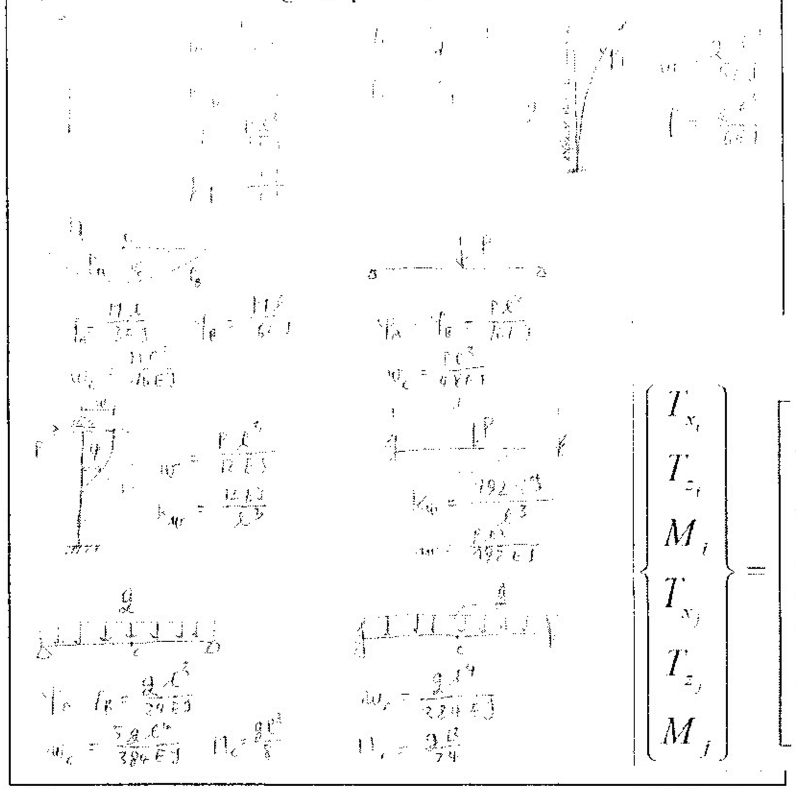
Analiza:
$N < kl \Rightarrow \omega > 0$ → nihanje z ω r.l. je stab.
$N = kl \rightarrow \omega = 0$ → nevtralno; brez nihanja (konstr. miruje
$N > kl \rightarrow \omega$ ni realen; ni nihanja

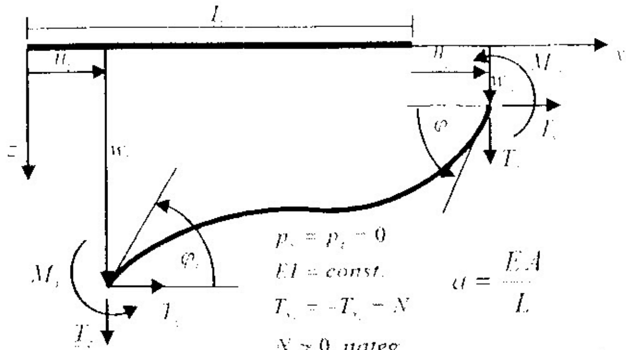
STABILNOST

- Vaje 2010
- Rešeni Izpiti

KOEFICIENTI TOGOSTI PO ELASTIČNI TEORIJI DRUGEGA REDA

Togosti po TPR





$p_x = p_z = 0$
$EI = const.$
$T_{x_i} = -T_{x_j} = N$
$N > 0$ nateg

$$a = \frac{EA}{L}$$

Rayleigh-Ritz

$$K_{ij} = \int (N P_{i,x} P_{j,x} - EI P_{i,xx} P_{j,xx})\,dx$$

$$\begin{Bmatrix} T_{x_i} \\ T_{z_i} \\ M_i \\ T_{x_j} \\ T_{z_j} \\ M_j \end{Bmatrix} = \begin{array}{cccccc} u_i & w_i & \varphi_i & u_j & w_j & \varphi_j \end{array} \left[\begin{array}{ccc|ccc} a & 0 & 0 & -a & 0 & 0 \\ 0 & b & -c & 0 & -b & -c \\ 0 & -c & d & 0 & c & e \\ \hline -a & 0 & 0 & a & 0 & 0 \\ 0 & -b & c & 0 & b & c \\ 0 & -c & e & 0 & c & d \end{array}\right] \cdot \begin{Bmatrix} u_i \\ w_i \\ \phi_i \\ u_j \\ w_j \\ \phi_j \end{Bmatrix}$$

Člen	TPR	element P3	element P4	P-Δ
b	$12\frac{EI}{L^3}$	$12\frac{EI}{L^3} + \frac{6N}{5L}$	$(12 + 1.2\bar{N} - 9\frac{\bar{N}^2}{\alpha_2})\frac{EI}{L^3}$	$12\frac{EI}{L^3} + \frac{N}{L}$
c	$6\frac{EI}{L^2}$	$6\frac{EI}{L^2} + \frac{N}{10}$	$(6 + 0.1\bar{N} - 4.5\frac{\bar{N}^2}{\alpha_2})\frac{EI}{L^2}$	$6\frac{EI}{L^2}$
d	$4\frac{EI}{L}$	$4\frac{EI}{L} + \frac{2NL}{15}$	$(4 + \frac{4}{30}\bar{N} - 2.25\frac{\bar{N}^2}{\alpha_2} - 1.75\frac{\bar{N}^2}{\alpha_1})\frac{EI}{L}$	$4\frac{EI}{L}$
e	$2\frac{EI}{L}$	$2\frac{EI}{L} - \frac{NL}{30}$	$(2 - \frac{1}{30}\bar{N} - 2.25\frac{\bar{N}^2}{\alpha_2} + 1.75\frac{\bar{N}^2}{\alpha_1})\frac{EI}{L}$	$2\frac{EI}{L}$

$\bar{N} = \frac{NL^2}{EI}$, $\alpha_1 = 30(\bar{N} + 42)$, $\alpha_2 = 70(\bar{N} + 90)$

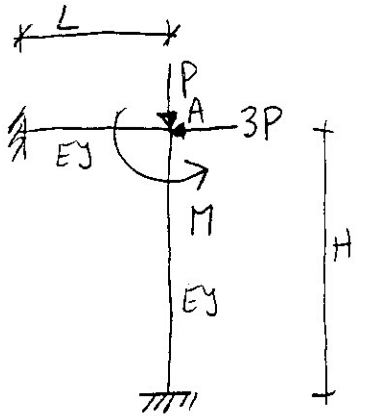
Funkcije stabilnosti

$$\omega^2 = \frac{|N|L^2}{EI}$$

	N	Vrednost
b	tlak	$\frac{\omega^3 \sin\omega}{2(1-\cos\omega) - \omega\sin\omega}\frac{EI}{L^3}$
	nateg	$-\frac{\omega^3 \sinh\omega}{2(\cosh\omega - 1) - \omega\sinh\omega}\frac{EI}{L^3}$
c	tlak	$\frac{\omega^2(1-\cos\omega)}{2(1-\cos\omega) - \omega\sin\omega}\frac{EI}{L^2}$
	nateg	$\frac{\omega^2(1-\cosh\omega)}{2(\cosh\omega - 1) - \omega\sinh\omega}\frac{EI}{L^2}$
d	tlak	$\frac{\omega\sin\omega - \omega^2\cos\omega}{2(1-\cos\omega) - \omega\sin\omega}\frac{EI}{L}$
	nateg	$\frac{\omega\sinh\omega - \omega^2\cosh\omega}{2(\cosh\omega - 1) - \omega\sinh\omega}\frac{EI}{L}$
e	tlak	$\frac{\omega^2 - \omega\sin\omega}{2(1-\cos\omega) - \omega\sin\omega}\frac{EI}{L}$
	nateg	$\frac{\omega^2 - \omega\sinh\omega}{2(1-\cosh\omega) - \omega\sinh\omega}\frac{EI}{L}$

STABILNOST – RAČUNSKI IZPIT 12.9.2011

1)



$E = 3000\ kN/cm^2$

$J = 10\,000\ cm^4$

$P = 300\ kN$ $H = 600\ cm$

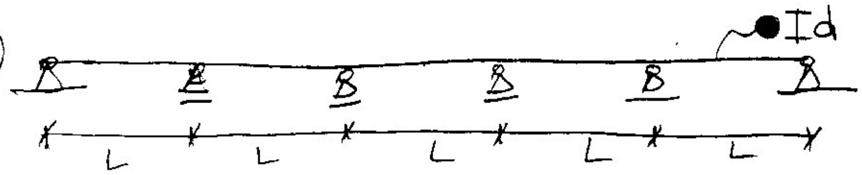
$M = 2000\ kNcm$ $L = 300\ cm$

Določi zasuk v točki A:

a) po TPR
b) po TDR (funkcije stabilnosti)

Določi elastično kritično obtežbo P_{cr} z metodo K_G.

2)



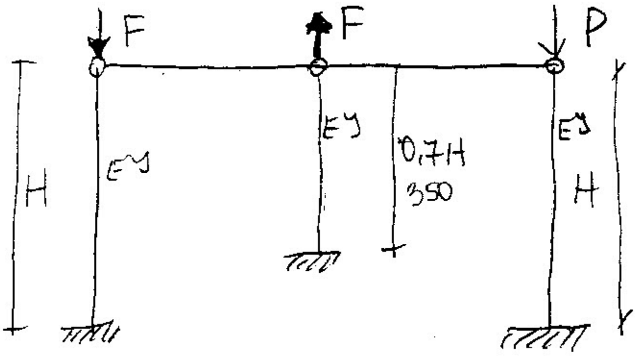
$\alpha_T = 10^{-5}/^\circ C$

$L = 300\ cm$

$E = 21000\ kN/cm^2$

Nosilec je bil zmontiran pri $-10^\circ C$. Določi tak premer polnega okroglega profila ●, da se bo elastično uklonil pri $50^\circ C$.

3)



$F = 2000\ kN$

$P_{cr} = ?$

$H = 500\ cm$

$E = 20\,000\ kN/cm^2$

$J = 10\,000\ cm^4$

Oceni elastično kritično obtežbo P_{cr} in jo določi z MKE (polinom 3 reda).

Klancek.si

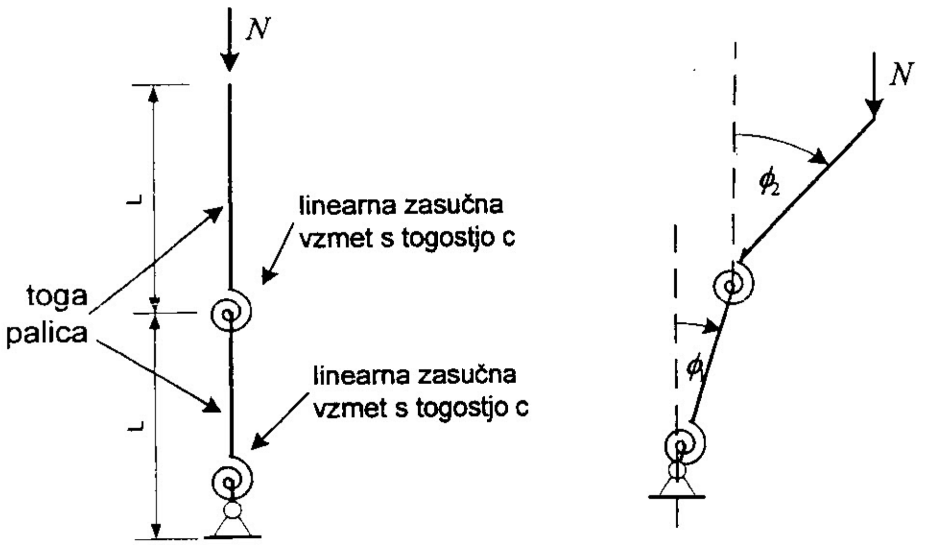
UNI-III letnik
STABILNOST KONSTRUKCIJ J

Teoretičen del:

Kaj izraža amplifakcijski faktor? Kako določimo kritično obtežbo konstrukcije na podlagi amplifakcijskega faktorja.(1 točka)

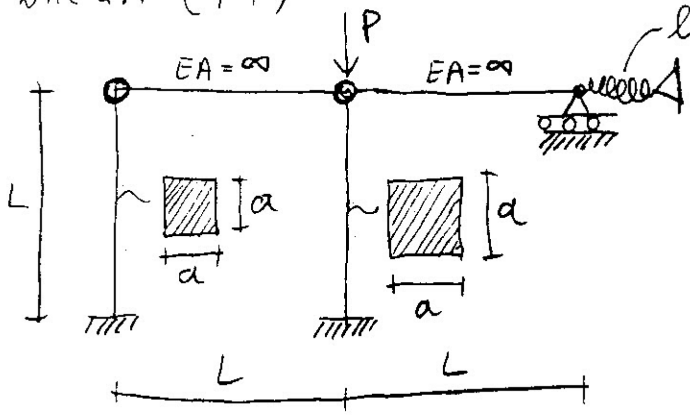
Uklon nosilca na elastični podlagi. (zasnova in predpostavke, lastnosti rešitve in karakteristični primeri)

Izpelji kritično uklonsko silo N_{cr} podane konstrukcije ob predpostavki majhnih pomikov z uporabo energijske metode.



STABILNOST KONSTRUKCIJ 29.8.2011

1 NALOGA (4T)



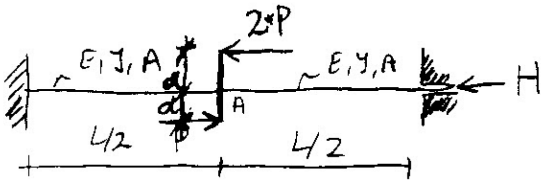
linearna vzmet s togostjo k

$L = 500$ cm
$E = 3000$ kN/cm^2
$a = 15$ cm

$P_{CR} = 300$ kN

Določi tako togost vzmeti k, da se bo konstrukcija elastično uklonila pri sili $P_{CR} = 300$ kN. (uporabi MKE, element P3).

2. NALOGA (3T)

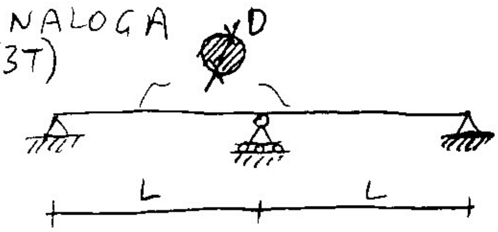


$L = 300$ cm
$H = 8000$ kN
$a = 100$ cm

$J = 10\,000$ cm^4
$P = 1000$ kN
$E = 21000$ kN/cm^2

Določi razliko momenta v točki A po TPR in z uporabo funkcij stabilnosti po TDR.

3. NALOGA (3T)

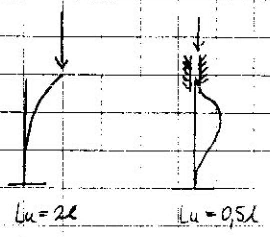


$E = 21000$ kN/cm^2
$\alpha_T = 10^{-5}/°C$
$L = 500$ cm

Nosilec je bil zmontiran pri −20°C. Določi tak premer okrogle palice D, da se bo nosilec uklonil pri temperaturi 100°C.

ZUPANC MAŠA 26108659

1\. NALOGA oceni elastično kritično silo P_{CR}.



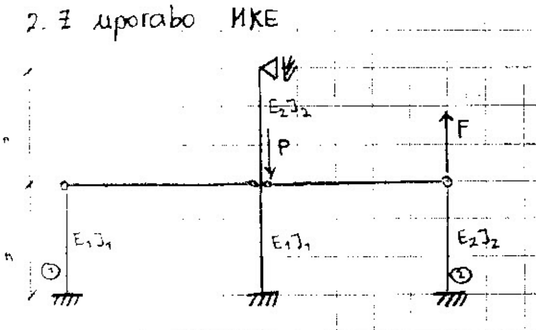
1\. OCENA:

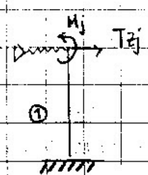
$$P_{CR} = \frac{\pi^2 EJ}{(2L)^2} = \frac{\pi^2 \cdot 3000 \cdot 50\,000}{(2 \cdot 500)^2} = 471{,}3 \text{ kN}$$

$$P_{CR} = \frac{\pi^2 EJ}{(0{,}5L)^2} = \frac{\pi^2 \cdot 3000 \cdot 50\,000}{(0{,}5 \cdot 500)^2} = 23\,687 \text{ kN}$$

$$471{,}3 \text{kN} < P_{CR} < 23\,687 \text{ kN}$$

2\. Z uporabo MKE



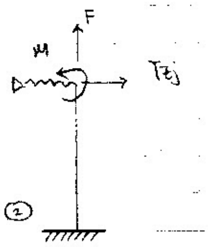


$$\begin{Bmatrix} T_{zj} \\ M_j \end{Bmatrix} = \begin{bmatrix} b - k_w & c \\ c & d \end{bmatrix} \begin{Bmatrix} w \\ \varphi \end{Bmatrix}$$

$$k_w = b - \frac{c^2}{d}$$

$$\left.\begin{aligned} b &= 66{,}7 \\ c &= 10\,000 \\ d &= 2\,000\,000 \end{aligned}\right\} k_w = 16{,}7$$

$k_w = \frac{3EJ}{l^3} = 16{,}7$ (OK ker ni osne sile)



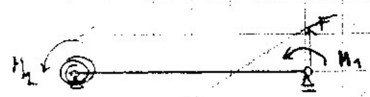
$$\begin{Bmatrix} T_{zj} \\ M_j \end{Bmatrix} = \begin{bmatrix} b - k_w & c \\ c & d \end{bmatrix} \begin{Bmatrix} w \\ \varphi \end{Bmatrix}$$

Rotacijska $k_\varphi = \frac{3EJ}{l} = 1\,500\,000$

$$k_w = b - \frac{c^2}{d}$$

(OSNA SILA!)

$$\left.\begin{aligned} b &= 46{,}87 \\ c &= 7005 \\ d &= 1\,402\,000 \end{aligned}\right\} k_w = 11{,}87$$



$$\cancel{\begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix} = \begin{bmatrix} d + k_\varphi & e \\ e & d \end{bmatrix} \begin{Bmatrix} \varphi_1 \\ \varphi_2 \end{Bmatrix}}$$

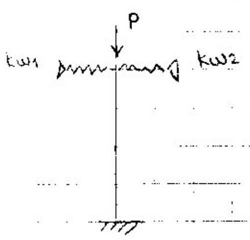
$$\cancel{(d - k_\varphi) d - e^2 = 0}$$

$$\cancel{d - k_\varphi = \frac{e^2}{d}}$$

$$\cancel{k_\varphi = d - \frac{e^2}{d}}$$

$$\left.\begin{aligned} d &= 840\,000 \\ e &= 420\,000 \end{aligned}\right\} \quad k_\varphi = ?$$

oz lahko



$$\begin{Bmatrix} T_{zj} \\ M_j \end{Bmatrix} = \begin{bmatrix} b + k_{w1} + k_{w2} & c \\ c & d \end{bmatrix} \begin{Bmatrix} w \\ \varphi \end{Bmatrix}$$

$$\det [K] = 0$$

$$(b + k_{w1} + k_{w2}) \cdot d - c^2 = 0$$

$$(b + k_{w1} + k_{w2}) = 66{,}7 + 16{,}7 + 11{,}87 + 0{,}004\, N_{CR} = 95{,}27 - 0{,}004\, N_{CR}$$

$$c = 10\,000 - 0{,}1\, N_{CR}$$

$$d = 2\,000\,000 - 40\, N_{CR}$$

$$N_{CR\,1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

$$\sqrt{D} = 798$$

$$952\,700 - 9{,}527\, N_{CR} - 40\, N_{CR} + 0{,}0004\, N_{CR}^2 - (4 \cdot 10^{12} - 16 \cdot 10^7 + 1600\, N_{CR}^2) = 0$$

$N_{CR1} = 52\,493$ //

$N_{CR2} = 50\,673$ //

$$-3{,}99 \cdot 10^{12} + 159\,999\,950\, P_{CR} - 1600\, N_{CR}^2 = 0 \quad / : (-10000) \mapsto \quad 3{,}99 \cdot 10^8 - 15999{,}9 + 0{,}16\, P_{CR}^2 = 0$$

2. NALOGA

Rayleigh – Ritz

TEŽA: $\rho \cdot a^2 \cdot g = 7850 \cdot 0{,}15^2 \cdot 10 = 1766{,}25$ kN/m

$(-P \cdot H + p \cdot x)$ $\qquad x(x-L)$

$$K_{ij} = \int (N\ P_{i,x}\ P_{j,x}) + (EI\ P_{i,xx}\ P_{j,xx})\,dx$$

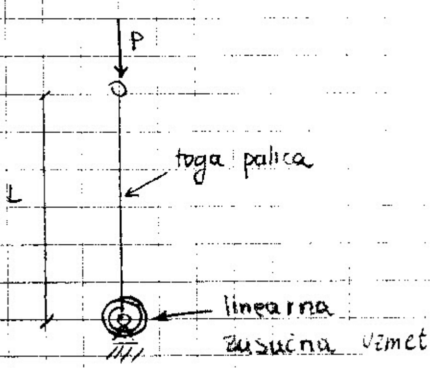
STABILNOST KONSTRUKCIJ (H) Teoretični del:

(1) Izpelji Rayleighev količnik. Kaj z njim določimo in kakšno je območje veljavnosti. Veljavnosti rešitev dobljenih z uporabo Rayleighevega količnika.

(2) Bočna zvrnitev (fenomen, predpostavke, klasične rešitve, ...)

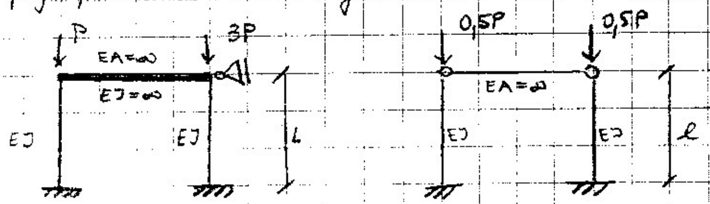
(3) Analiziraj obtežne poti (P/Φ) za primer:
a) Teorija prvega reda
b) Teorija drugega reda
c) Točna kinematika

(Analiziraj podano konstr na skici, ne nekaj na pamet !!!)

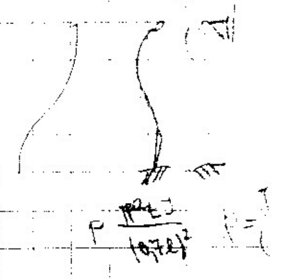


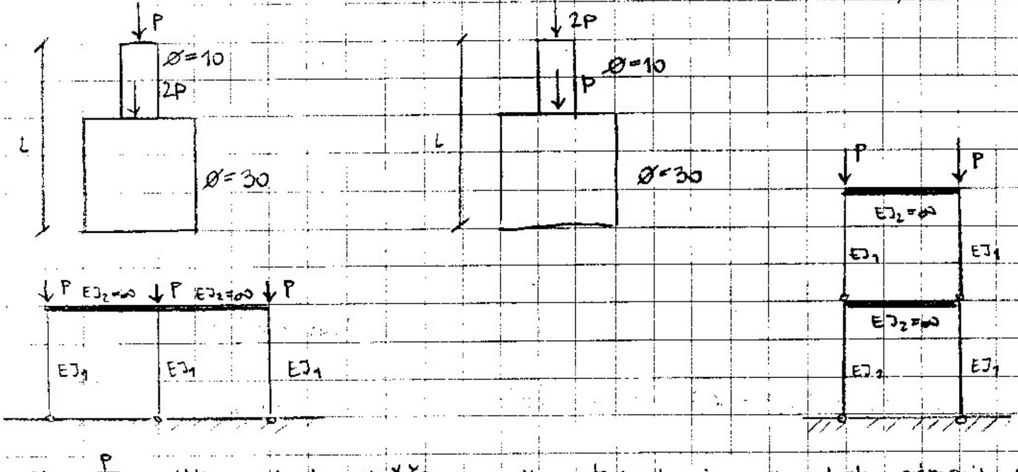
KROŽKANJE Silo P enakomerno povečujemo. Za naslednje pare konstr presodi v kateri konstr bo prej prišlo do elastičnega uklona

(1.)

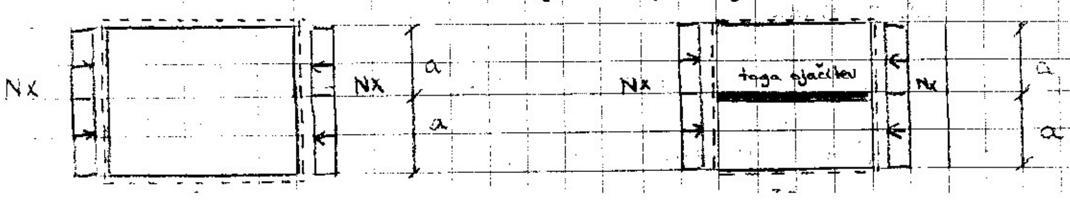


Možnosti
a) v levi
b) v desni
c) obe hkrati
d) v nobeni





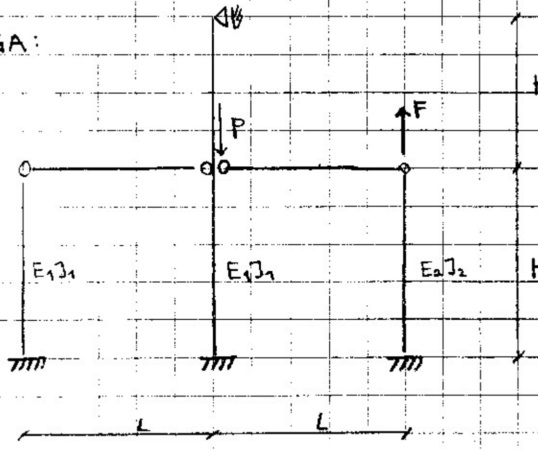
$N_x = \frac{P}{a}$ vrtljivo podprta plošča, upogibna togost je v obeh primerih enaka



STABILNOST 6.7.2011

Klancek.si

1. NALOGA:



$F = 50$ kN

$E_1 = 3000$ kN/cm²

$E_2 = 21\,000$ kN/cm²

$J_1 = 50\,000$ cm⁴

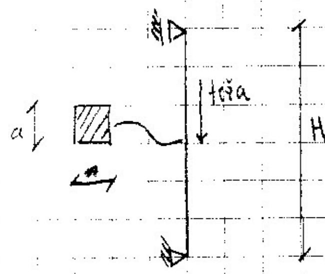
$J_2 = 5000$ cm⁴

$H = 300$ cm

$L = 500$ cm

Oceni in z MKE izračunaj ELASTIČNO KRITIČNO silo P_{CR}

2. NALOGA



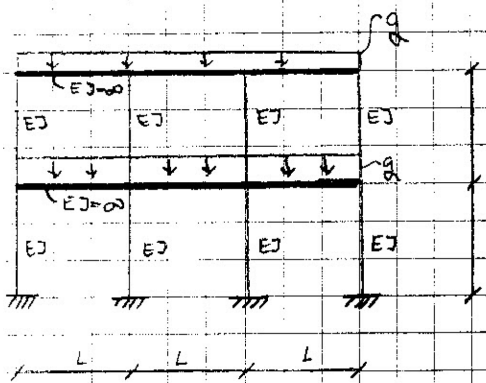
$a = 15$ cm

$E = 21\,000$ kN/cm²

$\rho = 7850$ kg/m³

Določi višino H pri kateri se nosilec ukloni zaradi lastne teže (Uporabi metodo Rayleigh – Ritz)

3. NALOGA



$L = 700$ cm

$H = 300$ cm

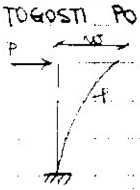
$E = 3000$ kN/m²

$J = 15\,000$ cm⁴

Z metodo horne določi elastično kritično obtežbo g_{CR}.

NA LISTU dopisano: Rayleigh-Ritz $k_{ij} = \int (N\, \varphi_{i,x} \varphi_{j,x} + EJ\, \varphi_{i,xx} \varphi_{j,xx})\, dx$

TOGOSTI PO TPR:

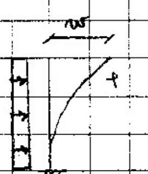


$w = \frac{Pl^3}{3EJ} = \left(b - \frac{c^2}{d}\right) P$

$k_w = 3EJ/l^3 = (b - c^2/d)$

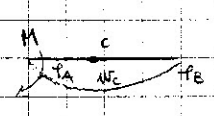
$\varphi = Pl^2/2EJ$

$k_\varphi = 2EJ/l^2$



$w = \frac{ql^4}{8EJ}$

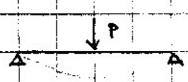
$\varphi = \frac{ql^3}{6EJ}$



$\varphi_A = \frac{Ml}{3EJ}$

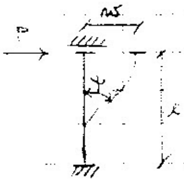
$w_c = \frac{Ml^2}{16EJ}$

$\varphi_B = \frac{Ml}{6EJ}$



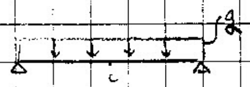
$\varphi_A = \varphi_B = \frac{Pl^2}{16EJ}$

$w_c = \frac{Pl^3}{48EJ}$



$w = \frac{Pl^3}{12EJ}$

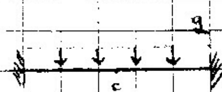
$k_w = 12EJ/l^3$



$\varphi_A = \varphi_B = \frac{ql^3}{24EJ}$

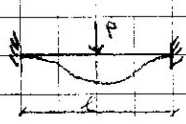
$w_c = 5ql^4/384EJ$

$M_c = ql^2/8$



$w_c = ql^4/384EJ$

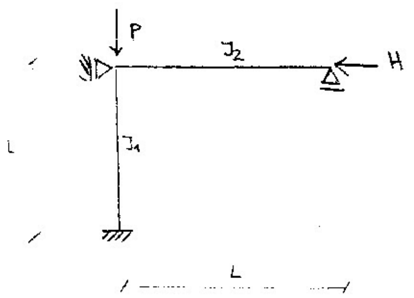
$ql^2/$



$k_w = \frac{192 \cdot EJ}{l^3}$

$w = \frac{Pl^3}{192EJ}$

MKE – Določi P_{CR} (P3)



$L = 600\ cm$

$J_1 = 8000\ cm^4$

$J_2 = 0{,}5\ J_1 = 4000\ cm^4$

$E = 20\,000\ kN/cm^2$

$H = 180\ kN$

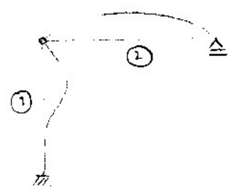
Ocena

$P_{CR} = \dfrac{\pi^2 EJ}{(0{,}7L)^2} =$ $\qquad$ $P_{CR} = \dfrac{\pi^2 EJ}{(0{,}5L)^2}$

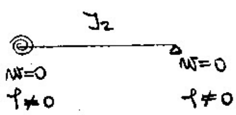
$P_{CR} = 8\,952{,}0$ $\qquad$ $P_{CR} = 17\,546$

Ocena : $8952 < P_{CR} < 17\,546$

Izračun



Palica 2 :



$$\begin{Bmatrix} M_i \\ M_j \end{Bmatrix} = \begin{bmatrix} d + k_\varphi & e \\ e & d \end{bmatrix} \begin{Bmatrix} \varphi_i \\ \varphi_j \end{Bmatrix}$$

$(d + k_\varphi)d - e^2 = 0$

$k_\varphi = \dfrac{e^2}{d} - d$

$k_\varphi = -378\,175$

$d = 533\,333 - 80N = 518\,93$

$e = 266\,667 + 20N = 270\,26$

$\boxed{\text{Ona je reševala } [M] = [d_1 + d_2]\,\phi_1}$

Palica 1 $\quad \{M_j\} = [d - k_\varphi]\{\phi_i\}$

$(d - k_\varphi) = 0$

$d = 1066\,667 - 80N$

$e = 533\,333 + 20N$

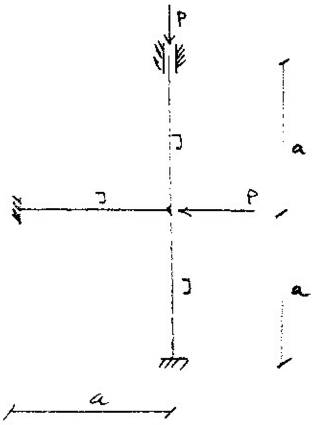
$(1066\,667 - 80N + 378\,175) = 0$

$N = \dfrac{1\,444\,842}{80} = 18\,060$

↳ Izven ocene

MKE (določi $P_{CR} \rightarrow$ P3) ✓ 17. 6. 2005

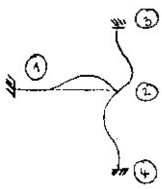
Klancek.si



$E = 3000\ kN/cm^2$

$J = 10\,000\ cm^4$

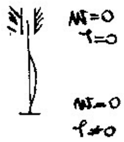
$a = 300\ cm$



HORIZONTALNA PALICA

$I.\ M_2 = d \cdot \varphi_2$

VERTIKALNA ZGORAJ :



$M_2 = d \cdot \varphi_2$ enako za spodnjo palico

$$I\ M_2 = d\ \varphi_2 \rightarrow d = \frac{4EJ}{L} + \frac{2NL}{15} = 400\,000 - 40 \cdot N$$

$$II\ M_2 = d \cdot \varphi_2 \rightarrow d = \frac{4EJ}{L} + \frac{2NL}{15} = 400\,000 - 40N$$

$$III\ M_2 = d \cdot \varphi_2 \rightarrow d = 400\,000 - 40 \cdot N$$

$$M = [d_I + d_{II} + d_{III}]\ \varphi_2$$

$$3 \cdot (400\,000 - 40N) = 0$$

$$N = 10\,000$$

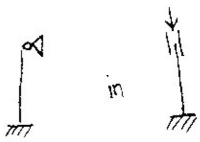
$$P_{CR} = \frac{\pi^2 EJ}{(0{,}7L)^2} =$$

$P_{CR} = 6714$

$$P_{CR} = \frac{\pi^2 EJ}{(2L)^2}$$

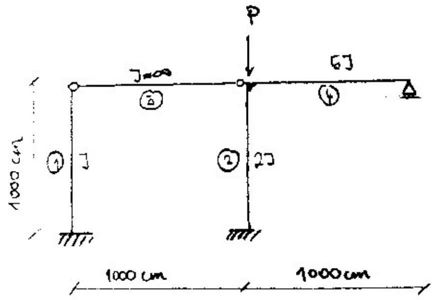
$P_{CR} = 822$

Ona izbrala



MKE – Določi P_{CR} (P3) + Ocena

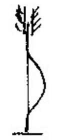
Klancek.si



$E = 21\,000\ kN/cm^2$

$J = 10\,000\ cm^4$

OCENA:



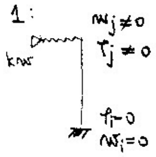
$P_{CR} = \frac{\pi^2 EJ}{(2 \cdot L)^2} \cdot 2$

$P_{CR} = 1036$

$P_{CR} = \frac{\pi^2 EJ \cdot 2}{(0{,}5L)^2}$

$P_{CR} = 16\,581$

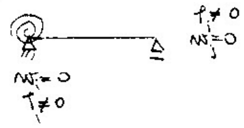
IZRAČUN: Palica 1:



$$\begin{Bmatrix} T_{zj} \\ M_j \end{Bmatrix} = \begin{bmatrix} b - k_w & c \\ c & d \end{bmatrix} \begin{Bmatrix} w_j \\ \varphi_j \end{Bmatrix}$$

ker ni osne sile: $k_w = \frac{3EJ}{\ell^3} = 0{,}63$

Palica 4:



$$\begin{Bmatrix} M_i \\ M_j \end{Bmatrix} = \begin{bmatrix} d + k_\varphi & e \\ e & d \end{bmatrix} \begin{Bmatrix} \varphi_i \\ \varphi_j \end{Bmatrix}$$

$J_4 = 5J$

$d = 4\,200\,000$

$e = 2\,100\,000$

$(d + k_\varphi) \cdot d - e^2 = 0$

$k_\varphi = d - \frac{e^2}{d} = 3\,150\,000$

Palica 2:

$$\begin{Bmatrix} T_{zj} \\ M_j \end{Bmatrix} = \begin{bmatrix} b + k_w & c \\ c & d + k_\varphi \end{bmatrix} \begin{Bmatrix} w_i \\ \varphi_i \end{Bmatrix}$$

$J_2 = 2J$

$c = 2\,520 - 0{,}1 \cdot N$

$d = 1\,680\,000 - 133{,}33\,N$

$b = 5{,}04 - 0{,}0012\,N$

$(b + k_w) \cdot (d + k_\varphi) - c^2 = 0$

$(5{,}67 - 0{,}0012N) \cdot (+4\,830\,000 - 133{,}33N) - (2520 - 0{,}1N)^2 = 0$

$27\,386\,100 + 755{,}9811N - 5796N + 0{,}16N^2 - [6\,350\,400 - 504N + 0{,}01N^2] = 0$

$0{,}15N^2 - 6\,048N + 21\,035\,700 = 0$

$N_{CR} = 3\,844{,}74$ ✓

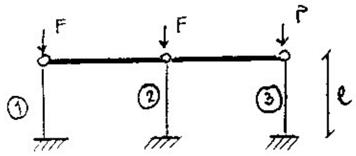
$N_{CR} = 36\,475{,}3$

✓

MKE – Določi elast kritično silo P_{CR} (P3) (17.5.2004)

Klancek.si

Ocena in izračun:



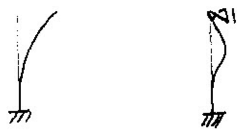
$F = 10\,kN$

$l = 300\,cm$

$E = 20\,000\,kN/cm^2$

$J = 10\,000\,cm^4$

OCENA:



$P_{CR} = \frac{\pi^2 EJ}{(2L)^2}$ $\quad$ $P_{CR} = \frac{\pi^2 EJ}{(0{,}7L)^2}$

NAJPREJ –

NATO +

$5483 < P_{CR} < 44\,760$

IZRAČUN: ELEMENT 1

$$\begin{Bmatrix} T_{zj} \\ M_j \end{Bmatrix} = \begin{bmatrix} b-kw & c \\ c & d \end{bmatrix} \begin{Bmatrix} w_j \\ \varphi_j \end{Bmatrix}$$

$b = \frac{12EJ}{L^3} + \frac{6N}{5L} = 89$

$c = \frac{6EJ}{L^2} + \frac{N}{10} = 13\,332$

$d = \frac{4EJ}{L} + \frac{2NL}{15} = 2666\,267$

$(b-kw)\cdot d - c^2 = 0 \longmapsto kw = b - \frac{c^2}{d}$

$kw = 22{,}34$

ELEMENT 2: isto $kw_2 = 22{,}34$

ELEMENT 3:
$$\begin{Bmatrix} T_{zj} \\ M_j \end{Bmatrix} = \begin{bmatrix} b+2kw & c \\ c & d \end{bmatrix}$$

$b = \frac{12EJ}{L^3} + \frac{6N}{5L} = 88{,}89 - 0{,}004\cdot P$

$c = \frac{6EJ}{L^2} + \frac{N}{10} = 13\,333 - 0{,}1P$

$d = \frac{4EJ}{L} + \frac{2NL}{15} = 2\,666\,667 - 40P$

$(b-2w)d - c^2 = 0$

$(88{,}89 - 0{,}004P + 44{,}68)\cdot(2666\,667 - 40P) - (13\,333 - 0{,}1P)^2 = 0$

$237\,040\,030 - 35556\cdot P - 10\,667P + 0{,}16P^2 + 119\,146\,682 - 1787{,}2\cdot P - \left(177\,768\,889 - 2666{,}6P + 0{,}01P^2\right) = 0$

$0{,}15P^2 - 45\,343{,}6P + 178\,417\,826 = 0$

$P_{CR\,1,2} = \frac{-b \pm \sqrt{D}}{2a}$ $\quad$ $\sqrt{D} = 44\,147$

$P_{CR\,1} = \frac{45\,343{,}6 + 44\,147}{2\cdot 0{,}15} = 289\,302$

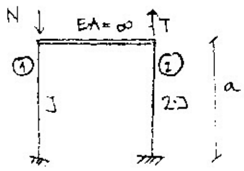
$P_{CR\,2} = \frac{45\,343{,}6 - 44\,147}{2\cdot 0{,}15} = +3\,989$

?

?

KRITIČNA OBTEŽBA N_{CR} Z MKE (polinom 3. reda)

Ocena in izračun! 2.12.2003



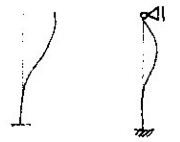
$E = 21000\ kN/cm^2$

$J = 5000\ cm^4$

$a = 100\ cm$

$T = 50\ kN$

OCENA:



$$N_{CR} = \frac{\pi^2 EJ}{(0{,}5L)^2} \qquad N_{CR} = \frac{\pi^2 EJ}{(0{,}7L)^2}$$

$$211491 < N_{CR} < 414523$$

PALICA 2

$$\begin{Bmatrix} T_{z1} \\ M_1 \end{Bmatrix} = \underbrace{\begin{bmatrix} b-k_w & c \\ c & d \end{bmatrix}}_{K} \begin{Bmatrix} w_1 \\ \phi_1 \end{Bmatrix}$$

$\det K = 0$

$(b-k_w)\cdot d - c^2 = 0$

$k_w = b - \frac{c^2}{d}$

$k_w = -6936{,}4$

$c = 6\frac{E(2J)}{L^2} + \frac{N}{10} = 126\,005$

$d = 4\frac{EJ}{L} + \frac{2NL}{15} = 1\,680\,667$

$b = 12\frac{EJ}{L^3} + \frac{6N}{5L} = 2520{,}6$

PALICA 1

$$\begin{Bmatrix} T_{z2} \\ 0 \end{Bmatrix} = \begin{bmatrix} b+k_w & c \\ c & d \end{bmatrix} \begin{Bmatrix} w_2 \\ \phi_2 \end{Bmatrix}$$

$b_1 = \frac{12EJ}{L^3} + \frac{6N}{5L} = 1260 - 0{,}012N$

$c_2 = 6\frac{EJ}{L^2} + \frac{N}{10} = 63000 - 0{,}1N$

$d_2 = 4\frac{EJ}{L} + \frac{2NL}{15} = 4200000 - 13{,}333\cdot N$

$(b-k_w)\cdot d - c^2 = 0$

$(1260 - 0{,}012N - 6936{,}4)\cdot(4200000 - 13{,}333\cdot N) - (63000 - 0{,}1N)^2 = 0$

$5292\cdot 10^6 - 16800N - 50400N + 0{,}16N^2 - 2{,}9133\cdot 10^{10} + 92483\cdot N - (3969\cdot 10^6 - 12600N + 0{,}01N^2) = 0$

$0{,}15N^2 + 37883N - 2{,}781\cdot 10^{10} = 0$

$N_{CR1,2} = \frac{-b \pm \sqrt{D}}{2a}$ $\qquad \sqrt{D} = 134\,614$

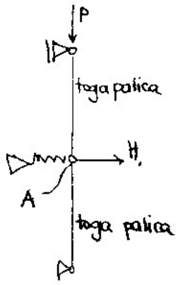
$N_{CR1} = \frac{-37883 + 134614}{2\cdot 0{,}15} = 322\,439$

$N_{CR2} = -574\,990$

//

?

METODA P-Δ določi horizontalni pomik tč A (7.5.2004)



k-elastična vzmet

$P = 10\,kN$

$k = 1\,kN/cm$

$H = 5\,kN$

$l = 200\,cm$

1. $u_1 = 0$

$\varphi_1 = \frac{u_1}{L} = 0$

$\Delta H_1 = P \cdot \varphi_1 = 0$

$u = \frac{H + \Delta H}{k} = \frac{5+0}{1} = 5$

2) $u = 5$

$\varphi = \frac{5}{L} = \frac{5}{200} = 0{,}025$

$\Delta H_1 = P \cdot \varphi = 10 \cdot 0{,}025 = 0{,}25$

$u_2 = \frac{H + \Delta H}{k} = \frac{5+0{,}25}{1} = 5{,}25$

3) $u = 5{,}25$

$\varphi = \frac{u}{L} = \frac{5{,}25}{200} = 0{,}02625$

$\Delta H = P \cdot \varphi = 10 \cdot 0{,}02625 = 0{,}2625$

$u_3 = \frac{H + \Delta H}{k} = \frac{5{,}2625}{1} = 5{,}262$

4) $u = 5{,}2625$

$\varphi_4 = \frac{u}{l} = \frac{5{,}2625}{200} = 0{,}02631$

$\Delta H = P\,\varphi_4 = 10 \cdot 0{,}02631 = 0{,}2631$

$u_4 = \frac{H + \Delta H}{k} = \frac{5{,}2631}{1} = 5{,}2631$

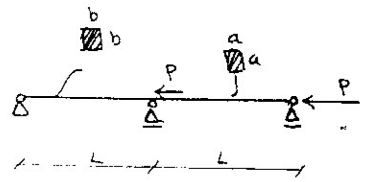
5) $u = 5{,}2631$

$\varphi_5 = \frac{u}{l} = \frac{5{,}2631}{200} = 0{,}02631$

$\Delta H = P \cdot \varphi_5 = 0{,}2631$

$u_5 = \frac{5{,}2631}{1} = 5{,}2631$

RAYLEIGH – RITZ (določi P_{CR} → polinom 3. reda) 7.5.2004 Klancek.si



$l = 5m$

$a = 3cm$

$b = 5cm$

$E = 20\,000\ kN/cm^2$

$N_1 = -2P$

$N_2 = -P$

Ničle: $w(0) = 0$

$w(L) = 0$

$w(2L) = 0$

$P = (x-0)(x-L)(x-2L)$

$P = x\cdot(x^2 - 2xL - Lx + 2L^2)$

$P = x^3 - 3x^2L + 2L^2x$

$P_{,x} = 3x^2 + 6xL + 2L^2$

$P_{,xx} = 6x + 6L$

$J_1 = \frac{b^4}{12} = \frac{5^4}{12} = 52{,}08$

$J_2 = \frac{a^4}{12} = \frac{3^4}{12} =$

$$K_{ij} = \int_0^L (N_i P_{i,x}\cdot P_{j,x} + EJ_1 P_{i,xx}\cdot P_{j,xx})\,dx + \int_L^{2L} (N_2 P_{i,x}\cdot P_{j,x} + EJ_2 P_{i,xx}\cdot P_{j,xx})\,dx$$

$$K_{ij} = \int_0^L (-2P)(3x^2+6xL+2L^2)\cdot(3x^2+6xL+2L^2) + EJ_1(6x+6L)(6x+6L)\,dx +$$

$$+\int_0^{2L} (-P)(3x^2+6xL+2L^2)(3x^2+6xL+2L^2) + EJ_2(6x+6L)(6x+6L)\,dx =$$

$$K_{ij} = -2P\int_0^L (9x^4 + 18x^3L + 6x^2L^2 + 18x^3L + 36x^2L^2 + 12xL^3 + 6x^2L^2 + 12xL^3 + 4L^4)\,dx + EJ_1\int_0^l (36x^2 + 72xL + 36L^2)\,dx$$

$$-P\int_L^{2L}(9x^4 + 36x^3L + 48x^2L^2 + 24xL^3 + 4L^4)\,dx + EJ_2\int_L^{2L}(36x^2 + 72xL + 36L^2)\,dx$$

$$K_{ij} = -2P\left(9\frac{L^5}{5} + 36\cdot\frac{L^4}{4}\cdot L + 48\frac{L^3}{3}L^2 + 24\frac{L^2}{2}\cdot L^3 + 4\cdot L\cdot L^4\right) + EJ_1\left(36\frac{L^3}{3} + 72\frac{L^2}{2}\cdot L + 36\cdot L\cdot L^2\right)$$

$$-P\left[9\frac{x^5}{5} + 36\cdot\frac{x^4}{4}\cdot L + 48\frac{x^3}{3}L^2 + 24\frac{x^2}{2}L^3 + 4\cdot xL^4\right]\Big|_L^{2L} + EJ_2\left[36\frac{x^3}{3} + 72\frac{x^2}{2}L + 36xL^2\right]\Big|_L^{2L}$$

$$K_{ij} = (-2P\cdot 42{,}8L^5 + E\cdot J_1\cdot 84\cdot L^3) - P\cdot[386{,}6\cdot L^5 - 42{,}8\cdot L^5] + EJ_2[360L^3 - 84L^3]$$

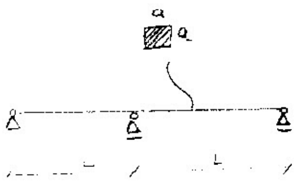
$$K_{ij} = -85{,}6\cdot P\cdot L^5 + EJ_1\cdot 84\cdot L^3 - P\cdot 343{,}8\cdot L^5 + EJ_2\cdot 276\cdot L^3 = 0$$

$$K_{ij} = -429{,}4\cdot P\cdot L^5 + EJ_1\cdot 84\cdot L^3 + EJ_2\cdot 276\cdot L^3 = 0$$

$$\boxed{P_{CR} = \frac{EJ_1\cdot 84\cdot L^3 + EJ_2\cdot 276\cdot L^3}{429{,}4\cdot L^5} = 0{,}34547}$$

$$P_{CR}(\text{EULER}) = \frac{\pi^2 EJ_2}{L^2} = 5{,}329$$

DOLOČI DIMENZIJO a, da se nosilec ukloni pri $T = 50°C$ 1.12.2004

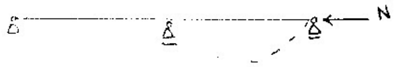


$E = 21\,000 \; kN/cm^2$

$\alpha_T = 10^{-5} \; /°C$

$L = 500 \, cm$

Nosilec je bil zmontiran pri 0°C

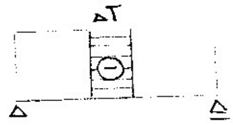


① Uklonska oblika

$$N_{CR} = \frac{\pi^2 E J}{L^2}$$

$$J = \frac{b h^3}{12} = \frac{a^4}{12}$$

② Osna sila zaradi enakomerne temp spremembe



$\Delta T = 50 - 0 = 50°C$

$$N = \alpha_T \, \Delta T \cdot EA = 10^{-5} \cdot 50 \cdot 21000 \cdot a^2 = 10{,}5 \cdot a^2$$

③ $N_{CR} = \frac{\pi^2 \cdot E J}{L^2}$; $J = \frac{a^4}{12}$, $N = 10{,}5 a^2$

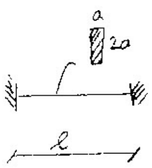
$$10{,}5 a^2 \; \frac{\pi^2 \cdot E \cdot a^4}{12 \cdot L^2}$$

$$a^2 = \frac{10{,}5 \cdot 12 \cdot L^2}{\pi^2 \cdot E}$$

$a^2 = 151{,}3$

$a = 12{,}3 \, cm$

Določi temperaturo, ko se bo palica uklonila (17.6.2005)



$\alpha_T = 10^{-4} \; K^{-1}$

$\ell = 200 \, cm$

$a = 10 \, cm$

$E = 21\,000 \; kN/cm^2$

$T_0 = 0°C$

Uklonska oblika

$$P_{CR} = \frac{\pi^2 E J}{(0{,}5 L)^2}$$

$J_y = \frac{b h^3}{12} = \frac{a \cdot (2a)^3}{12} = 6\,666{,}7$ $\quad J_z = 1\,666{,}7$

$A = a \cdot 2a = 2a^2 = 200$

$$N = \alpha_T \, \Delta T \, EA = 10^{-4} \cdot \Delta T \cdot 21000 \cdot 200 = 420 \, \Delta T$$

① $P_{CR} = \frac{\pi^2 E J}{(0{,}5 L)^2}$

$$420 \, \Delta T = \frac{\pi^2 \cdot 21000 \cdot 1666{,}7}{(0{,}5 \cdot 200)^2}$$

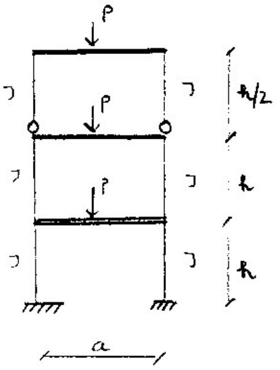
$$\boxed{\Delta T = 82{,}25 \, °C}$$

② $P_{CR} = \frac{\pi^2 E J}{(0{,}5 L)^2}$

$$420 \, \Delta T = \frac{\pi^2 \cdot 21000 \cdot 6\,666{,}7}{(0{,}5 \cdot 200)^2}$$

$\Delta T = 329$

METODA HORNE → določi P_{CR} 1.12.2004

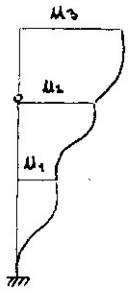


$E = 21\,000\ kN/cm^2$

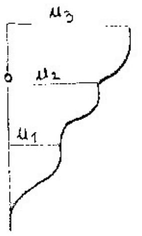
$h = 600\ cm$

$a = 1000\ cm$

$J = 5000\ cm^4$



1\. NAČIN
↓
Mi na pred



$w_3 = 3P$

$w_2 = 2P$

$w_1 = P$

$$\Delta\mu_3 = \frac{\cancel{3}\cdot P\cdot (h/2)^3}{2\cdot \cancel{3}EJ} = P\cdot 0{,}12857$$

$$\Delta\mu_2 = \frac{\cancel{2}\cdot P\,(h)^3}{\cancel{2}\cdot 12EJ} = P\cdot 0{,}1714$$

$$\Delta\mu_1 = \frac{1\cdot P\,(h)^3}{2\cdot 12EJ} = P\cdot 0{,}0857$$

$$\varphi_3 = \frac{\Delta\mu_3}{(h/2)} = 4{,}286\cdot 10^{-4}\,P$$

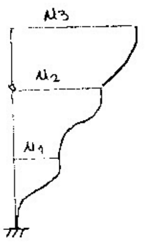
$$\varphi_2 = \frac{\Delta\mu_2}{h} = 2{,}857\cdot 10^{-4}\,P$$

$$\varphi_1 = \frac{\Delta\mu_1}{h} = 1{,}4283\cdot 10^{-4}\,P$$

$$\gamma_{CR} = \frac{1{,}00\cdot 10^4}{1{,}11\cdot P\cdot 4{,}286} = 2102\,P^{-1}$$

$$P_{CR} = \gamma_{CR}\cdot P = 2102$$

2\. NAČIN
↓
V rešenih vajah



$w_3 = P$

$w_2 = 2P$

$w_1 = 3P$

$$\mu_3 = \frac{P\cdot (h/2)^3}{2\cdot 3\cdot EJ} = 0{,}0428\cdot P$$

$$\mu_2 = \frac{2\cdot P\cdot h^3}{2\cdot 12\cdot EJ} = 0{,}1714\cdot P$$

$$\mu_1 = \frac{3\cdot P\cdot h^3}{2\cdot 12EJ} = 0{,}2571\,P$$

$$\varphi_1 = \frac{\mu_3-\mu_2}{(h/2)} = -4{,}287\cdot 10^{-4}\,P$$

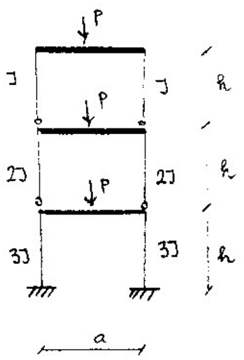
$$\varphi_2 = \frac{\mu_2-\mu_1}{h} = -1{,}4283\cdot 10^{-4}\,P$$

$$\varphi_3 = \frac{\mu_1-0}{h} = 4{,}285\cdot 10^{-4}\,P$$

✓

HORNE določi P_{CR}

Klancek.si

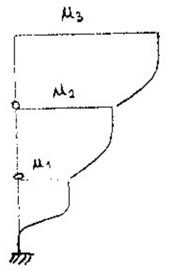


$E = 21000\ kN/cm^2$

$h = 500\ cm$

$a = 600\ cm$

$J = 5000\ cm^4$



$w_3 = P$

$w_2 = 2P$

$w_1 = 3P$

$$\mu_3 = \frac{P \cdot h^3}{2 \cdot 3EJ} = 0{,}1984\ P$$

$$\mu_2 = \frac{\cancel{2}P \cdot h^3}{\cancel{2} \cdot 3E(J \cdot 2)} = \frac{0{,}3968\ P}{2}$$

$$\mu_1 = \frac{3P \cdot h^3}{2 \cdot 12E(J \cdot 3)} = \frac{0{,}1488 \cdot P}{3}$$

$$f_3 = \frac{\mu_3 - \mu_2}{h} = 0$$

$$f_2 = \frac{\mu_2 - \mu_1}{h} = \frac{0{,}1488 \cdot P}{h} = 2{,}976 \cdot 10^{-4}\ P$$

$$f_1 = \frac{\mu_1}{h} = \frac{0{,}0496 \cdot P}{h} = 9{,}92 \cdot 10^{-5}\ P$$

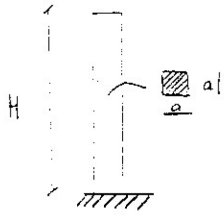
$$\psi_{max} = f_2 = 2{,}976 \cdot 10^{-4}\ P$$

$$\gamma_{CR} = \frac{1}{1{,}11 \cdot \psi_{max}} = 3027{,}0 \cdot P^{-1}$$

$$P_{CR} = \gamma_{CR} \cdot P = 3027{,}0 \cdot P^{-1}$$

RAYLEIGH – RITZ (določivišimo kjer se nosilec ukloni)

↳ zaradi lastne teže



$a = 10$ cm

$\rho = 7850$ kg/m³

$E = 21000$ kN/cm²

100 cm

$p = \rho \cdot g \cdot A = 7850 \, kg/m^3 \cdot 10 \, m/s^2 \cdot 0{,}1 m^2 = 785 \, N/m$

$= 780 \cdot \frac{kN}{1000} \cdot \frac{1}{100 \, cm}$

$= 0{,}00785 \, kN/cm$

NIČLE: $w(0) = 0$
$\varphi(0) = 0$

$P = (x-0)(x-0) = x^2$

$P_{,x} = 2x$

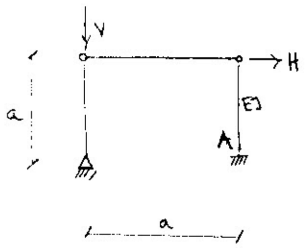
$P_{,xx} = 2$

$N = -p \cdot (H-x)$; $J = \frac{a^4}{12} = 833{,}333$

$K_{ij} = \int_0^H (-p(H-x) \cdot 2x \cdot 2x + EJ \cdot 2 \cdot 2 \, dx$

⋮

METODA P-Δ (določi moment v tč A ← vpliv TDR)
(določi V_{CR} z uporabo k_w)



$E = 3000\ kN/cm^2$

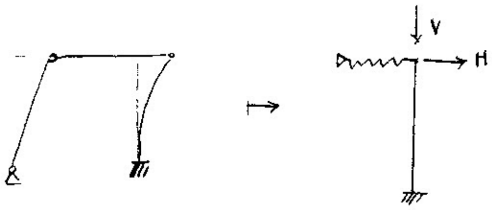
$J = 50\,000\ cm^4$

$a = 400\ cm$

$H = 150\ kN$

$V = 1000\ kN$

$EA = \infty$



(b+kw)

$k_w = b - \frac{c^2}{d}$

$k_w = 4{,}0234$

$b = 25{,}125$
$c = 5525$
$d = 1446\,667$

P-Δ
$b = 25{,}625$
$c = 5625$
$d = 1\,500\,000$
$k_w = 4{,}531$

① $w = 0$
$f = \frac{w}{h} = 0$
$\Delta H = f \cdot V = 0$
$w_1 = \frac{H + \Delta H}{k} = \frac{150}{4{,}0234} = 37{,}28$

② $w = 37{,}28$
$f_2 = \frac{37{,}28}{400} = 0{,}0932$
$\Delta H_2 = 0{,}0932 \cdot 1000$
$w_2 = \frac{150 + 93{,}205}{4{,}0234} = 60{,}447$

③ $w = 60{,}447$
$f_3 = 0{,}151$
$\Delta H_3 = 0{,}151 \cdot 1000$
$w_3 = \frac{150 + 301}{4{,}0234} = 74{,}84$

④ $w_1 = 74{,}84$
$f = 0{,}187$
$\Delta H = 187{,}104$
$w_4 = 83{,}78$ pomik se povečuje!

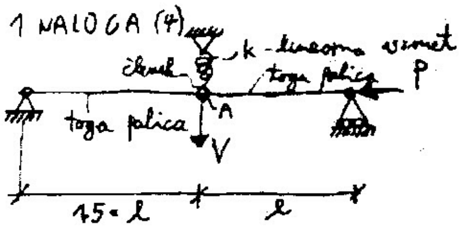
$M_A = c \cdot w_j$ $M_A = -H \cdot a + V \cdot a = 340\,000$

$c =$

NE ZNAM!

STABILNOST KONSTRUKCIJ - RAČUNSKI DEL 17. 6. 2005

1 NALOGA (4)



$P = 60$ kN

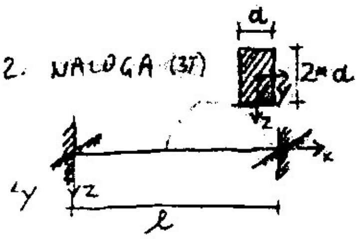
$k = 2$ kN/cm

$V = 10$ kN

$l = 300$ cm

Z METODO P-Δ DOLOČI VERTIKALNI POMIK V TOČKI A.

2. NALOGA (3T)



$\alpha_T = 10^{-4}\ K^{-1}$

$l = 200$ cm

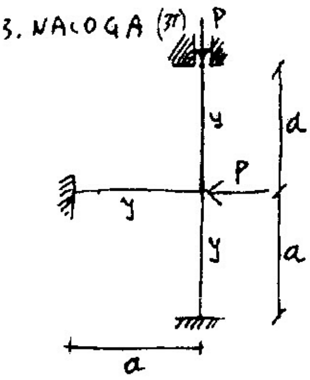
$a = 10$ cm

$E = 21000$ kN/cm²

$T_0 = 0°C$

KONST. JE BILA ZMONTIRANA PRI TEMPERATURI T_0. DOLOČI TEMPERATURO PRI KATERI SE BO ELASTIČNO UKLONILA.

3. NALOGA (3T)



$E = 3000$ kN/cm²

$J = 10\,000$ cm⁴

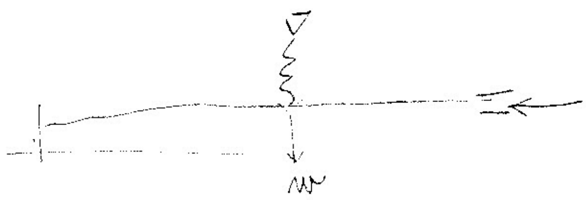
$a = 300$ cm

DOLOČI ELASTIČNO KRITIČNO SILO (P_{CR}) Z MKE (POLINOM 3 REDA - P3).

$\Delta V = 0$

f_1 f_2

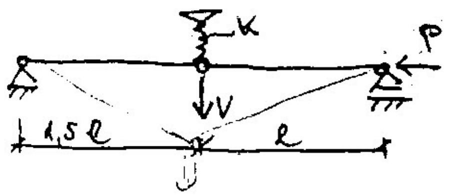
$\Delta H = f \cdot Y$



$$\Delta H = (N_i - N_{i-1}) \cdot \Psi_0 + \frac{1{,}2}{1{,}11} \left(N_1 \cdot \Psi_{1i} - N_2 \cdot \Psi_{2i} \right)$$

17.6.2005 (4)

1



$P = 60$ kN
$k = 2$ kN/cm
$V = 10$ kN
$l = 300$ cm

Je treba l ali 1,5 l ali 2,5 l? Kaj je treba gledat?

1) $w_1 = 0$
$\varphi_1 = 0$
$\Delta V_1 = 0$
$w_2 = \frac{V_1 + \Delta V_1}{k} = 5$ cm

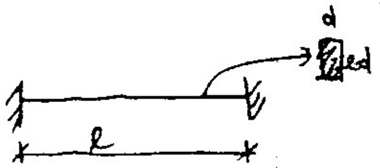
2) $w = 5$
$\varphi = \frac{w}{l} = \frac{5}{300} = 0{,}01\overline{6}$
$\Delta P = P \cdot \varphi \cdot 1{,}2 = 1{,}2$
$w = 5{,}6$ cm

3) $w = 5{,}6$ cm
$\varphi = 0{,}018\overline{6}$
$\Delta P = 1{,}344$
$w = 5{,}672$

4) $w = 5{,}672$
$\varphi = 0{,}0189066$
$\Delta P = 1{,}36128$
$w = 5{,}68064$

5) $w = 5{,}68064$
$\varphi = 0{,}0189354$
$\Delta P = 1{,}3633536$
$\underline{w = 5{,}6816768}$

2



$\alpha_T = 10^{-4}$
$l = 200$
$a = 10$
$E = 21000$
$T_0 = 0°C$

$J = \frac{d \cdot (2d)^3}{12} = 6666{,}\overline{6}$

$I_z = \frac{d^3 \cdot (2d)}{12} = \underline{1666{,}\overline{6}}$

$A = d \cdot 2d = 200$

$P_{cr} = N$

$P_{cr} = \frac{\pi^2 E I_{min}}{(0{,}5 l)^2}$

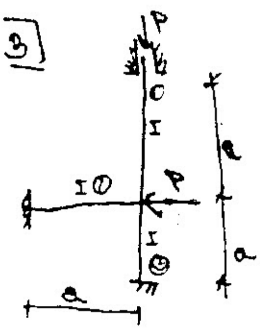
$N = \alpha_T \Delta T \cdot EA$

$$\alpha_T \Delta T \cdot EA = \frac{\pi^2 E I_{min}}{(0{,}5 l)^2}$$

$$\Delta T = \frac{\pi^2 E I_{min}}{\alpha_T \cdot EA (0{,}5 l)^2} = 82{,}25\,°C$$

$$\underline{\underline{t = 82{,}25°C}}$$

3



$E = 3000$
$I = 10000$
$a = 300$

$K = [d_1 + d_2 + d_3]$

$$d_3 = d_1 = d_2 = \frac{4EI}{a} - \frac{2aP}{15} = 400000 - 40P$$

$\det K = 0 \rightarrow P_{cr}$

$$3 \cdot 400000 - 3 \cdot 40 P = 0$$

$$\underline{\underline{P = 10000 \text{ kN}}}$$

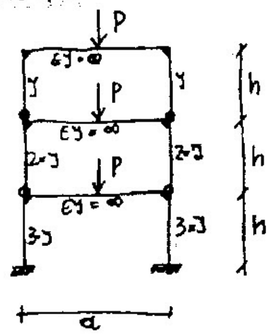
Ocena:

$P_{cr_{max}} = 13159{,}5$ kN

$P_{cr_{min}} = 6414{,}040$

STABILNOST KONSTRUKCIJ 28.6.

1. NALOGA



$E = 21\,000$ kN/cm²

$h = 500$ cm

$a = 600$ cm

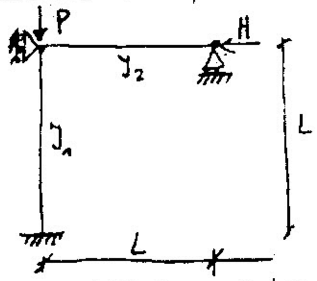
$J = 5000$ cm⁴

Ona $\gamma_{CR} = 2270{,}27 \cdot P^{-1}$

Moje $\gamma_{CR} = 3027{,}0 \cdot P^{-1}$

Z METODO HORNE DOLOČI ELAST. KRITIČNO OBTEŽBO P_{CR}.

2. NALOGA P_{CR} pride izven ocene



$L = 600$ cm

$J_1 = 8000$ cm⁴

$J_2 = 0.5\ J_1$

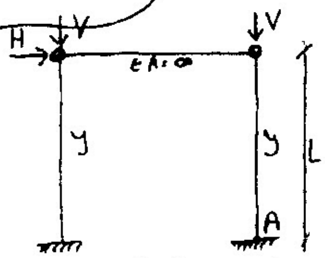
$E = 20\,000$ kN/cm²

$H = 180$ KN

OCENI ELASTIČNO KRITIČNO SILO P_{CR} TER DOLOČI P_{CR} Z MKE (POLINOM 3 REDA).

2

1. NALOGA



$J = 10\,000$ cm⁴

$E = 21\,000$ kN/cm²

$L = 1000$ cm

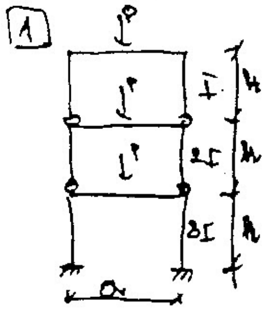
$V = 100$ KN

$H = 10$ KN

Z METODO P-Δ DOLOČI MOMENT V TOČKI A TER OCENI P_{CR} Z METODO K_G.

28.6.2005

1



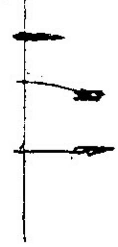
$E = 21000$
$h = 500$
$a = 600$
$I = 5000$

$W_1 = 3P$
$W_2 = 2P$
$W_3 = P$

$$\mu_1 = \frac{\frac{1}{2} 3P \cdot h^3}{2 \cdot 3EI \cdot 3} = 0{,}1984127\,P$$

$$\mu_2 = \frac{\frac{1}{2} 2P h^3}{3EI \cdot 2} = 0{,}1984127 \cdot P$$

$$\mu_3 = \frac{\frac{1}{2} P h^3}{3EI} = 0{,}1984127\,P$$



$\varphi_1 = 0{,}00039682540P = \varphi_{max}$

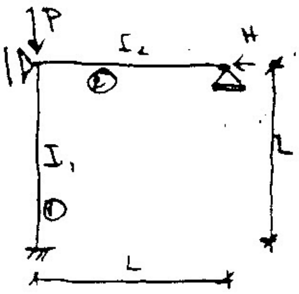
$\varphi_2 = 0$

$\varphi_3 = 0$

$$\gamma_{cr} = \frac{1}{\alpha \cdot \varphi_{max}} = \frac{1}{1{,}11 \cdot 0{,}000396 8P} = 2270{,}27 \cdot P$$

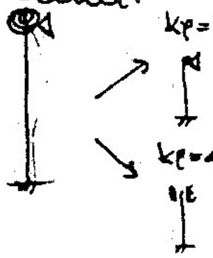
$$\underline{P_{cr} = 2270{,}27 \cdot P}$$

2



$L = 600$
$I_1 = 8000$
$I_2 = 4000$
$E = 20000$
$H = 180$

Ocena:



$k_\varphi = 0$

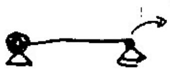
$$P_{cr,min} = \frac{\pi^2 E I_1}{(0{,}7L)^2} = 8952{,}02$$

$k_\varphi = \infty$

$$P_{cr,max} = \frac{\pi^2 E I_1}{(0{,}5L)^2} = 17545{,}86$$

$K_1 = d_1(P)$

$K_2 = d_2(H)$



je tlačen!

$K = d_1 + d_2$

$$K_2 = \begin{array}{c} \begin{matrix} \varphi_1 & \varphi_2 \end{matrix} \\ \begin{bmatrix} d_2 & e_2 \\ e_2 & d_2 \end{bmatrix} \end{array} \begin{matrix} \\ \varphi_1 \end{matrix}$$

$$d_1 = \frac{4EI_1}{L} - \frac{2 \cdot L \cdot P}{15} = 1066666{,}\bar{6} - 80P$$

$$d_2 = \frac{4EI_2}{L} - \frac{2LH}{15} = 533333{,}\bar{3} - 14400$$

$$K = \begin{bmatrix} d_1 + d_2 & e_2 \\ e_2 & d_2 \end{bmatrix}$$

$$1066666{,}\bar{6} - 80P + 533333{,}\bar{3} - 14400 = 0$$

$$\underline{P = 19.820\ kN}$$

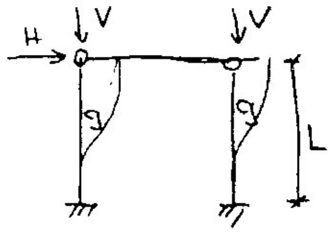
↳ Ni v skladu z oceno!?

3 Kako se lotit tega naloge?

2

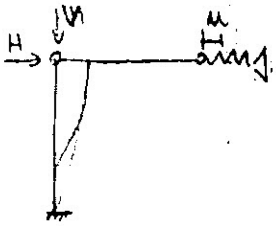
Klancek.si

3



$I = 10\,000\ cm^4$
$E = 21\,000\ \frac{kN}{cm^2}$
$L = 1000\ cm$
$V = 100\ kN$
$H = 10\ kN$

$k_w = \frac{3EI}{L^3} = 0{,}63$



1) $u = 0$
$\varphi = 0$
$\Delta H = 0$
$u = \frac{H}{k} = \frac{10}{0{,}63} = 15{,}873016$

2) $u = 15{,}873016$
$\varphi = 0{,}015873$
$\Delta H = \varphi \cdot V \cdot 1{,}2 = 1{,}90476$
$u = \frac{\Delta H + H}{k} = 18{,}896447$

3) $u = 18{,}896447$
$\varphi = 0{,}018896$
$\Delta H = 2{,}26757$
$u = 19{,}492339$

4) u
φ
ΔH
$u = 19{,}582033$

5) $u = 19{,}602827$

6) $19{,}606907$

7) $u = 19{,}60766$

$u = 19{,}61\ cm \qquad \varphi = 0{,}01961$

$M_A = C\ u$

$C = \frac{6EI}{L^2} - \frac{V}{10} = 1260 - 10 = 1250$

$M_A^{TDR} = 1250\ 19{,}61 = 24512{,}5\ kNcm = 245{,}125\ kNm$

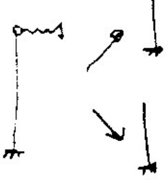
$M_A^{TPR} = 1260 \cdot 15{,}873 = 19999{,}98\ kNcm = 199{,}9998\ kNm$

$k_s = \frac{M^{TDR}}{M^{TPR}} = 1{,}225625$

$\gamma_{cr} = \frac{k_s \cdot \gamma}{k_s - 1} = 5{,}432183$

$P_{cr} = V \cdot \gamma_{cr} = \underline{543{,}213\ kN}$

Ocena

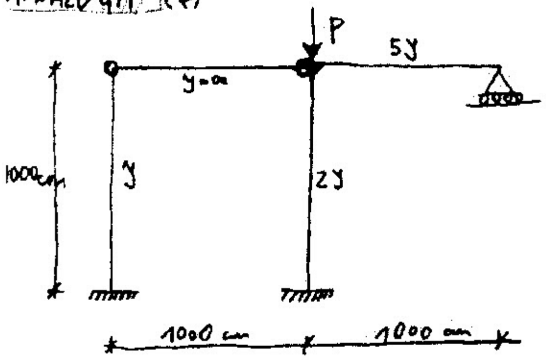


$P_{cr} = \frac{\pi^2 EI}{(0{,}7L)^2} = 4229{,}8\ kN$

$P_{cr} = \frac{\pi^2 EI}{(2L)^2} = 518{,}15\ kN$

STABILNOST KONSTRUKCIJ - RAČUNSKI DEL 1.12.2004

1. NALOGA (4)



$E = 21\,000$ kN/cm²

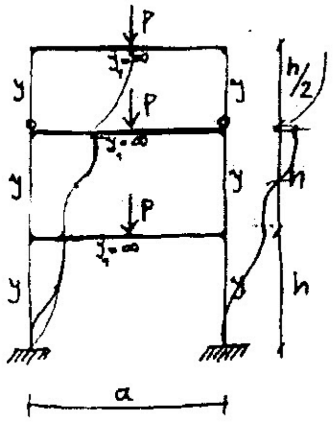
$J = 10\,000$ cm⁴

a) OCENI ZGORNJO IN SPODNJO MEJO ELASTIČNE KRITIČNE OBTEŽBE P_{cr}

b) DOLOČI P_{cr} Z MKE (POLINOM 3 REDA)

(R: $P_{cr,min} = 1036$ kN, $P_{cr,max} = 16581$ kN, $P_{cr,MKE} = 3844$ kN) ✓ OK

2. NALOGA (3)



$E = 21\,000$ kN/cm²

$h = 600$ cm

$a = 1000$ cm

$J = 5000$ cm⁴

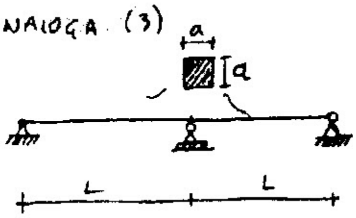
Moji in njeni rezult: $\psi_{max} = 0{,}00043 \cdot P$

$P_{CR} = 2102$

Z METODO HORNE DOLOČI ELASTIČNO KRITIČNO OBTEŽBO P_{cr}.

(R: $\psi_{max} = 0.00048$, $P_{cr} = 2121$ kN) ?

3. NALOGA (3)



$E = 21000$ kN/cm²

$\alpha_T = 10^{-5}$ /°C

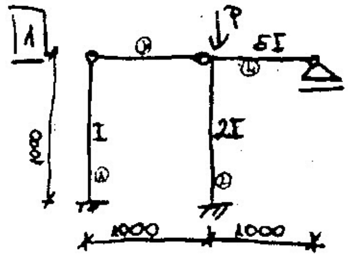
$L = 500$ cm

NOSILEC JE BIL ZMONTIRAN PRI 0°C. DOLOČI TAKO DIMENZIJO PREREZA "a", DA SE NOSILEC UKLONI PRI TEMPERATURI 50°C.

(R: $a = 12.3$ cm) OK

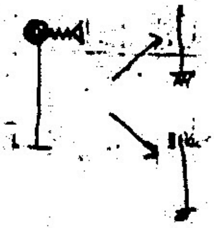
1.12.2004

1



E = 21000
I = 10000

Ocena:



$P_{cr_{min}} = \frac{\pi^2 E I 2}{(2L)^2} = 1036,81\ kN$

$P_{cr} = \frac{\pi^2 E I 2}{(0,5L)^2} = 1650,94\ kN$

① $k_w = \frac{3EI}{1000^3} = 0,63$

④ $k_\varphi = \frac{3E5I}{1000} = 3150000$

② $K_2 = \begin{bmatrix} b_2 & c_2 \\ c_2 & d_2 \end{bmatrix}$

$K = \begin{bmatrix} b_2 + k_w & c_2 \\ c_2 & d_2 + k_\varphi \end{bmatrix} \Rightarrow \det K = 0 \Rightarrow P_{cr}$

$b_2 = \frac{504}{\cancel{2,025}} - 0,0012P$

$c_2 = 1\cancel{2}60 - 0,1P$ (2020)

$d_2 = \cancel{840000}\ 1680000 - \frac{1000}{15}P$

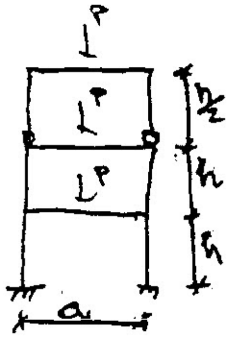
~~$0,15P_{cr}^2 - 4856P + 10080900 = 0$~~

~~$P_{cr} = 2888,32\ kN$~~

$0,15 \cdot P_{cr}^2 - 6048 P_{cr} + 2103\,5700 = 0$

$\underline{\underline{P_{cr,1} = 8844,74\ kN}}$ $\underline{\underline{P_{cr_2} = 36475,26}}$

2



E = 21000
h = 600
a = 1000
I = 5000

$W_1 = 3P$ $\quad M_1 = \frac{\frac{1}{2}3P \cdot h^3}{12EI} = 0,2571428\ P$

$W_2 = 2P$ $\quad M_2 = \frac{\frac{1}{2}P2\,h^3}{12EI} = 0,1714285\ P$

$W_0 = P$ $\quad M_0 = \frac{\frac{1}{2}P\,(h/2)^3}{3EI} = 0,0428571\ P$

Tu jest 12 i nie 3, ale?

$\varphi_1 = \frac{M_1}{h} = 0,0004285 = \varphi_{max}$

$\varphi_2 = \frac{M_2 - M_1}{h} = -0,0001428$

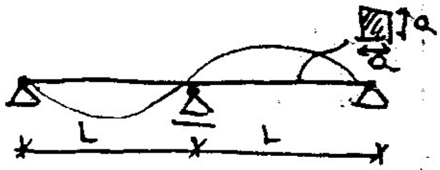
$\varphi_0 = \frac{M_0 - M_2}{(h/2)} = -0,0004285$

$\delta_{cr} = \frac{1}{1,11 \cdot \varphi_{max}} = 2102,45$

$\underline{\underline{P_{cr} = 2102,45\ kN}}$?

Rezultat nie OK.
Dlaczego?

9



$E = 21000$
$\alpha_T = 10^{-5}$
$L = 500$
$T = 50^\circ C$

$I = \frac{a^4}{12}$

$$P_{cr} = \frac{\pi^2 E I}{L_u^2} = \frac{\pi^2 E a^4}{12 \cdot L_u^2} = \frac{\pi^2 E a^4}{12 L^2}$$

$$N_T = \alpha_T \cdot t \cdot E A = \alpha_T \cdot t \cdot E \cdot a^2$$

$$P_{cr} = N_T$$

$$\frac{\pi^2 E a^4}{12 L^2} = \alpha_T t \cdot E a^2$$

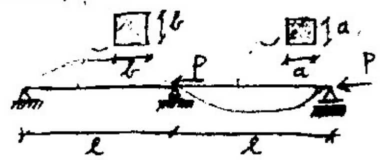
$$\pi^2 E a^2 = 12 L^2 \alpha_T t E$$

$$a^2 = \frac{12 L^2 \alpha_T t \cancel{E}}{\pi^2 \cancel{E}} = 151{,}98 \, cm^2$$

$$\underline{a = 12{,}33 \, cm}$$

STABILNOST KONSTRUKCIJ – RAČUNSKI DEL IZPITA – 7.5.2004

1. NALOGA 3T



$\ell = 5\,m$

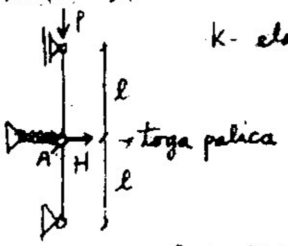
$a = 3\,cm$
$b = 5\,cm$

$E = 20000\ kN/cm^2$

S POMOČJO METODE RAYLEIGH-RITZ DOLOČI ELASTIČNO KRITIČNO OBTEŽBO P_{CR}. (POLINOM 3 REDA).

Moj $P_{CR} = 0{,}34547$

2. NALOGA 3T



k- elastična vzmet

$P = 10\ kN$

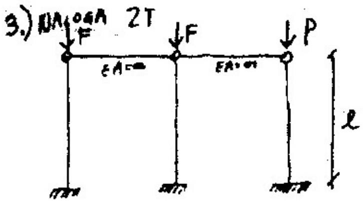
$k = 1\ kN/cm$

$H = 5\ kN$

$\ell = 200\ cm$

Z METODO P-Δ DOLOČI HORIZONTALNI POMIK VOZLIŠČA A

3.) NALOGA 2T



$F = 10\ kN$

$\ell = 300\ cm$

$E = 20.000\ kN/cm^2$

$J = 10\,000\ cm^4$

a) Z MKE DOLOČI ELASTIČNO KRITIČNO SILO P_{CR} (POLINOM 3 REDA)

b) OCENI KRITIČNO OBTEŽBO

Njen rezultat $P_{CR} = 16\,316$

Moj: $P_{CR1} = 289\,302$

$P_{CR2} = 3\,983$

$P = x(x-\ell)(x-2\ell)$

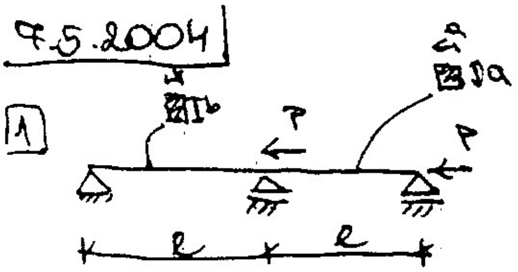
$\ell \qquad 2\ell$

ℓ

7.5.2004

(16)

1



$l = 500$
$a = 3$
$b = 5$
$E = 20000$

?

$$K = \int_0^{l} \left(N_b(x)\, P_{1x} \cdot P_{x} + EI_b P_{1xx} \cdot P_{xx} \right) dx + \int_{l}^{2l} \left(N_a(x)\, P_{1x} P_{x} + EI_a P_{1x} P_{x} \right) dx$$

$P = \sin \frac{\pi x}{L}$ → je P tak ok?

$N(x) = ?$ $\quad N(x) = P$ ali $N(x) = 2P$ ali

$N_a(x) = \frac{K_a}{K_a + K_b} \cdot 2P$ in $N_b = \frac{K_b}{K_a + K_b} \cdot 2P$ $\quad$?

$K_a = \frac{EA_a}{L} = 360$

$K_b = \frac{EA_b}{L} = 1000$

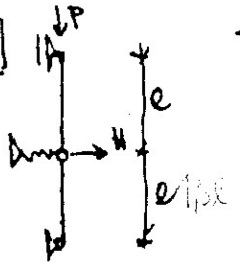
$P = x(x-l)(x-2l) =$
$= x^3 - 3lx^2 + 2l^2x$

$N_a = -0{,}2647058P$
$N_b = -1{,}7302941P$

$P_{1x} = 3x^2 - 6lx + 2l^2$

$P_{1xx} = 6x - 6l$

2



$P = 10$
$k = 1$
$H = 5$
$l = 200$

1) $u_1 = 0$
$\varphi_1 = 0$
$\Delta H_1 = 0$
$u_1 = \frac{H + \Delta H}{k} = 5$ cm

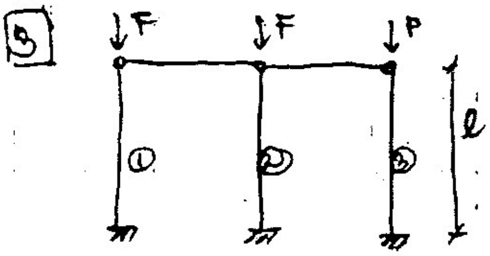
2) $u = 5$
$\varphi = \frac{5}{l} = 0{,}025$ Tu je l ali mogoče $2l$?
$\Delta H = 0{,}3$
$u = 5{,}3$ cm

3) $u = 5{,}3$
$\varphi = 0{,}0265$
$\Delta H = 0{,}318$
$u = 5{,}318$ cm

4) $u = 5{,}318$
$\varphi = 0{,}02659$
$\Delta H = 0{,}31908$
$u = 5{,}31908$ cm

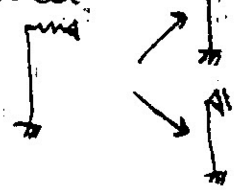
5) $u = 5{,}31908$
$\varphi = 0{,}0265954$
$\Delta H = 0{,}3191448$
$\underline{\underline{u = 5{,}319145 \text{ cm}}}$

③



$F = 20$
$l = 300$
$E = 20000$
$I = 10000$

Ocena



$P_{cr,min} = \frac{\pi^2 EI}{(2l)^2} = 5483,11\ kN$

$P_{cr,max} = \frac{\pi^2 EI}{(0,7l)^2} = 44760,11\ kN$

③ $K_3 = \begin{bmatrix} b_3 & c_3 \\ c_3 & d_3 \end{bmatrix}$

$K = \begin{bmatrix} b_3 + k_v \cdot 2 & c_3 \\ c_3 & d_3 \end{bmatrix}$

$b_3 = 88,\bar{8} - 0,004P$

$c_3 = 13333,\bar{3} - 0,1P$

$d_3 = 2666666,\bar{6} - 40P$

① in ② → nadomestna togost

$k_{v1} = k_{v2} = b_1 - \frac{c_1^2}{d_1} = 22,18222$

$b_1 = 88,\bar{8} - 0,04 = 88,84\bar{8}$

$c_1 = 13333,\bar{3} - 1 = 13332,\bar{3}$

$d_1 = 2666666,\bar{6} - 400 = 2666266,\bar{6}$

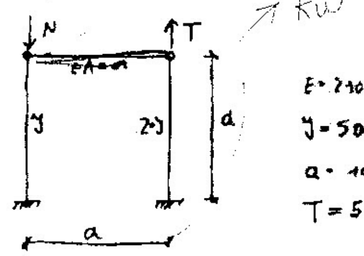
$\det K = 0$

$0,15 P_{cr}^2 - 13330,1037 P_{cr} + 1778564435,54 = 0$

$\underline{\underline{P_{cr} = 16316,21\ kN}}$

STABILNOST KONSTRUKCIJ - RAČUNSKI DEL IZPITA - 2.12.2013

1. NALOGA (2 T)



$E = 21000$ kN/cm²

$J = 5000$ cm⁴

$a = 100$ cm

$T = 50$ kN

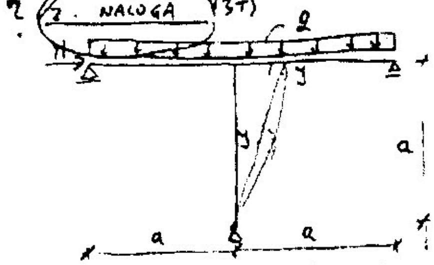
Njen rezultat: $P_{CR} = 77\ 232$ kN

Moj rezultat: $P_{CR1} = 1016\ 899$

$P_{CR2} = -36\ 345{,}8$

OCENI ELASTIČNO KRITIČNO OBTEŽBO N_{CR} TER DOLOČI N_{CR} Z MKE (POLINOM 3. REDA).

? 2. NALOGA (3T)



$g = 1$ kN/cm

$a = 500$ cm

$H = 50$ kN

$J = 30\ 000$ cm⁴

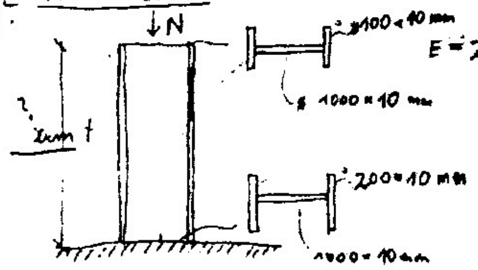
$\psi_0 = 0{,}05$ - začetna nepopolnost

$E = 21000$ kN/cm²

S P-Δ METODO DOLOČI RAZPORED NOTRANJIH SIL PO TDR TER OCENI ELASTIČNO KRITIČNO OBTEŽBO g_{CR} S K_G METODO.

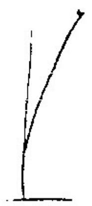
3. NALOGA (3T)

moj bo $h_1 = 1000$ cm



$E = 21000$ kN/cm²

Z METODO RALEIGH-RITZ DOLOČI ELASTIČNO KRITIČNO OBTEŽBO N_{CR}.



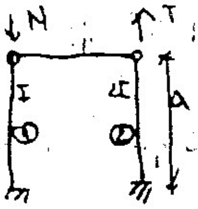
$P = x^2$

$x = 0$

$$I = \frac{bh^3}{12} + \frac{b_1 h_1^3}{12} - r^2 \cdot (b \cdot h)$$

2.12.2003

$\boxed{1}$



E = 21000
I = 5000
a = 100
T = 50

① $k_w = b_2 - \frac{c_2^2}{d_2} = 630,6$

$b_2 = (1260)\frac{1}{2} + 0,6$
$c_2 = (63000)\frac{1}{2} + 5$
$d_2 = (4200000)\frac{1}{2} + 666,\bar{6}$

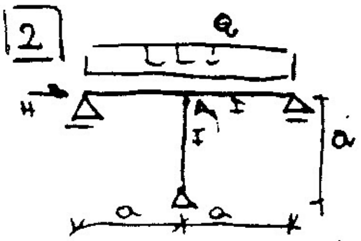
$$K = \begin{bmatrix} b_1 + k_w & c_1 \\ c_1 & d_1 \end{bmatrix}$$

$b_1 = 1260 - 0,012\,N$
$c_1 = 63000 - 0,1\,N$
$d_1 = 4200000 - 13,\bar{3}\,P$

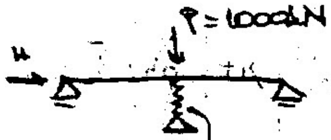
$$0,15\,P^2 - 630098 + 887152,0000 = 0$$

$\underline{P_{cr} = 77232,1\,kN}$

$\boxed{2}$



$q = 1\,kN/cm$ } $P = 1000$
$a = 500$
$H = 50$
$I = 30000$
$\varphi_0 = 0,05$ → kako upoštevati vpliv začetne nepopolnosti
$E = 21000$



k_w → kako določimo k

$k = \frac{EA}{L}$ → A ne poznamo

$k = \frac{3EI}{L^3} = 15,12$

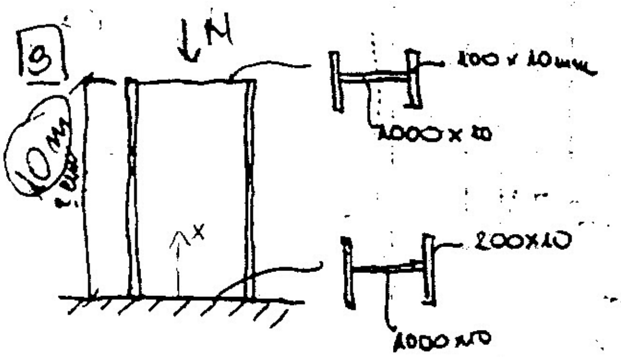
1) $w_1 = 0$
$\varphi_1 = \varphi_0 + \frac{w_1}{a} = 0,05$
$\Delta H = 1,2 \cdot P \cdot \varphi_1 = 60$
$w_2 = \frac{H + \Delta H}{k} = 7,275123$

2) $w_2 = 7,275123$
$\varphi_2 = \frac{w_2}{a} + 0,01455 + 0,05$
$\Delta H = 77,46$
$w_3 = 8,4298152$

3) $w_4 = 8,6132$
4) 8,6423
5) 8,6469
6) $\underline{8,648}$ cm

$Q_A = b \cdot w$
$M_A = c \cdot w$

3



$E = 21000$

$$I_{zgoraj} = \frac{100^3 \cdot 1}{12} + 2\,\frac{1^3 \cdot 10}{12} + (50{,}5)^2 (10 \cdot 1 \cdot 2)$$

$$I_{sp} = \frac{100^3 \cdot 1}{12} + 2\,\frac{1^3 \cdot 20}{12} + (50{,}5)^2 (1 \cdot 20 \cdot 2)$$

$I(x)$ → kako določiti I, če se ta spreminja

$$K = \int_0^h \left(N(x) \cdot P_{1x} \cdot P_{1x} + E I(x) \cdot P_{1xx} \cdot P_{1xx} \right)$$

$P = x^2$

$N = -N \left(-mg(H - x)\right)$ → ali je zraven še vpliv lastne teže

$I_y(x = h) = 32330 \text{ cm}^4$

$I_y(x = 0) = 18673{,}\bar{3} \text{ cm}^4$

$$I_z(x = h) = \frac{1^3 \cdot 100}{12} + 2\,\frac{10^3 \cdot 1}{12} = 175 \text{ cm}^4$$

$$I_z(x = 0) = \frac{1^3 \cdot 100}{12} + 2\,\frac{20^3 \cdot 1}{12} = 1341{,}67 \text{ cm}^4$$

$$I_z(x) = -\frac{1166{,}\bar{6}}{h}\, x + 1341{,}\bar{6}$$

$$K = \int_0^h \left(-N \cdot 2x \cdot 2x + E \cdot \left(-\frac{1166{,}\bar{6}}{h}\, x + 1341{,}\bar{6} \right) \cdot 2 \cdot 2 \right) dx =$$

$$= \left[N\, 4\, \frac{x^3}{3} + E \left(\frac{1166{,}\bar{6}}{h} \right) \frac{x^2}{2} \cdot 4 + E\, 1341{,}\bar{6} \cdot 4 \cdot x \right]_0^h =$$

$$= -N \cdot 4\, \frac{1000^3}{3} + E \left(-\frac{1166{,}\bar{6}}{1000} \right) \frac{1000^2}{2} \cdot 4 + E \cdot 1341{,}\bar{6} \cdot 4 \cdot 1000 \Rightarrow K = 0 \Rightarrow N_{cr}$$

$P = b \cdot w$

$T_R = C \cdot f$

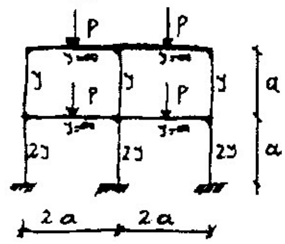
$w = \frac{P}{2F}$

$b = \frac{12 EI}{(h/2)^3}$

$\frac{12 \cdot 2^3 \cdot EI}{h^3}$

STABILNOST KONSTRUKCIJ – RAČUNSKI DEL 7. 9. 2005 14^{00} ure

1. NALOGA



$E = 3000\ kN/cm^2$

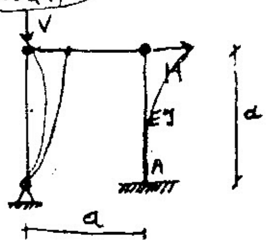
$J = 50000\ cm^4$

$a = 500\ cm$

$EA = \infty$

Z METODO HORNE DOLOČI ELASTIČNO KRITIČNO OBTEŽBO P_{cr}.

2. NALOGA



$E = 3000\ kN/cm^2$

$J = 50000\ cm^4$

$a = 400\ cm$

$H = 150\ kN$

$V = 1000\ kN$

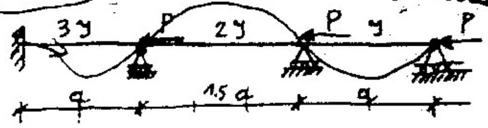
$EA = \infty$

$k_w = \frac{2EI}{a^3}$

$T_R = b \cdot w$

Z METODO P-Δ DOLOČI MOMENT V TOČKI A Z UPOŠTEVANJEM VPLIVA TDR, Z METODO k_g NATO DOLOČI ELASTIČNO KRITIČNO OBTEŽBO V_{cr}.

3. NALOGA



Kako razdeliš obtežbo?

$a = 500\ cm$

$E = 21000\ kN/cm^2$

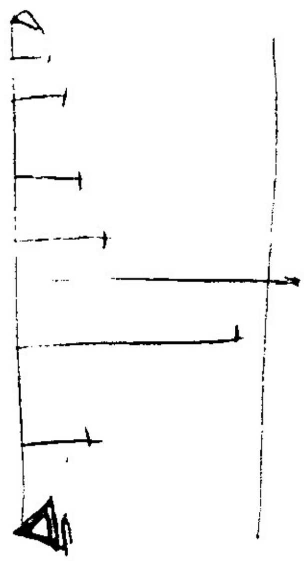
$J = 5000\ cm^4$

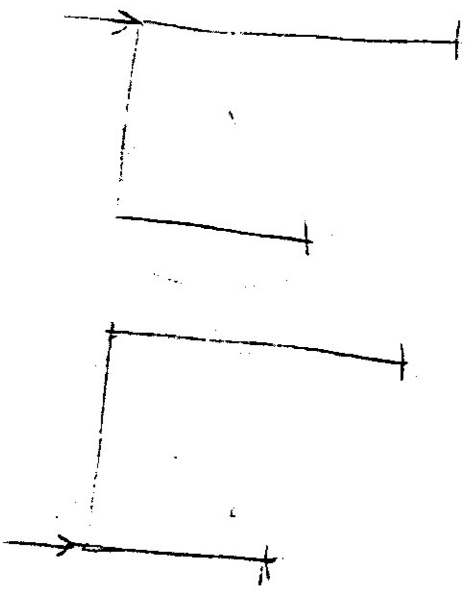
$EA = \infty$

OCENI IN NATO DOLOČI ELASTIČNO UKLONSKO SILO P_{cr}.

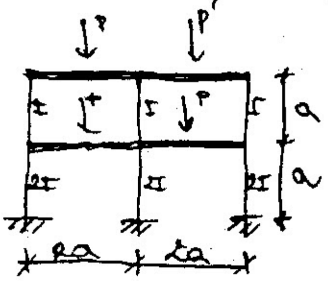
UPORABI METODO PO SVOJI IZBIRI.

$x^2 (x-a)(x-2{,}5a)(x-3{,}5a)$





1



Če P ni podano, se po metodi Horne računa enako, kot če bi bila P podana? (vzameš P=1)

$E = 3000$
$I = 50\,000$
$a = 300$

$W_1 = 4P$

$W_2 = 2P$

$$u_1 = \frac{\frac{1}{3}\,4P\cdot a^3}{12\,E\cdot 2I} = 0{,}0462962\,P$$

$$u_2 = \frac{\frac{1}{6}\,2P\cdot a^3}{12\cdot E\cdot I} = 0{,}0462962\,P$$

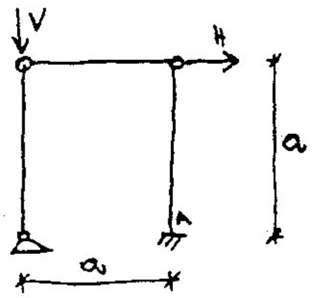
$$\psi_1 = \frac{u_1}{a} = 9{,}2592598\cdot 10^{-5}$$

$$\psi_2 = \frac{u_2 - u_1}{h} = 0$$

$$\lambda_{cr} = \frac{1}{1{,}11\cdot 9{,}258\cdot 10^{-5}} = 9729{,}72$$

$\underline{P = 9729{,}78\cdot P}$

2



$E = 3000$
$I = 50.000$
$a = 400$
$H = 150$
$V = 1000$

Kako z metodo P–Δ določiš moment v točki A?

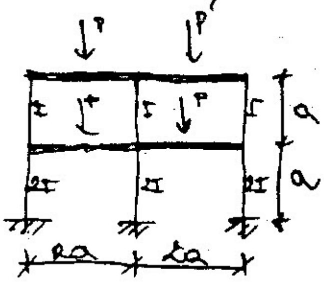
~~$M_{TPR} = H\cdot a = 150\cdot 400 = 60\,000$ kNcm~~

3 ?

7.9.2005

(11)

$\boxed{1}$



Če P ni podano, se po metodi Horne računa enako, kot če bi bila P podana? (vzemeš P=1)

$E = 8000$
$I = 50\,000$
$a = 500$

$W_1 = 4P$

$W_2 = 2P$

$$u_1 = \frac{\frac{1}{3}\,4P \cdot a^3}{12\,E \cdot 2I} = 0{,}0462962\,P$$

$$u_2 = \frac{\frac{1}{6}\,2P \cdot a^3}{12 \cdot E \cdot I} = 0{,}0462962\,P$$

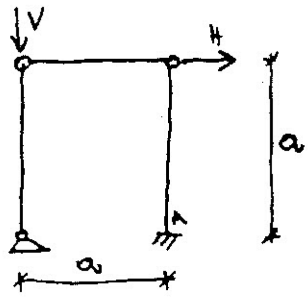
$$\psi_1 = \frac{u_1}{a} = 9{,}2592598 \cdot 10^{-5}$$

$$\psi_2 = \frac{u_2 - u_1}{h} = 0$$

$$\delta_{cr} = \frac{1}{1{,}11 \cdot 9{,}258 \cdot 10^{-5}} = 9729{,}78$$

$\underline{P = 9729{,}78 \cdot P}$

$\boxed{2}$



$E = 3000$
$I = 50\,000$
$a = 400$
$H = 150$
$V = 1000$

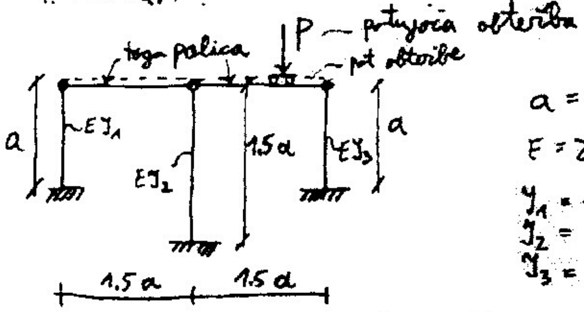
Kako z metodo P–Δ določiš moment v točki A ?

~~$M_{1r} = H \cdot a = 150 \cdot 400 = 60\,000$ kNcm~~

(3) ?

STABILNOST KONSTRUKCIJ 30.8.2005

1. NALOGA



$a = 500$ cm

$E = 21\,000$ KN/cm^2

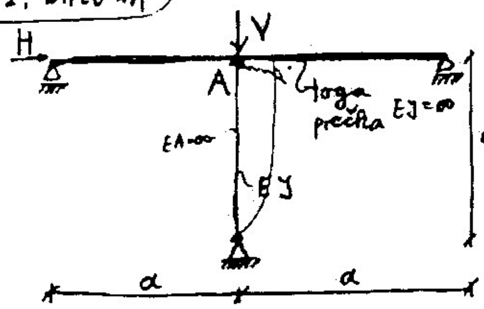
$J_1 = 10000$ cm^4

$J_2 = 2 J_1$

$J_3 = 0.5 J_1$

SILA P POTUJE PO OZNAČENI POTI. OCENI KRITIČNO OBTEŽBO P_{cR} IN JO DOLOČI Z MKE.

2. NALOGA



$V = 1000$ kN

$H = 50$ KN

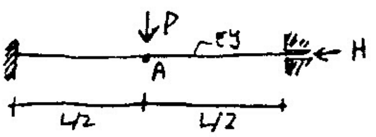
$a = 500$ cm

$J = 30\,000$ cm^4

$E = 21\,000$ KN/cm^2

S P-Δ METODO DOLOČI MOMENTE V TOČKI A PO TDR, TER OCENI ELASTIČNO KRITIČNO OBTEŽBO V_{cR}.

3. NALOGA



$J = 10\,000$ cm^4

$L = 2000$ cm

$P = 60$ KN

$H = 500$ KN

DOLOČI VERTIKALNI POMIK V TOČKI A !

a) PO TPR

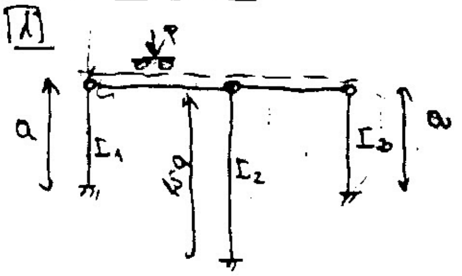
b) PO TDR (MKE polinom 3 reda)

c) Z UPORABO FUNKCIJ STABILNOSTI

30.8.2005

(12)

1



$a = 500$
$E = 21000$
$I_1 = 10000$
$I_2 = 20000$
$I_3 = 5000$

Ocena:

(1)

$$P_{cr,min}^{(1)} = \frac{\pi^2 E I_1}{(2a)^2} = 2072{,}62\ kN$$

$$P_{cr,min}^{(1)} = \frac{\pi^2 E I_1}{(0{,}7a)^2} = 16.919{,}32\ kN$$

Ali postavimo P nad steber 3, ker tak računamo po MKE?

(2) $P_{cr,min}^{(2)} = 1842{,}33\ kN$

$P_{cr,min}^{(2)} = 15089{,}40\ kN$

(3) $P_{cr,min}^{(3)} = 1036{,}31\ kN$

$P_{cr,min}^{(3)} = 8459{,}66\ kN$

$$K = \begin{bmatrix} b_3 + k_{w1} + k_{w2} & c_3 \\ c_3 & d_3 \end{bmatrix}$$

$b_3 = 10{,}08 - 0{,}0024\,P$

$c_3 = 2520 - 0{,}1P$

$d_3 = 840000 - \frac{1000}{15} P$

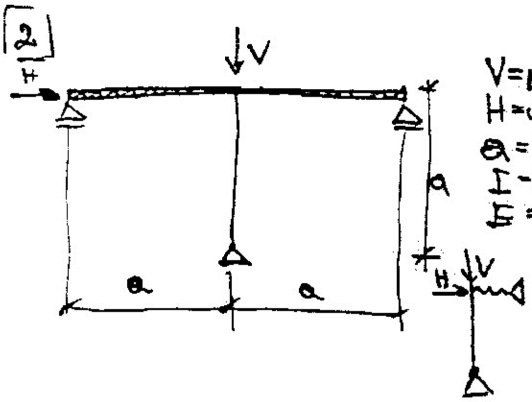
$k_{w1} = \frac{3EI_1}{a^3} = 5{,}04$

$k_{w2} = \frac{3EI_2}{(1{,}5a)^3} = 2{,}98\overline{6}$

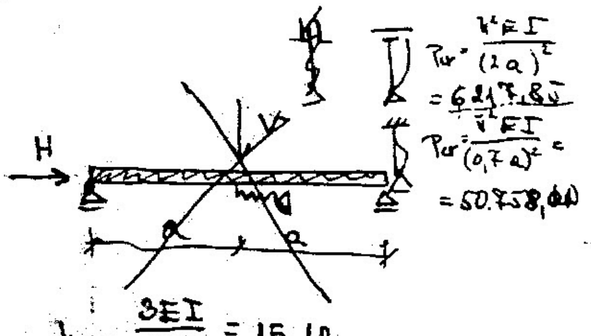
$0{,}15\,P^2 - 2719{,}\overline{11}\cdot P + 8859200 = 0$

$\underline{\underline{P_{cr,1} = 4258{,}56\ kN}}$ $P_{cr,2} = 13868{,}85\ kN$

2



$V = 1000\ kN$
$H = 500\ kN$
$a = 600\ cm$
$I = 30000\ cm^4$
$E = 21000\ kN/cm^2$



$P_{cr} = \frac{\pi^2 EI}{(2a)^2} = 6217{,}85$

$P_{cr} = \frac{\pi^2 EI}{(0{,}7a)^2} = 50.758{,}00$

$k_w = \frac{3EI}{a^3} = 15{,}12$

1) $\mu_1 = 0$ $\varphi_1 =$
$\varphi = 0$
$\Delta H = 0$
$\mu = 3{,}306783$

2) $\mu = 3{,}30687$
$\varphi = 0{,}0066$
$\Delta H = 7{,}9265$
$\mu = 3{,}821786$

3) $\mu = 3{,}915087$
4) $\mu = 3{,}9289223$
5) $\mu = 3{,}98042$
6) $\mu = 3{,}9307547$

$M = c \cdot \mu$

$c_{TDR} = \frac{6EI}{a^2} - \frac{V}{10} = 15020$

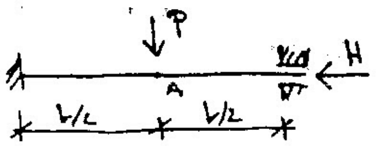
$\underline{M = 59.0399\ kNm}$

$k_s = \frac{\mu_{TDR}}{\mu_{TTR}} = 1{,}1886602$

$\lambda_{cr} = 6{,}3005$

$V_{cr} = 6300{,}5\ kN$

3



$I = 10000\ cm^4$
$L = 2000\ cm$
$P = 60\ kN$
$H = 5000\ kN$
E manjka ?

a) TPR

$$w_{TPR} = \frac{P}{b_{TPR} \cdot 2}$$

$$b_{TPR} = \frac{12EI}{\left(\frac{L}{2}\right)^3}$$

b) TDR

$$w_{TDR} = \frac{P}{2 b_{TDR}}$$

$$b_{TDR} = \frac{12EI}{\left(\frac{L}{2}\right)^3} - \frac{6H}{5\left(\frac{L}{2}\right)}$$

c) FS

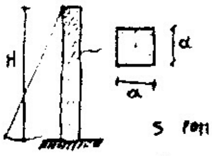
$$w_{FS} = \frac{P}{2 \cdot b_{FS}}$$

$$b_{FS} = \frac{\omega^2 \cdot (1-\cos\omega)}{2(1-\cos\omega)-\omega\sin\omega} \cdot \frac{EI}{L^2}$$

$$\omega = \sqrt{\frac{|H| \cdot \left(\frac{L}{2}\right)^2}{EI}}$$

STABILNOST KONSTRUKCIJ - RAČUNSKI DEL 30.11.2005

1. NALOGA



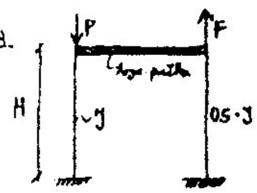
$a = 10$ cm

$\rho = 7850$ kg/m^3 (gostota jekla)

$E = 21000$ kN/cm^2

S POMOČJO METODE RAYLEIGH-RITZ DOLOČI VIŠINO H PRI KATERI SE STEBER ZARADI LASTNE TEŽE ELASTIČNO UKLONI. (KVADRATIČEN NASTAVEK)

2. NALOGA



$E = 3000$ kN/cm^2

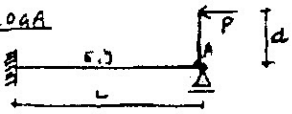
$J = 20000$ cm^4

$H = 500$ cm

$P = 5000$ kN

DOLOČI MINIMALNO VELIKOST SILE F TAKO, DA SE KONSTRUKCIJA NE BO ELASTIČNO UKLONILA. (MKE, polinom 3. reda)

3. NALOGA



$E = 21000$ kN/cm^2 $a = 30$ cm

$J = 1000$ cm^4 $P = 50$ kN

$L = 200$ cm

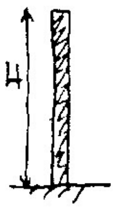
DOLOČI ZASUK KONSTRUKCIJE V TOČKI A:

a) PO TPR

b) PO TDR (polinom 3. reda, MKE)

30. 11. 2005

1



$A = 10^2\ cm^2$

$I = \frac{10^4}{12}\ cm^4$

$\rho = 7850\ kg/m^3$

$E = 21000\ kN/cm^2$

$N = -w(H-x)$

$w = \frac{78500\,N}{m^3} = 78{,}5\ \frac{kN}{m^3} = 78{,}5 \cdot 10^{-6}\ \frac{kN}{cm^3}$

$\varphi = x^2$

$\varphi_{,x} = 2x \qquad \varphi_{,xx} = 2$

$$K = \int_0^H \left(-78{,}5 \cdot 10^{-6}(H-x) \cdot 4x^2 + EI \cdot 4\right) dx =$$

$$= \left(-78{,}5 \cdot 10^{-6} H \cdot 4 \frac{x^3}{3} + 78{,}5 \cdot 10^{-6}\, 4\, \frac{x^4}{4} + EI \cdot 4x\right)\Big|_0^H =$$

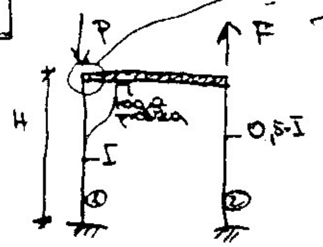
$$= -78{,}5 \cdot 10^{-6} \cdot \frac{4}{3} H^4 + 78{,}5 \cdot 10^{-6} H^4 + EI \cdot 4H$$

$K = 0 \rightarrow H$

$$78{,}5 \cdot 10^{-6}\left(1 - \frac{4}{3}\right) \cdot H^4 + 21000 \cdot \frac{10^4}{12} \cdot 4\ H = 0$$

$$H = 13881{,}9\ cm$$

2



Tu je samo ponik? Kosuka mi, če je prečka toga?

$E = 8000$
$I = 20000$
$H = 500$
$P = 5000$

~~nadomestna togost 1~~ $\cancel{T = b\,w + c\,\varphi}$
$\cancel{M = c\,w}$

$K_1 = b_I \qquad K_2 = b_{II} \qquad K = b_1 + b_2$

$$b_1 = \frac{12EI}{H^3} - \frac{6P}{5H} = 5{,}76 - 20$$

$$b_2 = \frac{12EI^{0,5}}{H^3} + \frac{6F}{5H} = \frac{5{,}76}{2} + 0{,}0024F = 2{,}88 + 0{,}0024 \cdot F$$

$$8{,}68 - 20 + 0{,}0024\,F = 0$$

$$\underline{F = 4733{,}32N}$$

3

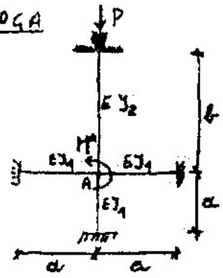
a) $\varphi_{TTE} = \frac{M}{d_{TTE}} = \frac{50\cdot 30}{\frac{4\cdot 21000\cdot 1000}{200}} = \frac{1500}{420000} = \underline{0{,}0035714}$

b) $d_{TDR} = d_{TTE} + \frac{2(-P)\cdot L}{15} = \cancel{4\ldots}\ 418666{,}67$

$\varphi_{TDR} = \frac{M}{d_{TDR}} = \underline{\underline{0{,}0035828}}$

STABILNOST KONSTRUKCIJ – RAČUNSKI DEL 25.1.2006

1 NALOGA



$a = 500$ cm

$b = 1000$ cm

$E = 21000$ kN/cm^2

$J_1 = 5000$ cm^4

$J_2 = 1000$ cm^4

$P = 2000$ kN

$M^A = 10000$ kNm

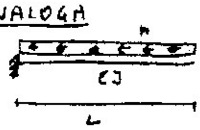
Določi zasuk konstrukcije v točki A:

a) po TIR (MKE)

b) po TDR (MKE)

Oceni elastično kritično silo P_{cr} z metodo K_S.

2. NALOGA



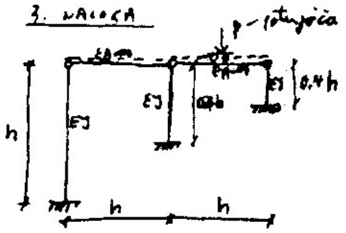
n – osna obtežba

$E = 20000$ kN/cm^2

$L = 200$ cm

Z metodo Rayleigh-Ritz določi elastično kritično obtežbo n_{cr}.

3. NALOGA



$J = 6000$ cm^4

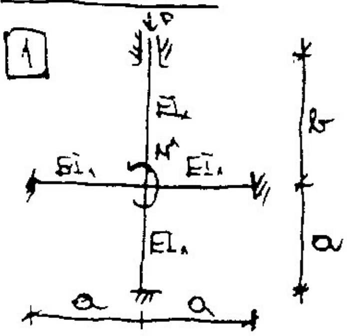
$h = 500$ cm

$E = 3000$ kN/cm^2

Sila P potuje po označeni poti. Oceni elastično kritično silo P_{cr} in jo določi po MKE.

25. 1. 2006

1



$a = 500$
$b = 1000$
$E = 21000$
$I_1 = 5000$
$I_2 = 1000$
$P = 2000$
$\hat{M} = 10000$

Ali pri takih konstrukcijah določiš najprej stupni "d" in potem $M = d \cdot \varphi \rightarrow \varphi$. ?

a) TPR

$$d_{TPR} = 3d_1 + d_2 = 3 \cdot \frac{4EI_1}{a} + \frac{4EI_2}{b} = 2520000 + 84000 = 2604000$$

$$\varphi_{TPR} = \frac{M}{d} = \frac{10\,000}{2604000} = 0{,}0038402$$

b) TDR

$$d_{TDR} = \frac{2 \cdot 4 \cdot EI_1}{a} + \frac{4EI_1}{a} - \frac{2Pa}{15} + \frac{4EI_2}{b} - \frac{2Pb}{15} =$$

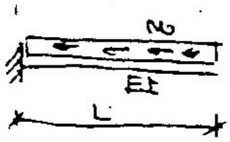
$$= 1680000 + 840000 - 133333{,}\bar{3} + 84000 - 266666{,}\bar{6} =$$
$$= 2204000$$

$$\varphi_{TDR} = \frac{M}{d_{TDR}} = 0{,}0045372$$

$$k_S = \frac{\varphi_{TDR}}{\varphi_{TPR}} = 1{,}1815023 \Rightarrow \gamma_{cr} = \frac{k_S \cdot \gamma}{k_S - 1} = \frac{1{,}1815023\gamma}{0{,}1815023} = 6{,}5096$$

$$P_{cr} = 6{,}5096 \cdot P = \underline{13019{,}14\,kN}$$

2



$E = 20000$
$L = 200$

$$N = -n(L - x)$$

$$\varphi = x^2 \qquad \varphi_{,x} = 2x \qquad \varphi_{,xx} = 2$$

$$K = \int_0^L \left(-n(L-x) \cdot x^4 + EI \cdot 4\right) dx = \int_0^L \left(n x^5 - nLx^4 + 4EI\right) dx =$$
$$= \left(n \frac{x^6}{6} - nL\frac{x^5}{5} + 4EIx\right)\Big|_0^L$$

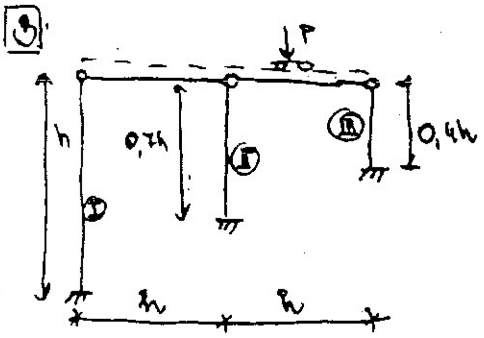
$$n_{cr}\frac{L^6}{6} - n_{cr} L \frac{L^5}{5} + 4EIL = 0$$

$$n_{cr}\left(\frac{L^6}{6} - \frac{L^6}{5}\right) = -4EIL$$

$$n_{cr} = \frac{-4EIL}{L^6\left(\frac{1}{6} - \frac{1}{5}\right)} = \frac{-4EI}{L^5\left(\frac{1}{6} - \frac{1}{5}\right)} = \underline{\underline{7{,}5 \cdot 10^{-6} \cdot I}}$$

I mi podan ali je postopek računa napačen ?

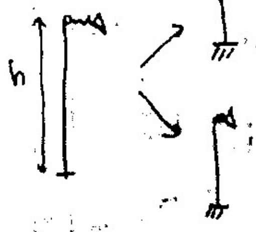
3



$I = 5000$
$h = 500$
$E = 3000$

Ali pri potujoči obtežbi računamo tako, da postavimo P na najbolj neugodno mesto (v tem primeru na najmanjši steber)?

P_{cr}



$P_{cr,min} = \frac{\pi^2 EI}{(2h)^2} = 148,04$ kN

$P_{cr,max} = \frac{\pi^2 EI}{(0,7h)^2} = 1208,52$ kN

Za krajša stebra vzamemo k_w?

$$K = \begin{bmatrix} b_I + k_{wI} + k_{wII} & c_I \\ c_I & d_I \end{bmatrix}$$

$$k_{wI} = \frac{3EI}{(0,7h)^3} = 1,0495627$$

$$k_{wII} = \frac{3EI}{(0,4h)^3} = 5,625$$

$$b_I = 1,44 - 0,0024P$$

$$c_I = 360 - 0,1P$$

$$d_I = 120000 - \frac{1000}{15}P$$

$$\det K = 0 \Rightarrow P_{cr}$$

$$0,15 P_{cr}^2 - 756,970847 P_{cr} + 844147,524 = 0$$

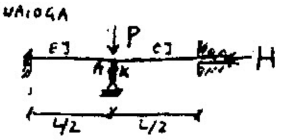
$P_{cr,1} = 1663,54$ kN $\quad P_{cr,2} = 3382,93$ kN

↓

Ne ujema se z oceno.

STABILNOST KONSTRUKCIJ - RAČUNSKI DEL 20.6.2006

1. NALOGA



- [illegible] vzmet $k = 1.5$ kN/cm

$E = 21000$ kN/cm^2
$J = 8000$ cm^4
$L = 1500$ cm
$P = 100$ kN
$H = 600$ kN

DOLOČI VERTIKALNI POMIK V TOČKI A:

a) PO TPR (MKE)
b) PO TDR (MKE)

Z METODO X_S DOLOČI KRITIČNO ELASTIČNO OBTEŽBO H_{cr}.

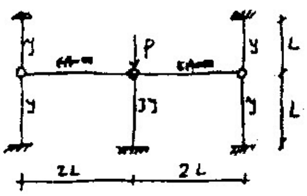
2. NALOGA

$\alpha_T = 10^{-5}$ /°C $J_o = \frac{\pi d^4}{64}$

$L = 500$ cm

NOSILEC JE BIL ZMONTIRAN PRI 10°C. DOLOČI TAK PREREZ POLNEGA OKROGLEGA PROFILA (d), DA SE BO ELASTIČNO UKLONIL PRI 100°C.

3. NALOGA

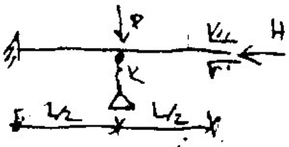


$L = 500$ cm
$J = 6000$ cm^4
$E = 15000$ kN/cm^2

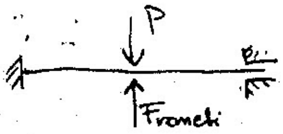
OCENI ELASTIČNO KRITIČNO OBTEŽBO P_{cr}. DOLOČI P_{cr} Z MKE (POLINOM 3 REDA).

20.6.2006

1



$E = 21000$
$I = 8000$
$L = 1500 \rightarrow \frac{L}{2} = 750$
$P = 100$
$H = 600$
$k = 1{,}5\ kN/cm$



$F_{vzmeti} = k \cdot w$

a) TPR če ni vzmeti $P = w \cdot 2b_{TPR}$

$$P - k \cdot w = w \cdot 2b_{TPR} \qquad b_{TPR} = \frac{12 \cdot EI}{(L/2)^3} = 4{,}778\overline{6}$$

$$100 = w \cdot 2b_{TPR} + kw$$
$$100 = w\,(2b_{TPR} + kw)$$
$$w_{TPR} = 9{,}0437719\ cm$$

b) TDR $P - k \cdot w = w \cdot 2b_{TDR}$ $b_{TDR} = \frac{12EI}{(\frac{L}{2})^3} - \frac{6H}{5\frac{L}{2}} = 3{,}818\overline{6}$

$$100 = w\,(2b_{TDR} + k)$$

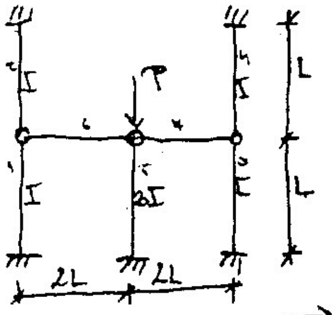
$$w_{TDR} = 10{,}844112\ cm$$

$$k_S = \frac{w_{TDR}}{w_{TPR}} = 1{,}2101269 \qquad \gamma_{cr} = \frac{k_S}{k_S - 1} = 5{,}759 \qquad \underline{H_{cr} = 3455{,}42\ kN}$$

2

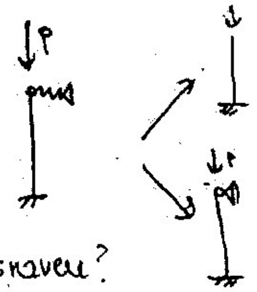
$$P_{cr} = N_T$$
$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad N_T = \alpha_T \cdot \Delta t \cdot EA$$
$$\frac{\pi^2 \cancel{E} \cdot \frac{\pi d^4}{64}}{L^2} = \alpha_T \cdot \Delta t \cdot \cancel{E} \cdot \pi \left(\frac{d}{2}\right)^2$$
$$\pi^3 \cdot d^4 = L^2 \cdot 64 \cdot \alpha_T \cdot \Delta t \cdot \pi \cdot \frac{1}{4}\, d^2$$
$$d^2 = \frac{1}{\pi^3} \cdot 500^2 \cdot 64 \cdot 10^{-5} \cdot 90 \cdot \pi \cdot \frac{1}{4}$$
$$\underline{\underline{d = 19{,}099\ cm}}$$

3



$L = 500\ cm$
$I = 6000\ cm^4$
$E = 15\,000\ kN/cm^2$

Ocena:



$P_{cr} = \frac{\pi^2 EI}{(2L)^2} = 888{,}3\ kN$

$P_{cr} = \frac{\pi^2 EI}{(0{,}7L)^2} = 7251{,}14\ kN$

je to ok? Ali se to ne šteje sploh?

$\underline{k_{w1} = k_{w2} = k_{w3} = k_{w4} = \frac{3EI}{L^3} = 2{,}16}$

⑤ $\varphi \neq 0$, $w \neq 0$

$$K_5 = \begin{bmatrix} b_5 & c_5 \\ c_5 & d_5 \end{bmatrix} \qquad K = \begin{bmatrix} b_5 + 4 \cdot k_w & c_5 \\ c_5 & d_5 \end{bmatrix}$$

$\det K = 0 \Rightarrow P_{cr}$

$b_5 = 8{,}64 - 0{,}0024\,P$

$c_5 = 2160 - 0{,}1\,P$

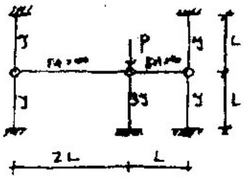
$d_5 = 720000 - 66{,}\bar{6}\,P$

$$0{,}15 \cdot P^2 - 2448 \cdot P + 7776000 = 0$$

$\underline{\underline{P_1 = 4320\ kN}}$ $\qquad P_2 = 12000\ kN$

STABILNOST KONSTRUKCIJ 27.6.2006

1 NALOGA



$L = 200$ cm

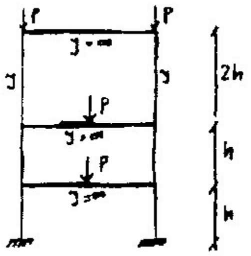
$J = ?$

$P = 1000$ kN

$E = 3000$ kN/cm²

DOLOČI TAK VZTRAJNOSTNI MOMENT PREREZA (J=?), DA SE BO KONSTRUKCIJA ELASTIČNO UKLONILA PRI SILI P=1000 kN

2 NALOGA



$P = 1000$ kN

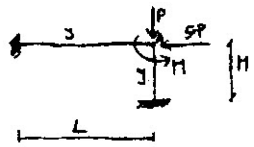
$E = 21000$ kN/cm²

$J = 5000$ cm⁴

$h = 400$ cm

Z METODO [illegible] DOLOČI KRITIČNI UKLONSKI OBTEŽNI FAKTOR μ_{cr}.

3. NALOGA



$E = 3000$ kN/cm² $P = 300$ kN

$J = 10000$ cm⁴ $M = 2000$ kNcm

$L = 600$ cm

$H = 300$ cm

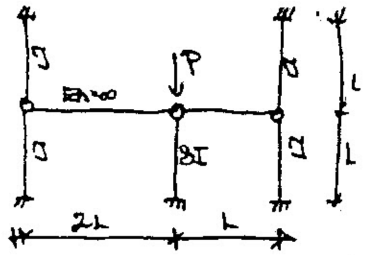
DOLOČI ZASUK V TOČKI A:

a) PO TPR

b) PO TDR (UPORABI FUNKCIJE STABILNOSTI)

27.6.2006 (4)

[1]



L=200
P=1000
E=8000
I=?

$$b = \frac{12EI}{L^3} - \frac{6P}{5L} = 0{,}0045 I - 6$$

$$c = 0{,}45 I - 100$$

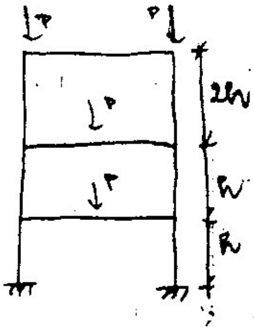
$$d = 60 \cdot I - 26666{,}667$$

$$K = \begin{bmatrix} b & c \\ c & d \end{bmatrix}$$

$$0{,}0675 I^2 - 890 I + 150000$$

$I_1 = 414{,}8269\ cm^4$ $\qquad I_2 = 5363{,}4508\ cm^4$

[2]



P=1000
E=21000
J=3000
h=400

$W_1 = 4P$
$W_2 = 3P$
$W_3 = 2P$

$$\mu_1 = \frac{\frac{1}{2} 4P \cdot h^3}{12EI} = 101{,}5873$$

$$\mu_2 = \frac{1\,3P \cdot h^3}{12EI} = 76{,}190476$$

$$\mu_3 = \frac{1\,2P \cdot (2h)^3}{12EI} = 406{,}34921 \quad \cancel{812{,}69841}$$

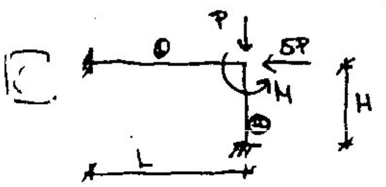
$\varphi_1 = 0{,}253968$

$\varphi_2 = -0{,}063492$

$\varphi_3 = 0{,}4126984 = \varphi_{max}$

$$\gamma_{cr} = \frac{1}{1{,}11 \cdot 0{,}4126984} = \underline{\underline{2{,}18295}}$$

[3]



E=3000
I=10000
L=600
H=300
P=300
M=2000

a) $$\varphi^{①}_{TPR} = \frac{M}{d_{TPR}} = \frac{2000}{\frac{4EI}{L}} = 0{,}01$$

$$\varphi^{②}_{TPR} = \frac{M}{d_{TPR}} = \frac{2000}{\frac{4EI}{H}} = 0{,}005$$

b) ?

①

$M = d \cdot \varphi \qquad \varphi_1 = \frac{M}{d}$

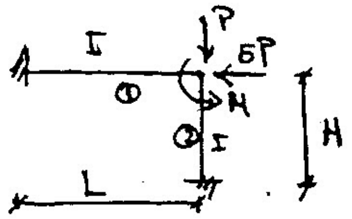
$d^{①}_{TDR} = 32628{,}768 \qquad \varphi^{①}_{TDR} = 0{,}0612937$

$\omega = \sqrt{18} = 4{,}2426407$

$d^{②}_{TDR} = 887855{,}81 \qquad \varphi^{②}_{TDR} = 0{,}0001565$

$\omega = 0{,}9486833$

3



$E = 3000$
$I = 10\,000$
$L = 600$
$H = 300$
$P = 300$
$M = 2000$

a) TPR :

$$M = (d_1 + d_2) \cdot \varphi \qquad \varphi_{TPR} = \frac{M}{d_1 + d_2}$$

$$d_1 = \frac{4EI}{L} = 200\,000$$

$$d_2 = \frac{4EI}{H} = 400\,000 \qquad \underline{\varphi_{TPR} = 0{,}00\overline{3}}$$

b) TDR (Pol. 3. reda)

$$d_1 = \frac{4EI}{L} - \frac{2 \cdot 5P \cdot L}{15} = 80000$$

$$d_2 = \frac{4EI}{H} - \frac{2 \cdot P \cdot H}{15} = 388\,000 \qquad \varphi_{TDR} = \frac{M}{d_1 + d_2} = 0{,}0042735$$

c) TDR (funkcije stabilnosti)

$$\omega_1 = \sqrt{\frac{5P \cdot L^2}{EI}} = \sqrt{18}$$

$$\omega_2 = \sqrt{\frac{P \cdot H^2}{EI}} = \sqrt{0{,}9}$$

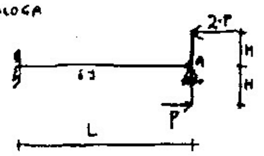
$d_1 = 32629{,}768$

$d_2 = 387855{,}81$

$$\underline{\varphi_{TDR} = 0{,}0047564}$$

STABILNOST KONSTRUKCIJ - RAČUNSKI DEL 29.4.

1. NALOGA



$E = 2000\ kN/cm^2$

$J = 40000\ cm^4$

$L = 1000\ cm$

$H = 400\ cm$

$P = 400\ kN$

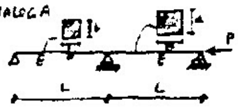
DOLOČI ZASUK V TOČKI A:

a) PO TPR

b) PO TDR (UPORABI FUNKCIJE STABILNOSTI)

c) DOLOČI KRITIČNO UKLONSKO SILO P_{kr} Z UPORABO METODE [illegible]

2. NALOGA



$L = 600\ cm$

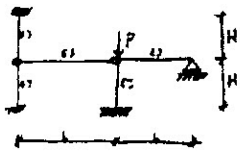
$a = 5\ cm$

$b = 4\ cm$ } [illegible]

$E = 20\,000\ kN/cm^2$

Z METODO RAYLEIGH - RITZ DOLOČI ELASTIČNO KRITIČNO SILO P_{kr}

3. NALOGA



$L = 700\ cm$

$H = 300\ cm$

$E = 20000\ kN/cm^2$

$J = 5000\ cm^4$

Z DKP DOLOČI ELASTIČNO KRITIČNO SILO P_{kr}.

$$-0{,}39P\left(9\frac{L^5}{5}-21600\frac{L^4}{4}+1728\cdot10^4\frac{L^3}{3}-5184\cdot10^6\frac{L^2}{2}+5184\cdot10^8L\right)+$$

$$21{,}\bar{3}\left(72000\frac{L^3}{3}-864\cdot10^6\frac{L^2}{2}+2592\cdot10^8\cdot L\right)\Bigg]+$$

$$+\Bigg[\left(-0{,}61P\left(9\frac{L^5}{5}-21600\frac{L^4}{4}\;\cdot\cdot\qquad\right)\right)+$$

$$+\;52{,}08\bar{3}\left(72000\frac{L^3}{3}-864\cdot10^6\frac{L^2}{2}+2592\cdot10^8L\right)\Bigg]=0$$

$$-{,}39P\cdot6{,}2208\cdot10^{13}+21{,}\bar{3}\cdot5{,}184\cdot10^{12}-0{,}61P\cdot6{,}2208\cdot10^{13}+52{,}08\bar{3}\cdot5{,}184\ldots$$

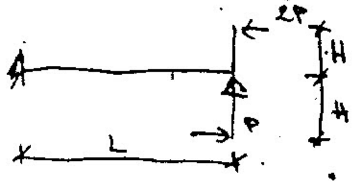
$$-P\cdot6{,}2208\cdot10^{+13}=-5{,}184\cdot10^{12}\left(21{,}\bar{3}+52{,}08\bar{3}\right)$$

$$P=6{,}118055\ kN$$

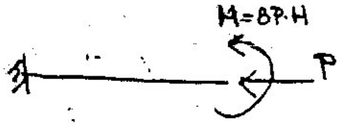
29.8.2006

(C)

1



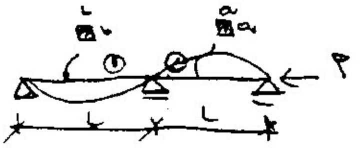
$E=8000$
$I=10000$
$L=1000$
$H=100$
$P=100$



TP2: $\varphi=\frac{M}{d_{TP2}}$ $\quad d_{TP2}=\frac{4EI}{L}=120000$ $\quad \varphi=\frac{8\cdot100\cdot100}{120000}=\underline{\underline{0{,}5}}$

TD2: $\varphi=\frac{M}{d_{TD2}}$ $\quad d_{TD2}=\frac{4EI}{L}-\frac{2PL}{15}=106666{,}67$ $\quad \varphi=\underline{\underline{0{,}5625}}$

2



$L=600$
$a=5 \quad A_2=25$
$b=4 \quad A_1=16$
$E=10000$

$I_1=\frac{b^4}{12}=21{,}\overline{3}$
$I_2=\frac{a^4}{12}=52{,}08\overline{3}$

$u_1=u_2$
$L_1\cdot\varepsilon_1=L_2\varepsilon_2$
$L_1\frac{N_1}{A_1E}=L_2\frac{N_2}{A_2E}$
$N_1=\frac{A_1}{A_2}\cdot N_2$
$P-N_2=\frac{A_1}{A_2}\cdot N_2$
$P-N_2=0{,}64N_2$
$P=(0{,}64+1)N_2$
$P=1{,}64N_2 \rightarrow \underline{N_2=0{,}609756P}$
$\underline{N_1=0{,}3902438P}$

$N_1+N_2=P$
$N_1=P-N_2$

$\cancel{\ldots=\sin\frac{\pi x}{L}}$
$\cancel{P_{1x}=\cos\frac{\pi x}{L}\cdot\frac{\pi}{L}}$
$\cancel{P_{1xx}=-\sin\left(\frac{\pi x}{L}\right)\cdot\left(\frac{\pi}{L}\right)^2}$

$$\cancel{K=\int_0^L\left(0{,}3902439P\cdot\cos^2\frac{\pi x}{L}\left(\frac{\pi}{L}\right)^2+EI\left(\frac{\pi}{L}\right)^4\sin^2\frac{\pi x}{L}\right)+\int_0^L\left(0{,}609756P\cdot\cos^2\frac{\pi x}{L}\left(\frac{\pi}{L}\right)^2+EI\left(\frac{\pi}{L}\right)^4\sin^2\frac{\pi x}{L}\right)=}$$

$$\cancel{=\int_0^L\left(0{,}00000001\ 0851552\cdot10^{-5}\cdot\cos^2\frac{\pi x}{L}+0{,}0003206\cdot\sin^2\frac{\pi x}{L}\right)+\int_0^L\left(0{,}0000\ 16716207\cdot10^{-5}\cdot\cos^2\frac{\pi x}{L}+0{,}0007829\cdot\sin^2\frac{\pi x}{L}\right)}$$

$$P=x\cdot(x-L)(x-2L)=(x^2-Lx)(x-2L)=x^3-Lx^2-2Lx^2+2L^2x$$

$$P=x^3-3Lx^2+2L^2x \qquad P_{1x}=3x^2-6Lx+2L^2$$

$$P_{1xx}=6x-6L$$

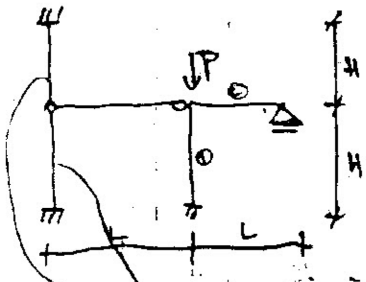
$$K=\int_0^L\left(-0{,}3902439P\cdot(3x^2-6Lx+2L^2)^2+EI_1(6x-6L)^2\right)dx+$$

$$+\int_L^{2L}\left(-0{,}609756P\cdot(3x^2-6Lx+2L^2)^2+EI_2(6x-6L)^2\right)dx$$

$$=\int_0^L\Big[\left(-0{,}39P\left(9x^4-21600x^3+17280000x^2-5184000000x+5184\cdot10^8\right)\right)+$$
$$+I_1\left(72000x^2-864\cdot10^6x+2592\cdot10^8\right)\Big]dx\quad+$$

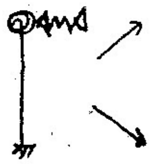
$$+\int_L^{2L}\Big[\left(-0{,}61P\left(9x^4-21600x^3+1728\cdot10^4x^2-5184\cdot10^6x+5184\cdot10^8\right)\right)+$$
$$I_2\left(72000x^2-864\cdot10^6x+2592\cdot10^8\right)\Big]dx \rightarrow$ na zadnji strani zapisa

3

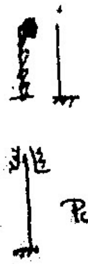


L=500
H=300
E=20000
I=5000

Ocena



$P_{kr} = \frac{\pi^2 EI}{(2H)^2} = 2744,56$



$P_{kr} = \frac{\pi^2 EI}{(0,5 \cdot H)^2} = 43864,9$

ali je še od teh dveh kr. stebrov?

② $k_\varphi = \frac{3EI}{L} = 600000$

① $\varphi \neq 0$, $w \neq 0$ $\quad K_1 = \begin{bmatrix} b & c \\ c & d \end{bmatrix}$ $\qquad K = \begin{bmatrix} b & c \\ c & d + k_\varphi \end{bmatrix}$

$b = \frac{12EI}{H^3} - \frac{6P}{5H} = 44,\bar{4} - 0,004P$

$c = \frac{6EI}{H^2} - 0,1P = 6666,\bar{6} - 0,1P$

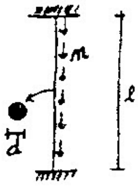
$d = \frac{4EI}{H} - \frac{2PH}{15} = 1333333,\bar{3} - 40P$

$0,15 \cdot P^2 - 8177,7776\,P + 44481472,8888 = 0$

$\underline{P_1 = 5660,1\,kN}$ $\qquad P_2 = 48858,4\,kN$

STABILNOST KONSTRUKCIJ - RAČUNSK DEL RPITA 6.9.2006

1. NALOGA (3T)



$\ell = 8.0$ m

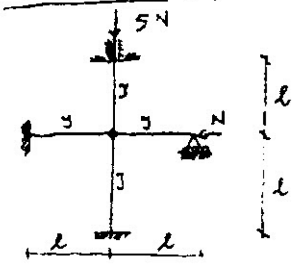
$d = 50$ mm

$E = 21000$ kN/cm^2

$J_{\bullet} = \frac{\pi d^4}{64}$

IZRAČUNAJ ELASTIČNO KRITIČNO OBTEŽBO m_{cr} S POMOČJO METODE RAYLEIGH-RITZ.

2. NALOGA (2T)



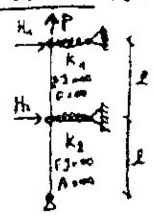
$E = 1000$ kN/cm^2

$J = 1000$ cm^4

$\ell = 4.0$ m

OCENI SPODNJO IN ZGORNJO MEJO ZA ELASTIČNO KRITIČNO OBTEŽBO N_{cr}. IZRAČUNAJ N_{cr} Z MKE.

3. NALOGA (3T)



$H_1 = H_2 = 20$ kN $P = 10$ kN

$\ell = 5.0$ m

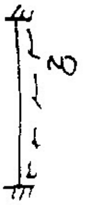
$k_1 = k_2 = 1$ kN/cm

Z METODO P-Δ DOLOČI HORIZONTALNI POMIK KONSTRUKCIJE PO TDR.
(PALICE SO TOGE, VZMETI SO ELASTIČNE)

6.9.2006

Klancek.si ⑤

1



$$N = n\left(\frac{l}{2} - x\right) = n(400 - x)$$

$$P = x^2(x-l)^2 = x^2(x^2 - 1600x + 640000)$$

$$P_{1x} = 4x^3 - 4800x^2 + 1280000x$$

$$P_{1xx} = 12x^2 - 9600x + 1280000$$

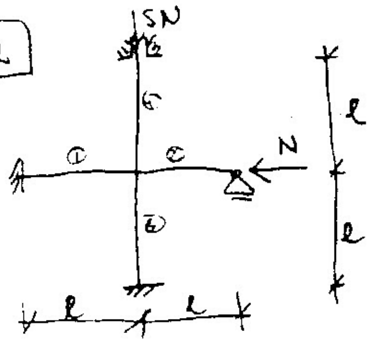
$$K = \int_0^l \left(n(400-x)(4x^3 - 4800x^2 + 1280000x)^2 + EI(12x^2 - 9600x + 1280000)^2 \right) dx$$

$$K = \int \Big[16 \cdot n \cdot x^7 + 44800\, x^6 - 18640000\, x^5 + 256 \cdot 10^8 x^4 - 6{,}5536 \cdot 10^{12} x^3 + 6{,}5536 \cdot 10^{14} x^2$$
$$+ EI \cdot 144 \cdot x^4 - EI\,230400\, x^3 + 12880000\, EI\, x^2 - 24576 \cdot 10^6\, EI\, x + 1{,}6384 \cdot 10^{12} \cdot EI \Big]$$

$$K = n\left(2\,x^8 + 6400\, x^7 - 8106666{,}7 \cdot x^6 + 51{,}2 \cdot 10^8 x^5 - 1{,}6384 \cdot 10^{12} x^4 \right)$$
$$+ EI\left(28{,}8\, x^5 - 57600\, x^4 + 4096 \cdot 10^4 x^3 - 12288 \cdot 10^6 x^2 + 1{,}6384 \cdot 10^{12}\right)$$

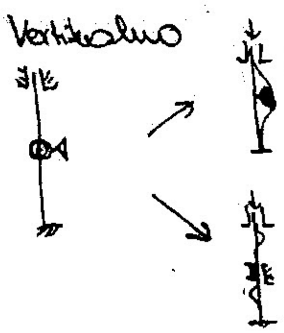
$$n = -8{,}47617 \cdot 10^{19} / -1{,}11848 \cdot 10^{23} = 0{,}0007578$$

2



$E = 1000$
$I = 1000$
$l = 400$

Vertikalno

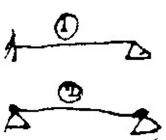


$$5P_{cr} = \frac{\pi^2 EI}{(2l \cdot 0{,}5)^2} = 61{,}685$$

$$P_{cr} = 12{,}337$$

$$5P_{cr} = \frac{\pi^2 EI}{(l \cdot 0{,}5)^2} = 246{,}74$$

$$P_{cr} = 49{,}348$$



$K_1 = d_1$

$K_2 = d_2$

$K_3 = d_3$

$K_4 = d_4$

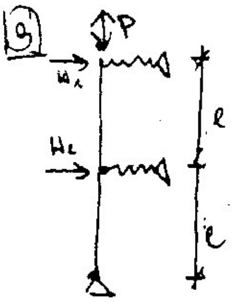
$$K = d_1 + d_2 + d_3 + d_4$$

$$d_1 = \frac{4EI}{L} - \frac{2 \cdot L \cdot N}{15} = 10000 - 53{,}\overline{3} \cdot N$$

$$d_3 = \frac{4EI}{L} - \frac{2 \cdot L \cdot 5N}{15} = 10000 - 266{,}\overline{6} \cdot N$$

$$K = -640\,N + 40000 \qquad \underline{N = 62{,}5\ kN}$$

9)



$P = 10\ kN$
$H_1 = H_2 = 20\ kN$
$\ell = 500\ cm$
$k_1 = k_2 = 1\ kN/cm$

1) $u_1 = 0$ $\quad \varphi_1 = 0$ $\quad \Delta H_1 = 0$ $\quad u_1 = \frac{20}{k} = 20$
$u_2 = 0$ $\quad \varphi_2 = 0$ $\quad \Delta H_2 = 10 \cdot 0 \cdot 1{,}2 = 0$ $\quad u_2 = 20$

2) $u_1 = 20$ $\quad \varphi_1 = 0{,}04$ $\quad \Delta H_1 = (10 \cdot 0{,}04 \cdot 1{,}2) = 0{,}48$ $\quad u_1 = 20{,}48$
$u_2 = 20$ $\quad \varphi_2 = 0$ $\quad \Delta H_2 = 0$ $\quad u_2 = 20$

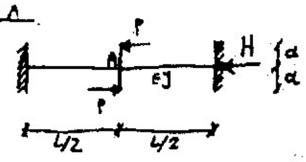
3) $u_1 = 20{,}48$ $\quad \varphi_1 = 0{,}04096$ $\quad \Delta H_1 = 0{,}50304$ $\quad u_1 = 20{,}50304$
$u_2 = 20$ $\quad \varphi_2 = -0{,}00096$ $\quad \Delta H_2 = -0{,}01152$ $\quad u_2 = 19{,}98848$

4) $u_1 = 20{,}50304$ $\quad \varphi_1 = 0{,}041006$ $\quad \Delta H_1 = 0{,}5044214$ $\quad u_1 = 20{,}504421$
$u_2 = 19{,}98848$ $\quad \varphi_2 = -0{,}0010281$ $\quad \Delta H_2 = -0{,}0123484$ $\quad u_2 = 19{,}987651$

$u_1 = 20{,}50\ cm$ $\qquad u_2 = 19{,}98\ cm$

STABILNOST KONSTRUKCIJ – 21.11.2006

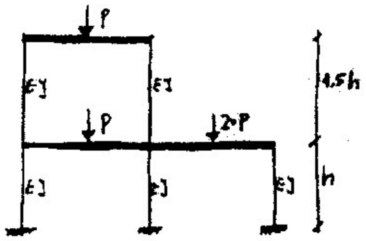
1. NALOGA



$L = 300$ cm
$H = 5000$ kN
$E = 3000$ kN/cm²
$J = 10000$ cm⁴
$P = 100$ kN
$d = 100$ cm

DOLOČI ZASUK V TOČKI "A" po:
a) TPR
b) TDR (funkcije stabilnosti)
DOLOČI ELASTIČNO KRITIČNO SILO H_{cr} PO K_G POSTOPKU.

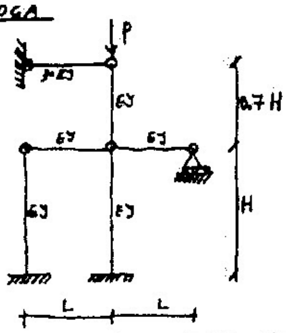
2. NALOGA



$P = 1000$ kN
$E = 21000$ kN/cm²
$J = 5000$ cm⁴
$h = 400$ cm

Z METODO HORNE DOLOČI KRITIČNI OBTEŽNI FAKTOR λ

3. NALOGA



$H = 200$ cm
$E = 3000$ kN/cm²
$J = 10000$ cm⁴
$L = 100$ cm

OCENI ELASTIČNO KRITIČNO OBTEŽBO KONSTRUKCIJE.
Z MKE DOLOČI ELASTIČNO KRITIČNO SILO P_{cr}.

21. 11. 2006

1

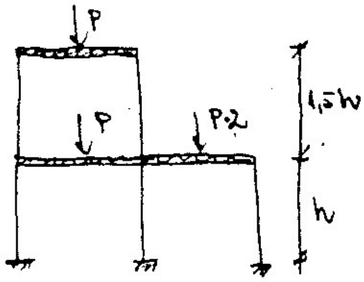
a) TPR: $H = d_{TPR} \cdot \varphi$ $\qquad \varphi_{TPR} = \dfrac{2 \cdot 100 \cdot 100}{\dfrac{4EI}{L/2}} = \underline{\underline{0{,}025}}$

b) TDR $\qquad d_{TDR} = \dfrac{\omega^{*} \sin\omega - \omega^2 \cos\omega}{2(1-\cos\omega) - \omega \cdot \sin\omega} \quad \dfrac{EI}{L/2}$

$\omega = \sqrt{\dfrac{|H| \cdot (\frac{L}{2})^2}{EI}} = 1{,}9864917$

$d_{TDR} = 694660{,}57 \qquad \underline{\underline{\varphi_{TDR} = 0{,}028791}}$

2



P = 1000
E = 21000
I = 5000
h = 400

$W_1 = 4P$
$W_2 = P$

$u_1 = \dfrac{\frac{1}{3} 4P \cdot h^3}{12EI} = 67{,}721868$

$u_2 = \dfrac{\frac{1}{2} P \cdot (h \cdot 1{,}5)^3}{4EI} = 85{,}714286$

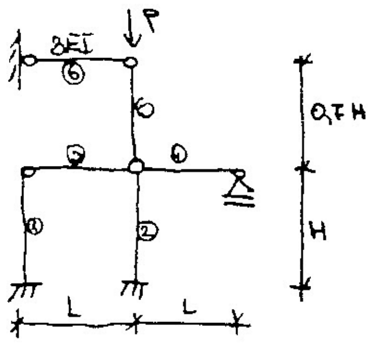
$\varphi_1 = \dfrac{u_1}{h_1} = 0{,}1693121 = \varphi_{max}$

$\varphi_2 = \dfrac{u_2 - u_1}{1{,}5h} = 0{,}0299828$

$\gamma_{cr} = \dfrac{1}{1{,}1 \cdot 0{,}1693121} = 5{,}36932$

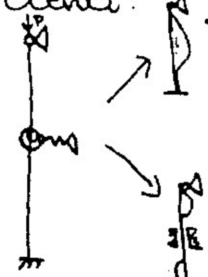
$\underline{\underline{P_{cr} = 5369{,}32\ kN}}$

2.2



H = 200
L = 100
E = 3000
I = 10000

Ocena:



$P_{cr,1} = \dfrac{\pi^2 EI}{(1{,}7H)^2 \cdot 0{,}7^2} = \underline{5227{,}1756}$

$P_{cr,2} = \dfrac{\pi^2 EI}{(0{,}7 \cdot 0{,}7H)^2} = \underline{30829{,}668\ kN}$

$k_{44} = \dfrac{3EI}{H^3} = 11{,}25$

②

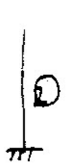
razdelitev sile P na el ② in ⑤

$u_2 = u_5$
$L_2 \varepsilon_2 = L_5 \cdot \varepsilon_5$
$L_2 \cdot \dfrac{N_2}{AE} = L_5 \dfrac{N_5}{AE}$

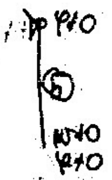
$N_2 \cdot L_2 = N_5 \cdot L_5 \qquad N_2 + N_5 = P \qquad N_5 = 0{,}588235 P$

$N_2 = \dfrac{L_5}{L_2}(P - N_2)$

$N_2\left(1 + \dfrac{L_5}{L_2}\right) = \dfrac{P L_5}{L_2} \qquad N_2 = \dfrac{P \cdot \frac{L_5}{L_2}}{(1 + L_5/L_2)} = 0{,}4117647 P$



$$K_2 = \begin{bmatrix} b_2 & c_2 \\ c_2 & d_2 \end{bmatrix}$$



$$K_5 = \begin{bmatrix} b_5 & -c_5 & -c_5 \\ -c_5 & d_5 & e_5 \\ -c_5 & e_5 & d_5 \end{bmatrix}$$

$$K = \begin{bmatrix} b_2 + b_5 + k_w & c_2 - c_5 & -c_5 \\ c_2 - c_5 & d_2 - d_5 & e_5 \\ -c_5 & e_5 & d_5 \end{bmatrix}$$

$N_2 = 0,4118\ P$
$N_5 = 0,5882\ P$

$$b_2 = \frac{12EI}{H^3} - \frac{6 \cdot 0,4118\ P}{5H} = 45 - 0,0024711\ P$$

$$b_5 = \frac{12EI}{(0,7H)^3} - \frac{6 \cdot 0,5882\ P}{5H \cdot 0,7} = 131,19534 - 0,0050417\ P$$

$$c_2 = \frac{6EI}{H^2} - 0,1 \cdot 0,4118 \cdot P = 4500 - 0,04118\ P$$

$$c_5 = \frac{6EI}{(0,7H)^2} - 0,1 \cdot 0,5882\ P = 9183,6735 - 0,05882\ P$$

$$d_2 = \frac{4EI}{H} - \frac{2H \cdot 0,4118\ P}{15} = 600000 - 10,98133\ P$$

$$d_5 = \frac{4EI}{0,7H} - \frac{2H \cdot 0,7 \cdot 0,5882\ P}{15} = 857142,86 - 10,97973\ P$$

$$e_5 = \frac{2EI}{0,7H} + \frac{0,5882 \cdot 0,7H \cdot P}{30} = 428571,43 + 2,744933\ P$$

$k_w = 11,25$

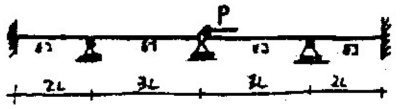
$$6,559107 \cdot 10^{-2}\ P^3 - 7639,44595\ P^2 + 308965195,87\ P - 3,5930229 \cdot 10^{13} = 0$$

$\underline{P_1 = 11.966,3}$

$P_{2,3} = 52252,26$

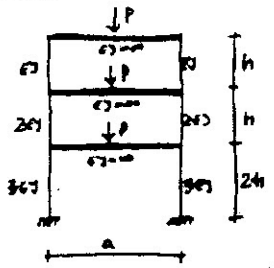
STABILNOST KONSTRUKCIJ 26.4.2007

1. NALOGA



OCENI IN NATO DOLOČI ELASTIČNO KRITIČNO OBTEŽBO P_{cr}.
UPORABI METODO PO SVOJI IZBIRI.

2. NALOGA



$E = 21000\ kN/cm^2$

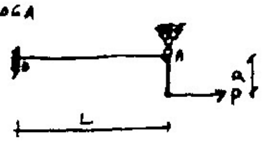
$h = 600\ cm$

$a = 1000\ cm$

$J = 5000\ cm^4$

Z METODO HORNE DOLOČI ELASTIČNO KRITIČNO OBTEŽBO P_{cr}.

3. NALOGA



$E = 21000\ kN/cm^2$

$J = 2000\ cm^4$

$L = 300\ cm$

$a = 100\ cm$

$P = 50\ kN$

DOLOČI ZASUK V TOČKI A IN MOMENT V TOČKI B

a) PO TPR

b) PO TDR (MKE, POLINOM 3 REDA)

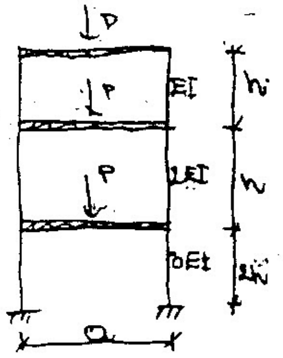
26. 4. 2007

③

Klancek.si

$\boxed{1}$?

$\boxed{2}$



E = 21000
h = 600
a = 1000
I = 5000

$W_1 = 3P$ $\quad H_1 = 3P$
$W_2 = 2P$ $\quad H_2 = 2P$
$W_0 = P$ $\quad H_0 = P$

$$u_1 = \frac{0.5 \cdot 3P \cdot (2h)^3}{12EI} = 2{,}0571429\,P \qquad \psi_1 = \frac{u_1}{h_1} = 0{,}00\cancel{84285}\,1714\,2P$$

$$u_2 = \frac{0{,}5\ 2P\ h^3}{12EI} = 0{,}1714285\,P \qquad \psi_2 = \frac{u_2 - u_1}{h_2} = -0{,}0031428\,P$$

$$u_3 = 0{,}0857142\,P \qquad \psi_3 = \frac{u_3 - u_2}{h_3} = -0{,}0001428\,P$$

$$\gamma_{cr} = \frac{K_{,1}}{1{,}11 \cdot 0{,}0017142\,P} \qquad P_{cr} = P \cdot 525{,}55\,\frac{1}{P}$$

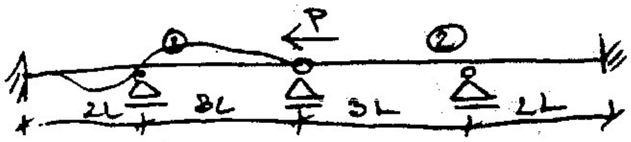
$$\gamma_{cr} = 525{,}55\,\frac{1}{P} \qquad P_{cr} = 525{,}55\ kN$$

$\boxed{3}$

T_{pr}: $\quad M = d_{rea}\ \varphi \qquad \varphi = \frac{M}{d_{rea}} = \frac{50 \cdot 100}{\frac{4EI}{L}} = \frac{5000}{560000} = \underline{0{,}0089285}$

T_{DR}: $\quad \varphi = \frac{5000}{560000 + \frac{3 \cdot 50 \cdot 300}{15}} = \underline{0{,}0088967}$

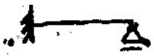
$\boxed{1}$



$N_{x1} = -0{,}5P$

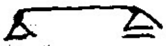
$N_{x2} = 0{,}5P$

Ocena



$P_{cr} = 2\cdot\frac{\pi^2 EI}{(2\cdot 0{,}7L)^2}$

$P_{cr} = 1{,}020408\,\frac{\pi^2 EI}{L^2}$



$P_{cr} = 2\,\frac{\pi^2 EI}{(3L)^2} = 0{,}\overline{2}\,\frac{\pi^2 EI}{L^2}$

$P = x^2(x-2L)(x-5L) = x^4 - 2Lx^3 - 5Lx^3 + 10L^2x^2 = x^4 - 7Lx^3 + 10L^2x^2$

$P_{1x} = 4x^3 - 21Lx^2 + 20L^2x$

$P_{1xx} = 12x^2 - 42Lx + 20L^2$

$$\int_0^{5L}\left(-0{,}5P\cdot(4x^3-21Lx^2+20L^2x)^2 + EI(12x^2-42Lx+20L^2)^2\right)dx =$$

$$= \int_0^{5L}\Big(-0{,}5P\cdot(16x^6-168Lx^5+601L^2x^4-840L^3x^3+400L^4x^2) +$$
$$+ EI\cdot(144x^4-1008Lx^3+2244L^2x^2-1680L^3x+400L^4)\Big)dx$$

$$= -0{,}5P\left(\frac{16}{7}x^7 - \frac{168}{6}Lx^6 + \frac{601}{5}L^2x^5 - 840\frac{1}{4}L^3x^4 + \frac{400}{3}L^4x^3\right)\Big|_0^{5L} +$$
$$+ EI\left(\frac{144}{5}x^5 - \frac{1008}{4}Lx^4 + \frac{2244}{3}L^2x^3 - 1680/2\,L^3x^2 + 400L^4x\right)\Big|_0^{5L}$$

$$-0{,}5P\left(178571{,}43L^7 - 437500L^7 + 375625L^7 - 131250L^7 + 16666{,}\overline{6}L^7\right) +$$
$$+ EI\left(90000L^5 - 157500L^5 + 93500L^5 - 21000L^5 + 2000L^5\right) = 0$$

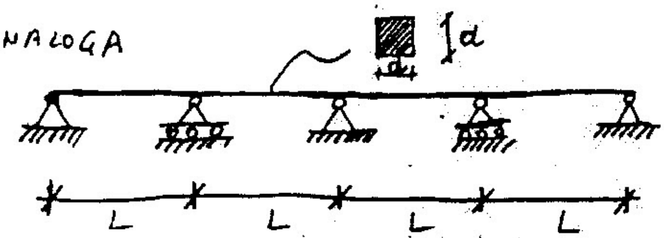
$$-0{,}5P\cdot 2113{,}096\cdot L^{\cancel{7}\,2} = -EI\cdot 7000\cancel{L^5}$$

$$P = \frac{EI\cdot 7000}{L^2\cdot 0{,}5\cdot 2113{,}096}$$

$$P = 6{,}62535\cdot\frac{EI}{L^2}$$

STABILNOST KONSTRUKCIJ 26.6.2007

1. NALOGA

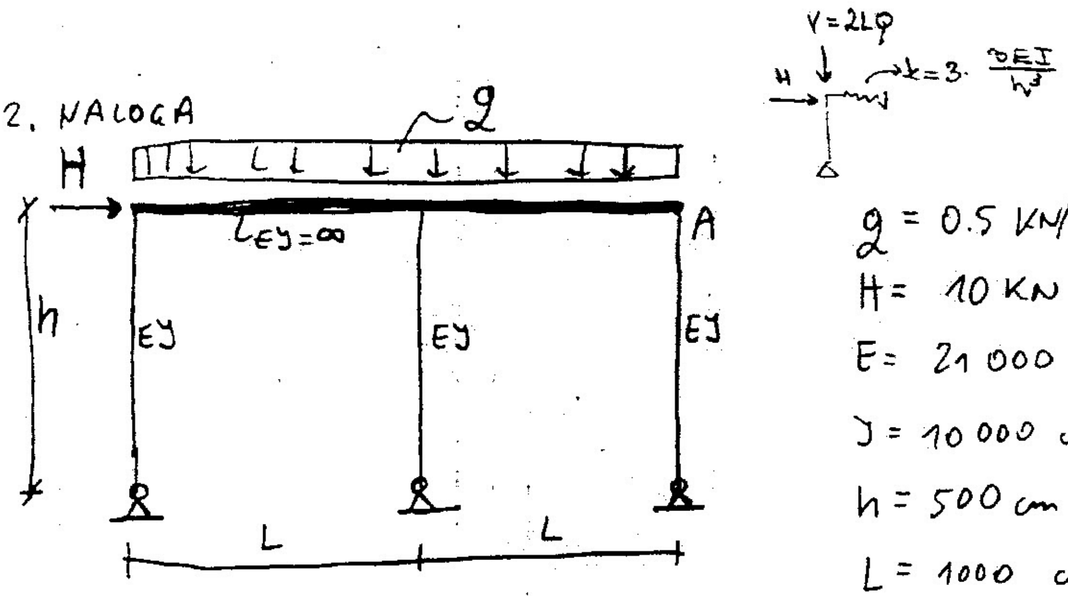


$\alpha_T = 10^{-5}/°C$

$L = 700 cm$

NOSILEC JE ZMONTIRAN PRI 0°C. DOLOČI TAKO DIMENZIJO PREREZA "a", DA SE BO ELASTIČNO UKLONIL PRI 200 °C.

2. NALOGA



$q = 0.5 kN/cm$

$H = 10 kN$

$E = 21\,000 kN/cm^2$

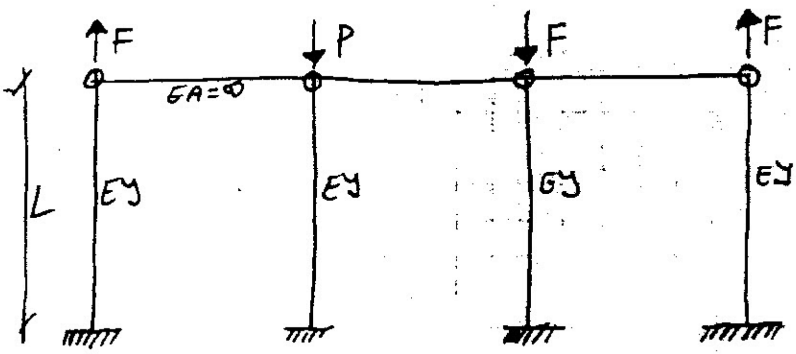
$J = 10\,000 cm^4$

$h = 500 cm$

$L = 1000 cm$

Z METODO P-Δ DOLOČI HORIZONTALEN POMIK V TOČKI A
Z METODO k_G OCENI KRITIČNO UKLONSKO OBTEŽBO q_{cr}.

3. NALOGA



$F = 10 kN$

$L = 300 cm$

$E = 3000 kN/cm^2$

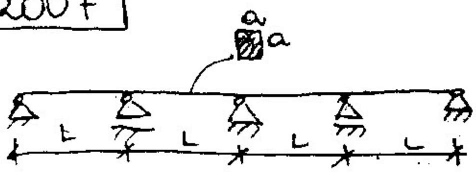
$J = 10\,000 cm^4$

Z MKE DOLOČI KRITIČNO UKLONSKO SILO P_{cr}.

26.6.2007

①

1



$L = 700\ cm$
$\alpha_T = 10^{-5}\ K^{-1}$
$\Delta T = 200°C$

$I = \frac{a^4}{12}$

$N_T = \alpha_T \cdot \Delta t \cdot E \cdot a^2$

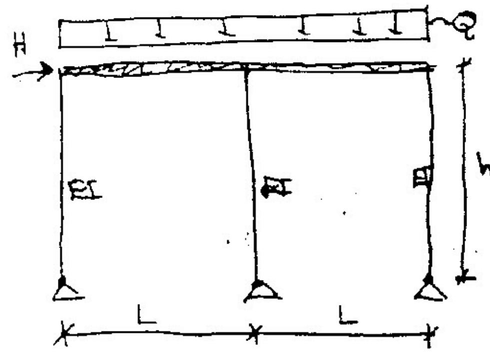
$P_{cr} = \frac{\pi^2 EI}{L^2}$

$$\alpha_t \cdot \Delta t \cdot E a^2 = \frac{\pi^2 E a^4}{12 \cdot L^2}$$

$$\frac{\alpha_t \cdot \Delta t \cdot \cancel{E} \cdot 12 L^2}{\pi^2 \cancel{E}} = a^2 \qquad a^2 = \frac{10^{-5} \cdot 200 \cdot 12 \cdot 700^2}{\pi^2}$$

$\underline{\underline{a = 34{,}5\ cm}}$

2



$Q = 0{,}5\ kN/cm$
$H = 10\ kN$
$E = 21000\ kN/cm^2$
$I = 10000\ cm^4$
$h = 500\ cm$
$L = 1000\ cm$

$V = 2L \cdot Q = 2000\ cm \cdot 0{,}5\ kN/cm = 1000\ kN$

$k = \frac{3EI}{h^3} \cdot 3 = 15{,}12$

1) $u = 0$
$\varphi_1 = 0$
$\Delta H = (P_z \cdot 0) \cdot 1{,}2 = 0$
$u = \frac{H + \Delta H}{K} = \frac{10}{15{,}12} = 0{,}6613756\ cm$

2) $u = 0{,}6613756$
$\varphi = 0{,}0013227$
$\Delta H = P_z \cdot \varphi \cdot 1{,}2 = V \cdot \varphi \cdot 1{,}2 = 1{,}5873$
$u = 0{,}7663559\ cm$

3) $u = 0{,}7663559$
$\varphi = 0{,}0015327$
$\Delta H = 1{,}83925$
$u = 0{,}7830194$

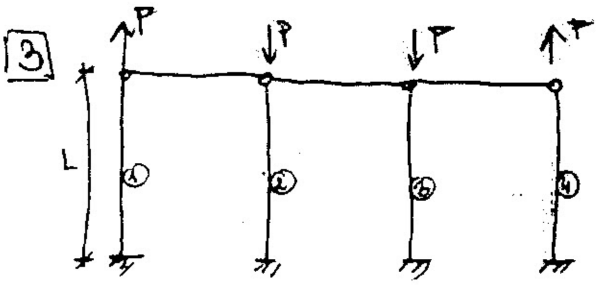
4) $u = 0{,}7830194$
$\varphi = 0{,}001566$
$\Delta H = 1{,}8792467$
$\underline{u = 0{,}7856644\ cm}$

$k_s = \frac{u_{TDR}}{u_{TTR}} = \frac{0{,}7856644}{0{,}6613756} = 1{,}1879248$

$\gamma_{cr} = \frac{k_s \cdot \gamma}{k_s - 1} = \frac{1{,}1879248 \cdot 1}{0{,}1879248} = 6{,}3212738$

$Q_{cr} = Q \cdot \gamma_{cr} = 3{,}1606287\ kN/cm^2$

3



$F = 10 kN$
$L = 300 cm$
$E = 3000 kN/cm^2$
$I = 10\,000 cm^4$

$P_{cr} = ?$

• togost nadomestne vzmeti

$$\begin{bmatrix} F \\ 0 \end{bmatrix} = \begin{Bmatrix} b_1 & c_1 \\ c_1 & d_1 \end{Bmatrix} \begin{Bmatrix} w \\ \varphi \end{Bmatrix}$$

$$T = b_1 \cdot w + c_1 \cdot \varphi$$

$$\underline{0 = c_1 w + d_1 \varphi} \Rightarrow \varphi = -\frac{c_1}{d_1} w$$

$$T = b_1 \cdot w - \frac{c_1^2}{d_1} \cdot w \quad ; \quad w = 1 \Rightarrow T = k_v \Rightarrow k_{w_1} = b_1 - \frac{c_1^2}{d_1}$$

$$k_{w_4} = k_{w_1}$$

$$k_{w_3} = b_3 - \frac{c_3^2}{d_3}$$

$$b_1 = \frac{12EI}{L^3} + \frac{6F}{5L} = 13,373\overline{3}$$

$$c_1 = \frac{6EI}{L^2} + \frac{F}{10} = 2001$$

$$d_1 = \frac{4EI}{L} + \frac{2FL}{15} = 400400$$

$$b_3 = \frac{12EI}{L^3} - \frac{6F}{5L} = 13,29\overline{3}$$

$$c_3 = \frac{6EI}{L^2} - \frac{F}{10} = 1999$$

$$d_3 = \frac{4EI}{L} - \frac{2FL}{15} = 399600$$

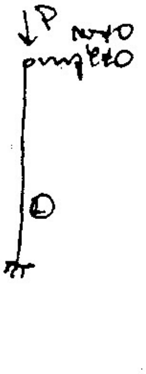
$$k_{w_1} = 3,3733$$

$$k_{w_2} = 3,2933$$

$$b_2 = \frac{12EI}{L^3} - \frac{6P}{5L} = 13,\overline{3} - 0,004 P$$

$$c_2 = \frac{6EI}{L^2} - 0,1P = 2000 - 0,1P$$

$$d_2 = \frac{4EI}{L} - \frac{2PL}{15} = 400\,000 - 40P$$



$$K = \begin{bmatrix} b_2 + 2k_1 + k_3 & c_2 \\ c_2 & d_2 \end{bmatrix}$$

$$K = \begin{bmatrix} 23,3733 - 0,004P & 2000 - 0,1P \\ 2000 - 0,1P & 400\,000 - 40P \end{bmatrix}$$

$\det K = 0$

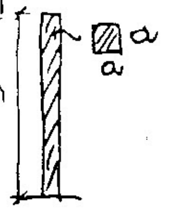
$$0,15 \cdot P^2 - 2134,932 P + 5349320 = 0$$

$$P = 3245,83 kN$$

$$P = 10987,05 kN$$

30. 11. 2005

1



$a = 10\,cm \rightarrow A = 10 \cdot 10\ cm^2 = 100\ cm^2$

$\rho = 7850\ kg/m^3$

$E = 21000\ kN/cm^2 \qquad I = \frac{a^4}{12}$

$\rho = 78500\ N/m^3 = 78{,}5\ kN/m^3 = 78{,}5 \cdot 10^{-3} \frac{kN}{dm^3} = 78{,}5 \cdot 10^{-6} \frac{kN}{cm^3}$

$N = -m(H-x) = -78{,}5 \cdot 10^{-4} \frac{kN}{cm} \cdot (H-x)$

$K = \int_0^h \left(-78{,}5 \cdot 10^{-4} \cdot (H-x) \cdot 2x \cdot 2x + EI \cdot 4\right) dx =$

$= \int_0^h \left(-314 \cdot 10^{-4} x^2 H + 314 \cdot 10^{-4} x^3 + 4EI\right) dx$

$\varphi = x^2 \rightarrow \varphi_{,x} = 2x \rightarrow \varphi_{,xx} = 2$

$K = \left(-314 \cdot 10^{-4} \cdot H \cdot \frac{x^3}{3} + 314 \cdot 10^{-4} \frac{x^4}{4} + 4EIx\right)\Big|_0^h$

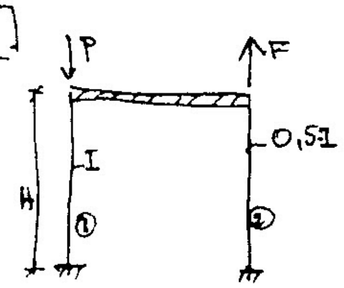
$K = -314 \cdot 10^{-4} \frac{H^4}{3} + 314 \cdot 10^{-4} \frac{H^4}{4} + 4EIH$

$314 \cdot 10^{-4} H^4 \left(\frac{1}{4} - \frac{1}{3}\right) + 4 \cdot 21000 \cdot \frac{10^4}{12} H = 0$

$314 \cdot 10^{-4} \left(\frac{1}{4} - \frac{1}{3}\right) H^3 = -\frac{4 \cdot 21000 \cdot 10^4}{12}$

$\underline{\underline{H = 2990{,}77\,cm}}$

2



$E = 3000$
$I = 20\,000$
$H = 500$
$P = 5000\,kN$

Ocena P_{cr}:

$P_{cr,min} = \frac{\pi^2 EI}{(0{,}5H)^2} = 9474{,}82\ kN$

$P_{cr,min} = \frac{\pi^2 EI}{(H)^2} = 2368{,}7\ kN$

$K_② = b_1$

$K_① = b_2$

$K = b_1 + b_2$

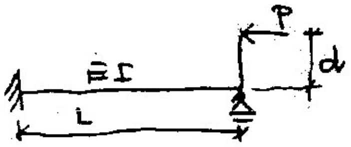
$b_1 = \frac{12EI}{H^3} - \frac{6P}{5H} = 5{,}76 - 12$

$b_2 = \frac{12EI\,0{,}5}{H^3} + \frac{6F}{5H} = 2{,}88 + 0{,}0024\,F$

$5{,}76 - 12 + 2{,}88 + 0{,}0024F = 0$

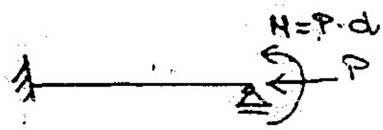
$\underline{\underline{F = 1400\ kN}}$

$\boxed{9}$



$E = 21000$
$I = 1000$
$L = 200$
$d = 30$
$P = 50$

$\Rightarrow$



a) TPR $\quad M = \alpha_{TPR}\,\varphi \qquad \varphi = \dfrac{M}{\alpha_{TPR}} = \dfrac{P \cdot d}{\dfrac{4EI}{L}} = \dfrac{1500}{420000} = \underline{0{,}0035714}$

b) TDR $\quad \varphi = \dfrac{M}{\alpha_{TDR}} = \dfrac{1500}{420000 - \dfrac{200 \cdot 200}{15}} = \underline{0{,}0035828}$